6 Chi square

The chi-square techniques to be introduced in this chapter are appropriate for use with data in the form of **frequencies**. In other words, they deal with the situation where we are simply counting the number of times something occurs. This happens, for example, if a test is made of the effectiveness of a new teaching method, and the information available is whether students pass or fail a test after the teaching experience. We would then be able to count the number of passes and of failures for the new method and might compare these with the number of passes and failures of another group taught by the old method (for this to be a true experiment it would be necessary for students to be randomly allocated to the two conditions).

Data in the form of measurements (e.g. of height) such as were used in the *t*-test described in the last chapter would not be appropriate for chi square. However, it may be possible to convert measures into counts, by appropriate grouping, so that chi square is appropriate. For example, if heights of a group of subjects are available, they could be converted into counts of how many are *tall* (say over 180 cm), how many *medium* (180–165 cm) and how many *short* (under 165 cm). Some detailed information is, of course, lost by this procedure, but it may be that the experimental hypothesis can still be adequately tested by the frequency data.

The 2 × 2 contingency table

In studying the possible relationship between smoking and lung cancer the data of Table 8 were obtained (fictitious data, if it is any comfort for smokers). This is an example of a 2 × 2 contingency table – sometimes called a fourfold table.

Table 8 Smokers and non-smokers having cancer or not having cancer (fictitious data)

	Smokers	*Non-smokers*	
cancer	23	3	26
no cancer	465	652	1117
	488	655	1143

It shows in compact form that, of an overall total of 1143 persons in the study, 488 were smokers and 655 were non-smokers. Of the smokers, 23 had developed lung cancer by a certain age, 465 had not. Of the non-smokers, 3 had developed lung cancer and 652 had not. We want to know if smokers are more likely to get cancer. We can see, of course, that the proportion of smokers developing cancer in our sample (23 out of 488 – just less than 5 per cent) is greater than the proportion of non-smokers developing cancer (3 out of 655 – less than $\frac{1}{2}$ per cent). But what we need to find out is how unlikely it is to get a difference in the proportions as large as is found here if only random effects are present.

The chi-square technique can be used to test the statistical significance of the difference in proportions. It can also be thought of in a rather different way – as a test of association between the categories used. Is smoking associated with lung cancer? Put more accurately, does membership of a given category on one dimension (e.g. smokers on the smoking/non-smoking dimension) tend to be associated with membership of a given category on the other dimension (e.g. lung cancer sufferers on the cancer/no-cancer dimension)?

If there is no association between the categories, the frequencies which would be expected can be worked out for each of the four 'cells' in the 2 × 2 table. As 26 persons develop lung cancer out of a total of 1143 (smokers and non-smokers together), one would expect that, *with no association*, the same proportion of smokers would develop cancer. So, of the 488 smokers, we expect that the proportion of $\frac{26}{1143}$ would develop cancer, i.e.

Expected number of smokers developing cancer $= 488 \times \dfrac{26}{1143}$
(if no association)

$= 11{\cdot}10$

As 1117 out of 1143 overall do not develop the disease, we expect that with no association, this same proportion of the 488 smokers would escape, i.e.

Expected number of smokers not developing cancer $= 488 \times \dfrac{1117}{1143}$
(if no association)

$= 476{\cdot}90$

Expected frequencies can be worked out in the same way for the other two cells in the table (try it). A general formula, which can be used for computing the expected frequency for a cell, is

$$\textbf{Expected frequency } (E) = \frac{\textbf{row total} \times \textbf{column total}}{\textbf{overall total}}$$

The table of expected frequencies is shown in Table 9.

Table 9 Expected frequencies calculated from the data of Table 8, on the basis of no association between the categories.

	Smokers	*Non-smokers*
cancer	11·10	14·90
no cancer	476·90	640·10

The chi-square statistic (χ^2) can be computed from the actual frequencies obtained, usually called the observed frequencies (O) and the expected frequencies (E), using the formula

$$\text{chi square (for } 2 \times 2 \text{ table) } \chi^2 = \Sigma\frac{(|O - E| - \frac{1}{2})^2}{E}$$

Here, the Σ refers to 'taking the sum of' the contributions from each of the four cells in the table.

As in the calculation of mean deviation (p. 48), $|O - E|$ means 'find the absolute value of $O - E$', i.e. always take the smaller from the larger (so you always end up with a positive difference). This difference is then reduced by subtracting $\frac{1}{2}$ and the result is squared. This is then divided by the expected frequency for that cell. The sum of these values for each of the cells is χ^2. You can see that the bigger the differences between O and E the bigger is χ^2. Hence a sufficiently large value of χ^2 leads to us accepting that there is a statistically significant association between the categories. Or, putting it in other words, that there is a statistically significant difference in proportions; one which is sufficiently unlikely to be due to random errors for us to reject the null hypothesis.

In our example,

$$\chi^2 = \Sigma\frac{(|23 - 11{\cdot}101 - \frac{1}{2})^2}{11{\cdot}10} + \frac{(|3 - 14{\cdot}90| - \frac{1}{2})^2}{14{\cdot}90}$$

$$+ \frac{(|465 - 476{\cdot}90| - \frac{1}{2})^2}{476{\cdot}90} + \frac{(|652 - 640{\cdot}10| - \frac{1}{2})^2}{640{\cdot}10}$$

$$= 11{\cdot}71 + 8{\cdot}72 + 0{\cdot}27 + 0{\cdot}20$$

$$= 20{\cdot}90$$

This can be assessed for significance by using Table G. However, in order to use this table, one has to know the appropriate degrees of freedom.

Degrees of freedom

In obtaining the expected frequencies, the column and row totals (and hence the overall total) are taken as fixed. This means that when one of the expected frequencies has been computed, the rest can be found by subtraction from the marginal totals. The implication of this is that a 2×2 table of this type has only one 'degree of freedom', i.e. only one of the frequencies can be considered as free to vary if we are to ensure that the marginal totals add up to the right value. Thus, from Table G, the table value is

$$\chi^2 = 3{\cdot}841$$

with 1 degree of freedom (1 d.f.), at the 5 per cent level.

We now go through the sub-routine which you should be familiar with from all the tests we have dealt with so far. As the χ^2 computed from the data exceeds the table value of χ^2 for the 5 per cent level, we have evidence that the IV has affected the DV. In other words that there is a statistically significant association between the IV (smoking or not smoking) and the DV (incidence of lung cancer).

Note that the table for χ^2 refers to the one-tailed test. This is appropriate for all the applications of χ^2 referred to in this chapter. We are concerned with just one tail (the upper tail) of the chi-square distribution because large discrepancies between observed and expected frequencies are reflected in large values of χ^2. These take us into that upper tail. The lower tail of the χ^2 distribution corresponds to observed and expected frequencies being closer together than is likely on a chance basis, which is not usually of experimental interest.

This is a rather different issue from that covered in the earlier discussion of one- and two-tailed tests (p. 75). There, the one-tailed test was concerned with situations where we start out by predicting the direction of the difference between two conditions, and need to have a very good reason for using it instead of the two-tailed test. With χ^2 the one-tailed test is the norm and simply refers to large differences between observed and expected frequencies, irrespective of the direction of the association.

Chi square and small samples

As the chi-square function is a continuous curve and the observed frequencies used in its estimation must take on whole number values, the actual sampling distribution is only approximated by the continuous function. The formula for χ^2 given above incorporates **Yates's correction** – subtraction of $\frac{1}{2}$ from the absolute value of $(O - E)$, which improves the approximation to a continuous function. It should be included for all cases of the use of χ^2 with 1 degree of freedom.

The smaller the sample size, the worse is the fit to a continuous distribution and below a certain size χ^2 should not be used. Although statisticians differ on the exact number below which χ^2 should not be used, a simple rule of thumb is: **do not use chi square if one or more of the *expected* frequencies falls below five**. Note that this is the expected rather than the observed frequency. In the smoking and lung cancer example it is permissible to use χ^2 because, although one of the observed frequencies is under five, all expected frequencies are above five.

An alternative small-sample test for data in the form of frequencies is called **Fisher's exact test**. This test can be used whatever the expected frequencies and is found in more advanced texts, such as Siegel and Castellan (1988).

Independence of observations

There are probably more inappropriate and incorrect uses of the chi-square test than of all the other statistical tests put together. For one thing it should only be used appropriately if each and every observation is independent of each and every other observation. Violations of this rule are very common. If, for example, we use a repeated measures design where participants are categorized as successful or not on some test before and after the experimental task, then we have pairs of scores rather than independent ones; and χ^2 should not be used.

In order to use χ^2 appropriately, each observation has to qualify for one and only one cell in the table. Another common misuse of the statistic occurs when an attempt is made to leave out some of the observations. Suppose, for example, that we have 'high', 'medium' and 'low' categories of performance on a task; then it is inappropriate to simply select out two of the categories for analysis. The frequencies in each of the categories must be included in the χ^2 analysis (a larger table than the 2×2 table will be needed, as discussed later in the chapter).

Interpretation of the result of a chi-square test

A statistically significant χ^2 is evidence for an association. Inspection of the data given above shows that the direction of this association is such that smokers are more likely to get lung cancer than non-smokers. Put differently, the proportion of smokers who get cancer (0·047) is significantly higher than the proportion of non-smokers who get cancer (0·005).

It is, of course, in attempting to interpret this result that we have to exercise caution. No details were given about the kind of study from which the data were obtained. We can safely assume that it was not a true experiment as this would have involved random assignment of persons to the 'smoking' and 'non-smoking' categories. Such an experiment would involve something like randomly assigning half of a group of non-smokers to begin smoking regularly. The ethics of that kind of study are, to say the least, somewhat dubious. However, in its absence, we are certainly in no position to say 'smoking *causes* lung cancer'.

Without random assignment there may well be many factors or variables which influence the outcomes of a study. As is well known, the existence of a causal relationship has been hotly disputed by many smokers and by cigarette manufacturers. One suggestion has been that people of a certain personality type are more like to smoke *and also* have cancer. Put more generally, the argument is that there is a third variable (personality type) which is associated with both the others.

This example can stand as a warning of the difficulties in interpreting the results of a statistical test. These difficulties are by no means restricted to χ^2, but they often crop up in a particularly awkward form with this test.

Step-by-step procedure

2 × 2 Chi square (test of association)

Use this test for *frequency data* only, i.e. for *counts* of different types of 'events'

Step 1 Draw up the 2 × 2 table, making sure that each event goes into one of the cells and into not more than one cell. The number in each cell is the **observed frequency** O for that cell

Step 2 Find the row totals, column totals and grand total

Step 3 Work out the **expected frequency** E for each cell separately using the formula

$$E = \frac{\text{row total} \times \text{column total}}{\text{grand total}}$$

Put the expected frequency for each cell into that cell

Worked example

2×2 Chi square (test of association)

A psychologist studying the symptoms of a random sample of 25 psychotics and 25 neurotics found that only 5 of the psychotics had suicidal feelings, whereas 12 of the neurotics had. Is there evidence for an association between the two psychiatric groups and the presence or absence of suicidal feelings?

Step 1

	Psychotics	*Neurotics*
suicidal	5	12
non-suicidal	20	13

Step 2

5	12	17
20	13	33
25	25	50

Step 3 For top left cell (cell A) $E = \frac{17 \times 25}{50} = 8{\cdot}5$

For top right cell (cell B) $E = \frac{17 \times 25}{50} = 8{\cdot}5$

For bottom left cell (cell C) $E = \frac{33 \times 25}{50} = 16{\cdot}5$

For bottom right cell (cell D) $E = \frac{33 \times 25}{50} = 16{\cdot}5$

8·5 5	8·5 12	17
16·5 20	16·5 13	33
25	25	50

2×2 Chi square (test of association)

Step 4 Work out the difference between O and E for each cell, taking the smaller of these from the larger in each case, i.e. obtain $|O - E|$

Step 5 Take away $\frac{1}{2}$ from $|O - E|$ for each cell $|O - E| - \frac{1}{2}$

Step 6 Square this for each cell $(|O - E| - \frac{1}{2})^2$

Step 7 Divide by the appropriate E value for that cell

$$\frac{(|O - E| - \frac{1}{2})^2}{E}$$

Step 8 Obtain χ^2 by adding all these contributions from the different cells

$$\chi^2 = \Sigma\frac{(|O - E| - \frac{1}{2})^2}{E}$$

This has 1 degree of freedom

Step 9 If the χ^2 obtained exceeds the table value (found in Table G) at the chosen level of significance, then there is evidence for an association between the categories

Step 10 Translate the result of the test back in terms of your experiment

Worked example – continued

2 × 2 Chi square (test of association)

	Step 4	Step 5
	$\lvert O - E \rvert$	$\lvert O - E \rvert - \frac{1}{2}$
Cell A	$\lvert 5 - 8{\cdot}5 \rvert = 3{\cdot}5$	$3{\cdot}5 - 0{\cdot}5 = 3$
Cell B	$\lvert 12 - 8{\cdot}5 \rvert = 3{\cdot}5$	$3{\cdot}5 - 0{\cdot}5 = 3$
Cell C	$\lvert 20 - 16{\cdot}5 \rvert = 3{\cdot}5$	$3{\cdot}5 - 0{\cdot}5 = 3$
Cell D	$\lvert 13 - 16{\cdot}5 \rvert = 3{\cdot}5$	$3{\cdot}5 - 0{\cdot}5 = 3$

	Step 6	Step 7
	$(\lvert O - E \rvert - \frac{1}{2})^2$	$\dfrac{(\lvert O - E \rvert - \frac{1}{2})^2}{E}$
Cell A	$3^2 = 9$	$9/8{\cdot}5 = 1{\cdot}0588$
Cell B	$3^2 = 9$	$9/8{\cdot}5 = 1{\cdot}0588$
Cell C	$3^2 = 9$	$9/16{\cdot}5 = 0{\cdot}5454$
Cell D	$3^2 = 9$	$9/16{\cdot}5 = 0{\cdot}5454$

Step 8 $\chi^2 = \Sigma \dfrac{(\lvert O - E \rvert - \frac{1}{2})^2}{E}$

$$= 1{\cdot}0588 + 1{\cdot}0588 + 0{\cdot}5454 + 0{\cdot}5454$$
$$= 3{\cdot}2084$$
$$= 3{\cdot}21 \text{ with 1 d.f.}$$

Step 9 From Table G,
$\chi^2 = 3{\cdot}84$
with 1 d.f., at the 5 per cent significance level

Step 10 There is not significant evidence for an association between psychotism/neuroticism and presence/absence of suicidal feelings at the conventional (5 per cent) significance level (or *alternatively*, the proportion of psychotics with suicidal feelings does not differ significantly from the proportion of neurotics with suicidal feelings)

Chi square in larger tables

Chi square can be used in tables with more than two rows and more than two columns. As with the 2 × 2 Chi square it can be regarded as a test of association between the attributes which make up the rows and those which make up the columns. The χ^2 formula is then simplified, as Yates's correction is not needed. Thus,

$$\chi^2 = \Sigma \frac{(O - E)^2}{E}.$$

As before, the summation sign indicates that the quantities

$$\frac{(O - E)^2}{E},$$

having been computed for each cell, should be added together for all the cells.

The calculation of expected frequencies is as for the 2 × 2 table, i.e.

$$E = \frac{\text{row total} \times \text{column total}}{\text{overall total}},$$

the reasoning behind this being exactly the same as for the 2 × 2 case.

The number of degrees of freedom can also be arrived at as in the 2 × 2 case. If the row totals are considered fixed, then the frequencies in one cell in any row is fixed when values have been given to frequencies of the other cells. The same applies to the columns, so that in a table with R rows and C columns, the total number degrees of freedom is

$$(R - 1) \times (C - 1).$$

This is perhaps clearer when displayed. Figure 23 shows it for a 3 × 4 table and a 2 × 6 table. When values have been given to the frequencies of the unshaded cells then, given fixed marginal totals, the values of frequencies for each of the shaded cells can be computed:

for the 3 × 4 table
degrees of freedom = (3 − 1) × (4 − 1) = 6

and for the 2 × 6 table
degrees of freedom = (2 − 1) × (6 − 1) = 5.

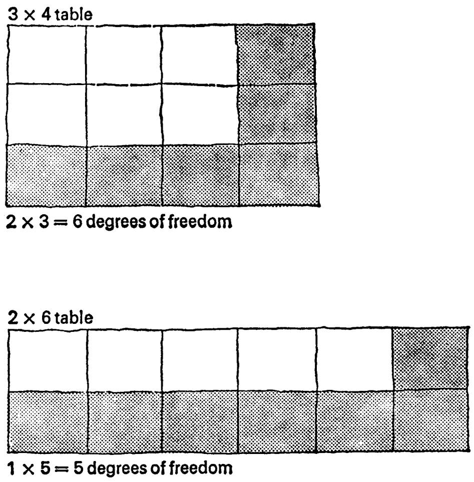

Figure 23 Degrees of freedom in large χ^2 tables

In interpreting larger tables it may be helpful to convert the observed cell frequencies into proportions. Table 10 shows this for the data in the worked example on p. 103.

Table 10 Contingency table with observed frequencies expressed as proportions

	Method A	*Method B*	*Method C*	*Overall*
pass	$\frac{50}{55} = 0{\cdot}91$	$\frac{42}{61} = 0{\cdot}69$	$\frac{56}{64} = 0{\cdot}88$	$\frac{148}{180} = 0{\cdot}82$
fail	$\frac{5}{55} = 0{\cdot}09$	$\frac{19}{61} = 0{\cdot}31$	$\frac{8}{64} = 0{\cdot}13$	$\frac{32}{180} = 0{\cdot}18$

However, while it is often easier to pick out what is happening from the proportions, remember that χ^2 must be calculated from the frequencies themselves.

Small samples in larger tables

Yates's correction is inappropriate for χ^2 with more than one degree of freedom, and its disappearance from the formula has been noted. It is recommended that the same rule of thumb be

Step-by-step procedure

Chi-square – tables larger than 2 × 2 (test of association)

Use this test for *frequency data* only, i.e. for *counts* of different types of 'events'

Step 1 Draw up the table, making sure that each event goes into one of the cells and into no more than one cell. The number in each cell is the **observed frequency** ***O*** for that cell

Step 2 Find the row totals, column totals and grand total

Step 3 Work out the **expected frequency** ***E*** for each cell separately using the formula

$$E = \frac{\text{row total} \times \text{column total}}{\text{grand total}}$$

Put the expected frequency for each cell into that cell

Worked example

Chi square – tables larger than 2 × 2 (test of association)

In an exam, varying numbers of students passed and failed after having been taught by one of three different methods. It is required to test for an association between the numbers passing and failing and the method of instruction (i.e. does the relative proportion of passes differ from method to method?)

Step 1

	Method A	*Method B*	*Method C*
pass	50	42	56
fail	5	19	8

Step 2

50	42	56	148
5	19	8	32
55	61	64	180

Step 3 For top left-hand cell,

$$E = \frac{148 \times 55}{180} = 45{\cdot}22$$

and so on for each of the cells:

45·22 50	50·16 42	52·62 56	148
9·78 5	10·84 19	11·38 8	32
55	61	64	180

Chi-square – tables larger then 2 × 2 (test of association)

Step 4 Work out $(O - E)$ for each cell

Step 5 Square this for each cell

Step 6 Divide by the appropriate E value for that cell

Step 7 Obtain χ^2 by adding all these contributions from the different cells

Step 8 Obtain the degrees of freedom = (number of rows − 1) × (number of columns − 1)

Step 9 If the χ^2 obtained exceeds the table value (found in Table G) at the chosen level of significance, then there is evidence for an association between the categories

Step 10 Translate the result of the test back in terms of the experiment

Worked example – continued

Chi-square – tables larger than 2 × 2 (test of association)

O	E	**Step 4** $(O - E)$	**Step 5** $(O - E)^2$	**Step 6** $(O - E)^2/E$
50	45·22	4·78	22·85	0·505
42	50·16	−8·16	66·59	1·328
56	52·62	3·38	11·42	0·217
5	9·78	−4·78	22·85	2·336
19	10·84	8·16	66·59	6·143
8	11·38	−3·38	11·42	1·004

Step 7 $\chi^2 = \Sigma\frac{(O - E)^2}{E}$

$$= 0{\cdot}505 + 1{\cdot}328 + 0{\cdot}217 + 2{\cdot}336 + 6{\cdot}143 + 1{\cdot}004$$
$$= 11{\cdot}53$$

Step 8 d.f. = (rows − 1) × (columns − 1) = 1 × 2 = 2

Step 9 From Table G, $\chi^2 = 5{\cdot}99$ with 2 d.f. at the 5 per cent significance level. As the χ^2 obtained (11·53) is greater than this table value, there is significant evidence for an association between the variables

Step 10 There is significant evidence for an association between method of instruction and relative proportion of passes. In other words, the relative proportions of passes differ significantly from one method to another

Note That the significance applies to the data taken as a whole. However inspection of the table suggests that it is Method B which differs from the other two applied as for small samples: **do not use chi square if one or more of the expected frequencies falls below five**.

This is a 'conservative' procedure in that circumstances may arise when the approximation to the chi-square distribution is adequate with smaller expected frequencies than this.

A common strategy when small expected frequencies are obtained is to pool categories in order to get the pooled expected frequencies above the magic number of five. There are difficulties in doing this, however. Possibly the hypothesis that one is interested in testing cannot be tested if categories are lumped together in this way. And even if this is still possible, a *post hoc* pooling (i.e. after the observed frequencies have been obtained) will affect the randomness of the sample. Exact tests exist as for the 2×2 case, but are difficult to compute for larger tables.

The obvious procedure to adopt is to take a large enough sample for the expected frequencies to be over five. If appropriate, categories can be combined in *a priori* fashion (i.e. before collecting the results). Finally, if one does end up with some low expected frequencies then it is probably preferable to go ahead with χ^2 adding a caveat that there may be a relatively poor approximation to the exact probabilities.

Chi square as a test of goodness of fit

We have so far considered χ^2 as a test of association. When used in this way the expected frequencies are calculated directly from the observed frequencies by assuming independence between the categories. It is also possible to use χ^2 in a rather different way where the expected frequencies are obtained from predictions based on theoretical considerations. When the χ^2 statistic is computed in this way, it becomes a test of the **goodness of fit** between the observations and the theory.

Chi square can, for instance, be used to determine whether a given set of data may be regarded as a sample from a normal population, and hence to decide whether or not a particular statistical test which assumes a normal distribution could be validly used in some situation.

We will restrict ourselves in this chapter to a simple case which occurs quite frequently. This is where, in a single-row χ^2, we are testing the hypothesis of equal probability of occurrence of the different alternatives (this is equivalent to testing the goodness of fit to a 'rectangular' distribution – a histogram of the distribution would be a rectangle with all the bars the same length). Thus, the expected frequency for each cell is obtained simply by dividing the total number of observations by the number of cells in the row. Degrees of freedom are one less than the number of cells, the reasoning being as before. The formula used is unchanged, and Yates's correction should be employed in the one degree of freedom case. The same rule of thumb for expected frequencies applies.

Step-by-step procedure

$1 \times N$ Chi square (test of goodness of fit)

Use this test for *frequency data* only, i.e. for *counts* of different types of 'events'

Step 1 Draw up the $1 \times N$ table, making sure that each event goes into one of the cells and into not more than one cell. The number in each cell is the **observed frequency** O for that cell

Step 2 Find the **total frequency**

Step 3 Work out the **expected frequency** E for each cell separately, using the theoretical distribution to be fitted. For the special case of the rectangular distribution we are considering, there is an equal probability of occurrence of the different alternatives.

$$E = \frac{\text{total frequency}}{N} \text{ for each cell}$$

Worked example

1 × *N* Chi square (test of goodness of fit)

A sample of 250 people in the street were asked to 'say any number from 0 to 9 inclusive'. Do the results show any evidence for number preference?

Step 1	*Digit*	*Observed frequency*
	0	18
	1	31
	2	29
	3	36
	4	17
	5	20
	6	20
	7	35
	8	14
	9	30

Step 2 Total frequency = 250

Step 3 In the absence of number preference, all observed frequencies will have the same expected value

i.e. $E = \frac{250}{10} = 25$

$1 \times N$ Chi square (test of goodness of fit)

Step 4 Work out $(O - E)$ for each cell
NB for the 1 d.f. case, Yates's correction must be applied, i.e. take $(|O - E| - \frac{1}{2})$ for each cell

Step 5 Square this for each cell

Step 6 Divide by E

Step 7 Obtain χ^2 by adding all these contributions from the different cells

Step 8 Obtain the degrees of freedom = (number of cells − 1)

Step 9 If the χ^2 obtained exceeds the value found in Table G at the chosen level of significance, then there is evidence for a divergence between the theoretical and observed distributions

Step 10 Translate the result of the test back in terms of the experiment

Worked example – continued

1 × *N* Chi square (test of goodness of fit)

O	E	Step 4 $(O - E)$	Step 5 $(O - E)^2$	Step 6 $(O - E)^2/E$
18	25	−7	49	1·96
31	25	6	36	1·44
29	25	4	16	0·64
36	25	11	121	4·84
17	25	−8	64	2·56
20	25	−5	25	1·00
20	25	−5	25	1·00
35	25	10	100	4·00
14	25	−11	121	4·84
30	25	5	25	1·00

Step 7 $\chi^2 = \Sigma\frac{(O - E)^2}{E} = 1{\cdot}96 + 1{\cdot}44 + 0{\cdot}64 + 4{\cdot}84 + 2{\cdot}56 + 1{\cdot}00 + 1{\cdot}00 + 4{\cdot}00 + 4{\cdot}84 + 1{\cdot}00 = 23{\cdot}28$

Step 8 d.f. = (number of cells − 1) = 9

Step 9 From Table G, $\chi^2 = 16{\cdot}92$ at the 5 per cent significance level with 9 d.f. As the observed χ^2 exceeds the table χ^2 at the 5 per cent level, there is evidence for a significant departure from equal choices of the different digits

Step 10 The results of the experiment show significant evidence for number preferences