5 Interest Rate Futures: Introduction

Answers to Questions and Problems

1. A 90-day T-bill has a discount yield of 8.75 percent. What is the price of a \$1,000,000 face value bill?

   Applying the equation for the value of a T-bill, the price of a \$1,000,000 face value T-bill is \$1,000,000 − DY(\$1,000,000)(DTM)/360, where DY is the discount yield and DTM = days until maturity. Therefore, if DY = 0.0875 the bill price is:

   $$\text{Bill Price} = \$1{,}000{,}000 - \frac{0.0875\ (\$1{,}000{,}000)\ (90)}{360} = \$978{,}125$$

2. The IMM Index stands as 88.70. What is the discount yield? If you buy a T-bill futures at that index value and the index becomes 88.90, what is your gain or loss?

   The discount yield = 100.00 − IMM Index = 100.00 − 88.70 = 11.30 percent. If the IMM Index moves to 88.90, it has gained 20 basis points, and each point is worth \$25. Because the price has risen and the yield has fallen, the long position has a profit of \$25(20) = \$500.

3. What is the difference between position day and first position day?

   First position day is the first day on which a trader can initiate the delivery sequence on CBOT futures contracts. With the three day delivery sequence characteristics of T-bond futures, for example, first position day is the second to last business day of the month preceding the contract's expiration month. For example, May 30 is the first position day for the JUN contract, assuming that May 30–June 1 are all business days. Position day is functionally the same, but it is not the first day on which a trader can initiate the sequence. For example, assuming June 10–12 are all business days, the position day could be June 10, with actual delivery occurring on June 12.

4. A \$100,000 face value T-bond has an annual coupon rate of 9.5 percent and paid its last coupon 48 days ago. What is the accrued interest on the bond?

   $$\text{Accrued Interest} = \$100{,}000\ (0.095/2)(48/182.5) = \$1{,}249.32.$$

   Note that we assume that the half-year has 182.5 days. There are specific rules for determining the number of days in a half-year.

5. What conditions are necessary for the conversion factors on the CBOT T-bond contract to create favorable conditions for delivering one bond instead of another?

There is one market condition under which the conversion factor method creates no bias: the yield curve is flat and all rates are 6 percent. Under any other circumstance, the conversion factor method will give incentives to deliver some bonds in preference to others.

6. The JUN T-bill futures IMM Index value is 92.80, while the SEP has a value of 93.00. What is the implied percentage cost-of-carry to cover the period from June to September?

   For the JUN contract the implied invoice amount is:

   $$\text{Bill Price} = \$1{,}000{,}000 - 0.0720(\$1{,}000{,}000)(90)/360 = \$982{,}000$$

   Paying this amount in June will yield $1,000,000 in September when the delivered T-bill matures. Therefore, the implied interest rate is:

   $$\textit{Implied Cost-of-Carry} = \frac{\$1{,}000{,}000}{\$982{,}000} - 1 = 0.018330$$

   Therefore, the implied interest rate to cover the June–September period is 1.8330 percent. (The information about the SEP futures is just a distraction.)

7. A spot 180-day T-bill has a discount yield of 9.5 percent. If the implied bank discount rate for the next three months is 9.2 percent, what is the price of a futures that expires in three months?

   To exclude arbitrage, the strategy of holding the 180-day T-bill must give the same return as investing for the first three months at the repo rate and taking delivery on the futures to cover the second three month period to make up the 180-day holding period.

   Assuming $1,000,000 face values, the price of the 180-day bill must be:

   $$\text{Bill Price} = \$1{,}000{,}000 - 0.095(\$1{,}000{,}000)(180)/360 = \$952{,}500$$

   This is a ratio of face value to price of 1.049869. With a bank discount yield of 9.2 percent, a bill that pays $1,000,000 in 90 days must have a price of:

   $$\text{Bill Price} = \$1{,}000{,}000 - 0.092\,(\$1{,}000{,}000)(90)/360 = \$977{,}000$$

   giving a ratio of face value to price of 1.023541. Therefore, the ratio of the $1,000,000 face value to the price of the futures, X, must satisfy the following equation:

   $$1.049869 = 1.023541X$$

   $X = 1.025722$. Therefore, the futures price must be $1,000,000/1.025722 = $974,923, or $974,925 rounded to the nearest $25 tick.

8. For the next three futures expirations, you observe the following Eurodollar quotations:

MAR	92.00
JUN	91.80
SEP	91.65

   What shape does the yield curve have? Explain.

   These IMM Index values imply Eurodollar add-on yields of 8, 8.2, and 8.35 percent, respectively. These rates apply to the following periods: March–June, June–September, and September–December, respectively. Essentially, we may regard these futures rates as forward rates. If forward rates increase with futurity, the yield curve must be upward sloping.

9. Assume that the prices in the preceding problem pertain to T-bill futures and the MAR contract expires today. What should be the spot price of an 180-day T-bill?

To avoid arbitrage, the spot price of an 180-day T-bill must give the same return as taking delivery on the futures today and taking a long position in the JUN contract with the intention of taking delivery of it as well. For convenience, we assume a T-bill with a face value of $1,000,000.

With the strategy of two 90-day positions, a trader would need to take delivery of both one full JUN contract and enough bills on the MAR contract to pay the invoice amount on the JUN contract. For the JUN contract, the IMM Index value implies a delivery price of

$$\$1,000,000 - 0.0820(\$1,000,000)(90)/360 = \$979,500.$$

For the MAR contract, the delivery price is

$$\$1,000,000 - 0.08(\$1,000,000)(90)/360 = \$980,000.$$

But the trader requires only $979,500 (or 97.95 percent) of the JUN contract. Therefore, for the short-term strategy, the current price of $1,000,000 in September is 0.9795($980,000) = $959,910. To avoid arbitrage, the 180-day bill must also cost $959,910, implying a discount yield of 0.08018.

10. The cheapest-to-deliver T-bond is a 10 percent bond that paid its coupon 87 days ago and it is priced at 105-16. The conversion factor of the bond is 1.0900. The nearby T-bond futures expires in 50 days and the current price is 98-00. If you can borrow or lend to finance a T-bond for a total interest outlay of 2 percent over this period, how would you transact? What if you could borrow or lend for the period at a total interest cost of 3 percent? What if you could borrow for the period at a total interest cost of 3 percent and earn 2 percent on an investment over the whole period? Explain.

To know how to respond to these quotations requires knowing the invoice amount that can be obtained for the bond and comparing this with the cost of carrying the bond to delivery on the futures. For convenience, we assume a face value that equals the contract size of $100,000. First, the accrued interest (assuming a 182.5-day half-year) is:

$$AI = (87/182.5)(0.5)(0.10)\$100,000 = \$2,383.56$$

At expiration, the accrued interest will be:

$$AI = (137/182.5)(0.5)(0.10)\$100,000 = \$3,753.42$$

For this bond and the futures price of 98-00, the invoice amount will be:

$$\text{Invoice Amount} = 0.9800(\$100,000)(1.09) + \$3,753.42 = \$110,573.42$$

Buying the bond and carrying it to delivery (at 2 percent interest for the period) costs:

$$(\$105,500 + \$2,383.56)(1.02) = \$110,041.23$$

Because the cost of acquiring and carrying the bond to delivery is less than the expected invoice amount, the trader could engage in a cash-and-carry arbitrage. Buying the bond and carrying it to delivery costs $110,041.23 and nets a cash inflow of $110,573.42. This gives an arbitrage profit. (Notice that the actual invoice amount is unknown, but transacting at the futures price of 98-00 guarantees the profit we have computed. This profit may be realized earlier depending upon the daily settlement cash flows.)

If the cost of carrying the bond for these next 50 days is 3 percent instead of 2 percent, the total cost of acquiring and carrying the bond will be:

$$(\$105,500 + \$2,383.56)(1.03) = \$111,120.07$$

This cost exceeds the expected invoice amount, so the cash-and-carry trade will not work for a 3 percent total cost-of-carry over the period.

Ignoring the short seller's options to choose the deliverable bond and the delivery date within the delivery month, the following reverse cash-and-carry strategy will be available with the 3 percent financing rate. The trader can buy the futures, borrow the bond and sell it short, and invest the proceeds to earn $111,120.07 by delivery. The short, we assume, obligingly delivers the same bond on the right day for the invoice amount of \$110,573.42, and the profit is:

$$\$111,120.07 - \$110,573.42 = \$546.65$$

If the trader can borrow at 3 percent and lend at 2 percent, these prices create no arbitrage opportunities. The cash-and-carry strategy is too expensive, because buying and carrying the bond costs \$111,120.07, more than the invoice amount of \$110,573.42. The reverse cash-and-carry strategy is also impractical, because it nets only \$110,041.23, less than the invoice amount of \$110,573.42.

11. You expect a steepening yield curve over the next few months, but you are not sure whether the level of rates will increase or decrease. Explain two different ways you can trade to profit if you are correct.

If the yield curve is to steepen, distant rates must rise relative to nearby rates. If this happens we can exploit the event by trading just short-term instruments. The yield on distant expiration short-term instruments must rise relative to the yield on nearby expiration short-term instruments. Therefore, one should sell the distant expiration and buy the nearby expiration. This strategy could be implemented by trading Eurodollar or T-bill futures.

As a second basic technique, one could trade longer term T-bonds against shorter maturity T-notes. Here the trader expects yields on T-bonds to rise relative to yields on T-notes. Therefore, the trader should sell T-bond futures and buy T-note futures. Here the two different contracts can have the same expiration month.

12. The Canadian invasion of Alaska has financial markets in turmoil. You expect the crisis to worsen more than other traders suspect. How could you trade short-term interest rate futures to profit if you are correct? Explain.

Greater than expected turmoil might be expected to result in rising yields on interest rate futures. To exploit this event, a trader could sell futures outright. A second result might be an increasing risk premium on short-term instruments. In this case, the yield differential between Eurodollar and T-bill futures might increase. To exploit this event, the trader could sell Eurodollar futures and buy T-bill futures of the same maturity.

13. You believe that the yield curve is strongly upward sloping and that yields are at very high levels. How would you use interest rate futures to hedge a prospective investment of funds that you will receive in nine months? If you faced a major borrowing in nine months, how would you use futures?

If you think yields are near their peak, you will want to lock-in these favorable rates for the investment of funds that you will receive. Therefore, you should buy futures that will expire at about the time you will receive your funds. The question does not suggest whether you will be investing long-term or short-term. However, if the yield curve is strongly upward sloping, it might favor longer term investment. Consequently, you might buy T-bond futures expiring in about nine months.

If you expect to borrow funds in nine months you may not want to use the futures market at all. In the question, we assume that you believe rates are unsustainably high. Trading to lock-in these rates only ensures that your borrowing takes place at the currently very high effective rates. Given your beliefs, it might be better to speculate on falling rates.

14. The spot rate of interest on a corporate bond is 11 percent, and the yield curve is sharply upward sloping. The yield on the T-bond futures that is just about to expire is 8 percent, but the yield for the futures contract that expires in six months is 8.75 percent. (You are convinced that this difference is independent of any difference in the cheapest-to-deliver bonds for the two contracts.) In these circumstances, a corporate finance officer wants to lock-in the current spot rate of 11 percent on a corporate bond that her firm plans to offer in six months. What advice would you give her?

Reform your desires to conform to reality. The yield curve is upward sloping and the spot corporate rate is 11 percent. Therefore, the forward corporate rate implied by the yield curve must exceed 11 percent.

Trading futures now to lock-in a rate for the future locks in the rate implied by the yield curve, and that rate will exceed 11 percent. Consequently, she must expect to lock in a rate above the current spot rate of 11 percent.

15. Helen Jaspers was sitting at her trading desk watching the T-bill spot and futures market prices. Her firm was very active in the T-bill market, and she was eager to make a trade. The quote on the T-bill having 120 days from settlement to maturity was 4.90 percent discount yield. This bill could be used for the September 20 delivery on the September T-bill futures contract which was trading at 95.15. The quote on the T-bill maturing September 20, having 29 days between settlement and maturity, was 4.70 percent discount yield.

A. Compute the T-bill and futures prices per dollar of face value.

T-bills are quoted in bank discount yield (DY), and T-bill futures are quoted in 100 −DY. These must be converted to dollars using the following formula:

$$P=\left(1-\frac{\text{DY}\times n}{360}\right)FV$$

where n is the number of days to settlement to maturity and FV is the face value.

Compute prices per dollar of face value:

120-day T-bill quoted at 4.90% DY: $$P_{120}=\left(1-\frac{.049\times 120}{360}\right)\$1=\$0.9837$$

29-day T-bill quoted at 4.70% DY: $$P_{29}=\left(1-\frac{.047\times 29}{360}\right)\$1=\$0.9962$$

The futures contract on its delivery date will call for a T-bill with 91 days from settlement to maturity. The discount yield on this T-bill is currently:

$$\text{DY}=100-95.15=4.85\%$$

The dollar price of the T-bill is:

$$F_{0,t}=\left(1-\frac{.0485\times 91}{360}\right)\$1=\$0.9877$$

B. Compute the implied repo rate. Could the implied repo rate be used to tell Helen where arbitrage profits are possible? If so, how?

The implied repo rate, C, is $C=(F_{0,t}/S_0)-1$

In this case, C is:

$$C=(0.9877/0.9837)-1=0.4066\%$$

What does this tell us? It tells us the return we get over the next 29 days if we buy the 120-day T-bill and deliver it against the September futures contract. This implied repo rate is compared to the 29-day borrowing/lending rate to point out opportunities for arbitrage. If C is greater than the 29-day financing rate, then the appropriate arbitrage is a cash-and-carry. If C is less than the 29-day financing rate, then a reverse cash-and-carry would be appropriate. The borrowing/lending rate over this 29-day time period is:

$$(FV/P_{29})-1=(1/0.9962)-1=0.3814\%$$

Since the implied repo rate is greater than the cost of financing, we have the possibility of cash-and-carry arbitrage.

C. What would be the arbitrage profit from a $1 million transaction?

A cash-and-carry arbitrage would call for borrowing for 29 days, buying the 120-day bill and selling the 120-day bill forward using the futures contract. On the delivery date, the 120-day bill, then having 91 days to maturity would be delivered against the futures contract. The proceeds would be used to pay off the 29-day borrowing.

Date	Cash Market	Futures Market
Today	Borrow $.9837 million for 29 days at 4.70% DY; Buy $1 million FV 120-day T-bill at a price of $.9837 million. Net investment = $0	Sell $1 million 91-day T-bills for September 10 delivery
September 10	Pay off borrowing; amount due = 0.9837/0.9962 = $.9874 million	Deliver 91-day T-bill against the futures contract; receive $.9877 million
	Net Profit = ($0.9877 − $0.9874) million = $300	

16. Angela Vickers has the responsibility of managing Seminole Industries' short-term capital position. In three weeks, Seminole will have a cash inflow that will be rolled over into a $10,000,000 90-day T-bill. There is a T-bill futures contract that calls for delivery at the same time as the anticipated cash inflow. It is trading at 94.75. There have been signs that the financial markets are calming and that interest rates might be falling.

A. What type of hedge might Angela employ?

Angela might employ a long hedge of the anticipated $10 million cash inflow. This would be accomplished by buying 10 T-bill futures contracts with a delivery date matching the anticipated cash inflow. The rate she would be locking in is:

$$100 - 94.75 = 5.25\% \text{ discount yield}$$

B. Three weeks in the future, interest rates are actually higher. The 90-day T-bill discount yield is 6.00 percent. Calculate Seminole's net wealth change if the position is left unhedged.

Three weeks before the cash inflow, Vickers would have been able to lock-in a 5.25 percent discount yield using the futures contract. This would have allowed her to purchase $10 million of face value for:

$$\text{Anticipated Price} = \left(1 - \frac{.0525 \times 90}{360}\right)\$10{,}000{,}000 = \$9{,}868{,}750$$

When it comes time to actually invest, interest rates have risen to 6 percent discount yield. Then the price of $10 million of face value costs:

$$\text{Realized Price} = \left(1 - \frac{.06 \times 90}{360}\right)\$10{,}000{,}000 = \$9{,}850{,}000$$

Seminole had an opportunity gain of $18,750.

C. Calculate Seminole's net wealth change if the position is hedged.

If the position had been hedged, Seminole would have been long futures that were bought at 5.25 percent discount yield, or $9,868,750. This results in a loss of $18,750. The loss in the futures market is offset by the opportunity gain in the cash market so that the net wealth change is $0.

Date	Cash Market	Futures Market
Today	Anticipate the purchase of $10 million in T-bills for $9,868,750	Buy $10 million of T-bill futures at $9,868,750
3 weeks	Buy $10 million in T-bills for $9,850,000	Sell $10 million T-bill futures contracts at $9,850,000
	Opportunity gain = $18,750	Futures loss = −$18,750
	Net wealth change = $0	

D. Was the hedge a mistake?

In hindsight, Seminole would have been better off unhedged, but hindsight is 20/20. *Ex ante*, the concern was the risk of falling interest rates. Seminole viewed the 5.25 percent discount yield as acceptable for the investment and wanted to guarantee it. Therefore, *ex ante*, the hedge would not have been a mistake.

17. Fred Ferrell works for ABC Investments. As part of ABC's investment strategy, Fred is charged with liquidating $20 million of ABC's T-bill portfolio in two months. Fred has identified $20 million of T-bills that would be deliverable against the March T-bill futures contract at the time of liquidation. The price of the futures contract is 94.50. Fred is losing sleep at night over concerns about future economic uncertainty that could lead to a rise in interest rates.

A. What action can Fred take to reduce ABC's exposure to interest rate risk?

Fred could enter a short hedge using 20 March T-bill futures contracts. Since ABC anticipates liquidating $20 million in T-bills, ABC can lock in a liquidation price based on the 100 − 94.50 = 5.5 percent discount yield.

B. At the time of liquidation, the price of the 90-day T-bill has risen to 5.25 percent discount yield. Compute the change in ABC's net wealth that has occurred if Fred failed to hedge the position.

If Fred does not hedge the position, ABC's net wealth change will be an opportunity gain or loss. ABC had the opportunity to lock-in a 5.5 percent discount yield but did not. The proceeds of $20 million face value at 5.5 percent are:

$$\text{Anticipated Proceeds} = \left[1 - \frac{0.055 - 90}{360}\right] - \$20{,}000{,}000 = \$9{,}868{,}750$$

The realized price is calculated using 5.25 percent discount yield:

$$\text{Realized Proceeds} = \left(1 - \frac{.0525 \times 90}{360}\right) - \$20{,}000{,}000 = \$19{,}737{,}500$$

The difference is the opportunity gain/loss:

$$\text{Realized Proceeds} - \text{Anticipated Proceeds} = \$19{,}737{,}500 - \$19{,}725{,}000 = \$12{,}500 \text{ gain}$$

C. Compute the change in ABC's net wealth that has occurred, assuming Fred hedged the position.

If Fred had hedged the position, he would have sold $20 million of face value using the futures contracts at a price of $19,725,000, and he would have closed that position at a price of $19,737,500. There would have been a $12,500 loss in the futures market. The net wealth change of ABC would then be zero, because the loss in the futures market would have exactly offset the opportunity gain in the cash market.

Date	Cash Market	Futures Market
Today	Anticipate selling \$20 million of T-bills for \$19,725,000	Sell \$20 million T-bill futures at \$19,725,000
March	Sell \$20 million T-bills for \$19,737,500	Buy \$20 million T-bill futures at \$19,737,500
	Opportunity gain = \$12,500	Loss = −\$12,500
	Net wealth change = \$0	

18. Alex Brown is a financial analyst for B.I.G. Industries. He has been given responsibility for handling the details of refinancing a \$500 million long-term debt issue that will be rolled over in May (5 months from today). The new 8 percent, 30-year debt, with a face value of \$500 million, is anticipated to have a 75-basis-point default risk premium over the yield on the 30-year T-bond. The 30-year T-bond is currently trading at 5.62 percent. Alex sees this risk premium as typical for corporate debt of a quality similar to B.I.G.'s debt. Alex looks at the June T-bond and notices that it is trading at 123-25. He is concerned that changing interest rates between now and May could have a negative impact on the refinancing cash flow.

A. Assuming no interest rate changes, what are B.I.G.'s anticipated proceeds from refinancing?

If interest rates do not change over the next five months, B.I.G. Industries can expect to price their bonds at:

$$YTM_{B.I.G.} = 5.62\% + 0.75\% = 6.37\%$$

The proceeds of \$500 million face with an 8 percent coupon, 30 years to maturity, and a yield to maturity of 6.37%, would be:

$$\text{Anticipated proceeds} = \sum_{t=1}^{60} \frac{.04(500{,}000{,}000)}{\left(1+\frac{.0637}{2}\right)^t} + \frac{500{,}000{,}000}{\left(1+\frac{.0637}{2}\right)^{60}} = \$608{,}443{,}959$$

B. What can Alex do to reduce the refinancing risks faced by B.I.G. Industries?

Alex could reduce the risk by selling T-bond futures. Alex's concern is rising interest rates. As interest rates rise, the proceeds from the debt issue are diminished. These diminished proceeds could be offset by profits made on the short position in the futures market because as rates rise, the futures prices fall and the short position makes money. Naively, Alex could sell \$500 million in the T-bond futures contract to hedge the risk.

C. At the time of refinancing, the 30-year T-bond yield is 5.80 percent, the T-bond futures price is 121-09, and B.I.G.'s new debt issue is priced to yield 6.75 percent. Compute the realized proceeds from the refinancing.

When the debt is issued, it is issued with a yield to maturity of 6.75 percent. Interest rates have risen. The proceeds are given by:

$$\text{Realized proceeds} = \sum_{t=1}^{60} \frac{.04(500{,}000{,}000)}{\left(1+\frac{.0675}{2}\right)^t} + \frac{500{,}000{,}000}{\left(1+\frac{.0675}{2}\right)^{60}} = \$579{,}955{,}566$$

This is an opportunity loss of \$28,488,392.

D. Assuming Alex sold \$500 million in T-bond futures at 123-25 to hedge the refinancing and liquidated the futures position when the refinancing took place, find the profit from the futures trade, and evaluate the net wealth change due to the change in the refinancing rate and the futures trade.

The futures profit is:

$$\text{Futures Profit} = \$500{,}000{,}000\left[\left(123+\frac{25}{32}\right)\% - \left(121+\frac{9}{32}\right)\%\right] = \$12{,}500{,}000$$

This profit only partially offsets the opportunity loss in the cash market. The net wealth change is:

$$\text{Net Wealth Change} = -\$28{,}488{,}392 + \$12{,}500{,}000 = -\$15{,}988{,}392$$

Date	Cash Market	Futures Market
Today	Anticipate issuing debt at 6.37% for proceeds of $608,443,959	Sell $500 million in T-bond futures at nominal price of $618,906,250
May	Issue debt with yield of 6.75% for proceeds of $579,955,566 Opportunity loss = −$28,488,392	Buy $500 million in T-bond futures at nominal price of $606,406,250 Gain = $12,500,000
	Net wealth change = −$15,998,392	

E. Discuss possible reasons why the net wealth change is not zero.

There are at least two reasons that the hedge did not do a better job of offsetting the cash market risk:

Cross-hedge: — This was a cross-hedge for several reasons. First, Alex was hedging the interest rates of a corporate bond, but the hedging instrument was a T-bond. Second, the cheapest-to-deliver T-bond may not be a 30-year bond. It may have as little as 15 years to maturity. This will make the price sensitivity of the futures contract and the corporate bond differ. Third, the delivery date on the T-bond futures contract is June, but the refinancing is taking place in May.

Faulty expectations: — First, Alex expected the default yield spread to stay fixed at 75 basis points, but it increased with the rise in interest rates to 6.75 %, giving a rise of 95 basis points. Second, Alex expected the pricing of the T-bond futures contract to react to interest rates in the same manner as the 30-year T-bond, but in reality the reaction of the T-bond futures price was not as strong as the reaction of the 30-year T-bond price.

While the hedge was not perfect, it did offset some of the refinancing price risk.