## Chapter 14: Price Fixing and Repeated Games

## Learning Objectives:

Students should learn to:

1. Explain why firms often prefer to work together instead of competing as in noncooperative oligopoly models.
2. Understand the various laws outlawing explicit cooperation among firms in many countries.
3. Understand why there are strong incentives for cartel members to cheat on cartel chosen output quotas.
4. Use the simple linear one-shot oligopoly model with two firms to show:
(a) Both firms do better if they join the cartel as compared to perfect competition or a 2 firm Cournot oligopoly.
(b) A firm that deviates from the 2 -firm cartel is always better off if the other firm continues to behave like a cartel member after the defection.
5. Distinguish one-shot from repeated games and discuss some of the strategies a firm might choose to employ in a repeated game to discourage cartel members from defecting. The student will be able to explain the need for discounting in analyzing repeated games, and be able to solve simple discounting problems.
6. Set up a cartel model as a two-period repeated game and derive the appropriate payoff matrix.
7. Solve a multiple period finite repeated game by backwards induction. The student will be able to solve a repeated game with an infinite horizon using discounting principles.
8. Understand the basic issues inherent in optimal antitrust policy and enforcement.
9. Recognize that the precise measures of the cost of collusion require a model to determine the price in the absence of collusion, as well as understand the role that formal econometrics can play in eliciting such a measure.

## Suggested Lecture Outline:

Spend two fifty-minute long lectures on this chapter.

## Lecture 1:

1. Reasons for cartels and why they may fail
2. Comparison of competitive, monopoly and N -firm oligopoly models
3. Incentives to cheat on a cartel
4. Introduction to repeated games

## Lecture 2:

1. Repeated games and associated strategies
2. Discounting in repeated games
3. Simple two-period repeated games
4. Simple repeated games with a finite horizon
5. Repeated games with an infinite horizon and discounting formulas

## Suggestions for the Instructor:

1. Students like to hear collusion stories. This is a good place to make the class real world. The classic 50's pricing scheme is still a good place to start. It may be useful to talk about producer organizations, pricing sheets, standard discounts, annual conventions, peer pressure and the like in motivating the ideas of cartel discipline.
2. It may be helpful to have the students rework a 2 -firm Cournot model problem prior to these lectures.
3. One will need to review discounting in order to consider the material on repeated games. A problem set the week before the lecture may be a good idea.
4. The best way to introduce repeated games is to consider the one defection - forever punished example. This makes clear the idea of retaliation without a lot of complications.
5. One might assign the problems that are the base for the Cournot and Bertrand cartel games prior to the lecture so that the students are familiar with the one-shot solutions.
6. Be sure to show the students how to construct the payoff matrices from the results of the Cournot and Bertrand models.

## Solution to End of the Chapter Problems:

Problem 1
(a) $Q_{1}=Q_{2}=40 \Rightarrow P=260-2(80)=100$
$\pi_{1}^{\text {Courrot }}=\pi_{2}^{\text {Courrot }}=(100-20)(40)=3200$
(b) $Q^{\text {Monopoly }}=\frac{260-20}{2(2)}=60 \Rightarrow P^{\text {Monopoly }}=260-2(60)=140$

Therefore, profit of each firm in a cartel is
$\pi_{1}^{\text {Cartel }}=\pi_{2}^{\text {Carrel }}=(140-20)(30)=3600$

## Problem 2

Without loss of generality, suppose Firm 2 cheats, but Firm 1 maintains its cartel quantity of 30 . Then, the optimal choice for Firm 2 can be found from its best response function.
$Q_{2}^{\text {Cheating }}=\frac{1}{4}(260-20-2(30))=45$
Therefore, the market price is $260-2(30+45)=110$. As a result, the profit of the cheating firm is: $\pi_{2}^{\text {Cheating }}=(110-20)(45)=4050$

## Problem 3

If Firm 2 cheats, then it earns 4050 for one period, but earns its Cournot profit; 3200, for all periods afterwards. On the other hand, if Firm 2 does not cheat, it can continue earning its cartel profit for ever. Hence, the collusive outcome can be sustained if

$$
3600+\delta(3600)+\delta^{2}(3600)+\ldots \geq 4050+\delta(3200)+\delta^{2}(3200) \Rightarrow \frac{3600}{1-\delta} \geq 4050+\frac{3200 \delta}{1-\delta}
$$

$\Rightarrow \delta \geq 0.53$, where $\delta$ is the probability adjusted discount factor.

## Problem 4

(a) With Bertrand price competition $P_{1}=P_{2}=20 \Rightarrow Q_{1}=Q_{2}=60, \pi_{1}=\pi_{2}=0$
(b) $Q^{\text {Monopoly }}=\frac{260-20}{2(2)}=60 \Rightarrow P^{\text {Monopoly }}=260-2(60)=140$

Therefore, profit of each firm in a cartel is
$\pi_{1}{ }^{\text {Cartel }}=\pi_{2}^{\text {Cartel }}=(140-20)(30)=3600$

## Problem 5

Without loss of generality, let Firm 1 charges \$140, but Firm 2 cheat. Firm 2 needs to undercut Firm 1 only slightly to capture almost the entire monopoly profit. At the limit, Firm 2 captures the entire monopoly profit by cheating. Therefore, $\pi_{2}^{\text {Cheating }}=7200$.

## Problem 6

If Firm 2 cheats, then it earns 7200 for one period, but earns its Bertrand profit; 0 , for all periods afterwards. On the other hand, if Firm 2 does not cheat, it can continue earning its cartel profit for ever. Hence, the collusive outcome can be sustained if
$3600+\delta(3600)+\delta^{2}(3600)+\ldots \geq 7200+\delta(0)+\delta^{2}(0) \Rightarrow \frac{3600}{1-\delta} \geq 7200$
$\Rightarrow \delta \geq \frac{1}{2}$, where $\delta$ is the probability adjusted discount factor.

## Problem 7

Comparing the discount factors, it can be seen that it is more difficult to sustain a cartel under Cournot competition, since it requires a larger discount factor.

## Problem 8

(a) Recall that for Cournot model with $n$ identical firms, with marginal cost $c$, demand intercept $a$ and slope $-b$
$Q_{1}=Q_{2}=\ldots=Q_{n}=\frac{(a-c)}{(n+1) b}, \pi_{1}=\pi_{2}=\ldots=\pi_{n}=\frac{(a-c)^{2}}{(n+1)^{2} b}$
Therefore, $Q_{1}=Q_{2}=\ldots=Q_{4}=24, \pi_{1}=\pi_{2}=\ldots=\pi_{n}=1152, P=68$
(b) $Q^{\text {Monopoly }}=\frac{260-20}{2(2)}=60 \Rightarrow P^{\text {Monopoly }}=260-2(60)=140$

Therefore, $Q_{1}=Q_{2}=\ldots=Q_{4}=15$ and the profit of each firm in a cartel is
$\pi_{1}^{\text {Cartel }}=\pi_{2}^{\text {Cartel }}=\ldots=\pi_{4}^{\text {Cartel }}=(140-20)(15)=1800$

## Problem 9

(a) Without loss of generality, suppose Firm 4 cheats, but all other firms maintain their cartel quantities. Then, the optimal choice for Firm 4 can be found from its best response function.
$Q_{4}^{\text {Cheating }}=\frac{1}{4}(260-20-2(15+15+15))=37.5$
Therefore, the market price is $260-2(37.5+45)=95$. As a result, the profit of the cheating firm is: $\pi_{2}^{\text {Cheating }}=(95-20)(37.5)=2812.5$

## Problem 10

If Firm 2 cheats, then it earns 2812.5 for one period, but earns its Cournot profit; 1152, for all periods afterwards. On the other hand, if Firm 2 does not cheat, it can continue earning its cartel profit for ever. Hence, the collusive outcome can be sustained if

$$
1800+\delta(1800)+\delta^{2}(1800)+\ldots \geq 2812.5+\delta(1152)+\delta^{2}(1152) \Rightarrow \frac{1800}{1-\delta} \geq 2812.5+\frac{1152 \delta}{1-\delta}
$$

$\Rightarrow \delta \geq 0.61$, where $\delta$ is the probability adjusted discount factor.

## Problem 11

Comparing the discount factors, it can be seen that it is more difficult to sustain a cartel under Cournot competition, when there are more firms.

## Problem 12

(a) Weighted average of the marginal cost is
$\bar{c}=(.32)(.7)+(.32)(.7)+(.14)(.8)+(.14)(.8)+(.04)(.85)+(.04)(.85)=.74$
The value of the Herfindahl Index is

$$
H=2(.32)^{2}+2(.14)^{2}+2(0.04)^{2}=0.2472
$$

Therefore,

$$
\frac{P^{*}-0.74}{P^{*}}=\frac{0.2472}{1.55}=0.16 \Rightarrow(1-0.16) P^{*}=0.74 \Rightarrow P^{*}=0.88
$$

We now need to find out what would have been the total sales under Cournot equilibrium. For simplicity, assume that total sales under a Cournot equilibrium is $Q^{*}$. Then ADM's profit under a Cournot equilibrium $=(0.88-0.70)(0.32) Q^{*}=(0 . .0576) Q^{*}$

Assume a constant elasticity of demand so that $\eta=1.55$ at all output levels. The cartel price of $\$ 1.12$, reflects a $27 \%$ increase in price over the (imperfectly) competitive level. Since $\eta=1.55$, the monopoly output of 100,000 tons should reflect a $1.55 \times 27$, or a 42 percent decrease in volume. In other words, the Cournot output would have been about172 thousand tons. Hence, ADM's profit would have been: $(0.0576 /$ pound $) *(172 \text { thousand tons })^{*}(2200$ pounds per ton $)$ or roughly $\$ 21.8$ million.
(b) ADM's annual profit under the cartel

$$
\begin{aligned}
& =(1.12-0.70)(0.32)(2200)(100,000)=(0.42)(0.32)(2200)(100.000) \\
& =(42)(32)(22)(1000)=(29568)(1000)=\$ 29.568 \text { million. }
\end{aligned}
$$

## Problem 12

The probability-adjusted discount factor is $\rho=0.5 / 1.16=0.43$. This must satisfy the condition in equation (14.7). In turn, this implies that $\$ 20.20$ million $\geq 0.57 * \pi^{D}$ or, ADM's profit from cheating on the cartel for one period exceeded $\$ 35.44$ million..

