## Chapter 6: Price Discrimination and Monopoly: Nonlinear Pricing

## Learning Objectives:

Students should learn to:

1. Differentiate the three types of price discrimination.
2. Explain graphically, in words, and algebraically how a monopolist can extract surplus from consumers, how this encourages the monopolist to expand output, and the potential efficiency gains from such surplus extraction.
3. Find the perfectly discriminating demand (offer) schedule from discrete data and aggregate over consumers to get the total demand at any particular price.
4. Explain the concepts of block pricing and two-part tariffs and compute such prices for simple examples graphically and algebraically. The student will be able to compute total and consumer surplus for linear demand schedules in order to compute these optimal tariffs.
5. Show how second-degree price discrimination leads to volume discounts and various ways to implement such discounts in a way to maximize profits. The student will be able to analyze block pricing schemes utilizing self-selection mechanisms. The student will be able to compute consumer surplus in analyzing incentive compatibility problems of various block pricing schemes.
6. Determine the optimal block pricing scheme in a model with two types of consumers.
7. Differentiate volume discounts based on cost from those based on price discrimination.

## Suggested Lecture Outline:

Spend two fifty minute long lectures on this chapter.

## Lecture 1:

1. Perfect price discrimination and the appropriation of surplus
2. Types of price discrimination and conditions to implement them
3. First degree price discrimination
4. Two-part tariffs and block pricing
5. Graphical and numerical examples

## Lecture 2:

1. Idea and prevalence of volume discounts
2. Capturing some consumer surplus when perfect discrimination is not possible
3. Second degree price discrimination
4. Incentive compatible pricing schemes
5. Numerical examples

## Suggestions for the Instructor:

1. Emphasize that the monopolist who can appropriate more surplus will produce more. An easy way to do this might be to assume that marginal cost is zero so that consumer surplus is equal to total surplus and the area below price and behind quantity is producer profit. Start with the MR=MC=0 solution and move to the right. Show that as the
monopolist is able to get some of the consumer surplus, she may expand output even though price falls.
2. The classic Disneyland example of Oi is a great way to discuss two-part tariffs though Disneyland no longer uses such pricing.
3. When discussing the first degree price discrimination model, also derive the competitive solution as a benchmark.
4. A useful homework problem may be to ask the students to think of other ways to implement first degree price discrimination.
5. In introducing second degree price discrimination ask students about their experience with volume discounts. Also ask if any of them have purchased a large volume of a product to get the discount and then tried to resell some of it at a profit to others.
6. Discuss common examples of second degree price discrimination such as buy 1 , get the second for $1 / 2$ price; 3 for the price of 2 , 2 for $\$ 5.00$ as opposed to $\$ 2.50$ each, delivery free on purchases over $\$ 500$ (although this may also have higher per unit costs), bus passes, etc. Also discuss the fact that pricing a product 2 for $\$ 5$ but allowing the purchase of 1 item for $\$ 2.50$ is not really a volume discount.

## Solutions to the End of the Chapter Problems:

## Problem 1

The demand for college education may depend on both price (tuition) and income. If we assume that income raises the demand for education, then richer students will demand more education (than the poorer students) at any price level. If the price elasticity for the rich students is less than the poor students (as would be in the case with a linear demand curve), then a higher price should be charged to the rich students. In this sense, college financial aid is probably closer to first degree price discrimination.

## Problem 2

(a) The consumers who join must have a higher demand elasticity for groceries than those who do not. Since they are willing to buy more groceries as a given price, the co-op would like to give them a quantity discount.
(b) The pricing option offered to members is classic, two-part pricing that will impart a quantity discount for members who buy more products. The difference in the (marginal) price per unit between members and non-members may be regarded as third-degree price discrimination, however, as price is reduced to the group with more elastic demand.

## Problem 3

This is an example of second degree price discrimination, involving multipart pricing. Since the consumers are heterogeneous and the company cannot identify who is who, it offers multiple pricing policies and let the consumers self select.

## Problem 4

(a) If the club owner cannot price discriminate, she will consider the aggregate demand, which is

$$
Q^{S+A}=(18-3 P)+(10-2 P)=28-5 P
$$

The corresponding inverse demand function is $P=\frac{28}{5}-\frac{1}{5} Q^{S+A}$ and the marginal revenue
is $M R=\frac{28}{5}-\frac{2}{5} Q^{S+A}$.
Equate marginal revenue with marginal cost, that is, $\frac{28}{5}-\frac{2}{5} Q^{S+A}=2 \Rightarrow Q^{S+A}=9$
Therefore, $P=\frac{28}{5}-\frac{1}{5} Q^{S+A}=\frac{28}{5}-\frac{9}{5}=\frac{19}{5}$.
Her profit without price discrimination: $\pi=(P-2) Q=\left(\frac{19}{5}-2\right)(9)=\frac{(9)^{2}}{5}=16.2$
(a) If the club owner can price discriminate, she will equate marginal revenue and marginal cost for each group. That is,

$$
\begin{aligned}
& 6-\frac{2}{3} Q^{S}=2 \Rightarrow Q^{S}=6 \Rightarrow P^{S}=4 \\
& 5-Q^{A}=2 \Rightarrow Q^{A}=3 \Rightarrow P^{A}=3.5
\end{aligned}
$$

Hence, total profit with price discrimination $\pi^{S}+\pi^{A}=(4-2)(6)+(3.5-2)(3)=12+4.5=16.5$. Therefore, her profit is higher with price discrimination.

## Problem 5

In this scenario, she can practice two-part pricing. For each group, the number of token will be equal to quantity demanded at price $\$ 2$, which is the marginal cost of a drink. Number of tokens for students $=18-3(2)=12$, and the number of tokens for the adults $=10-2(2)=6$. Now, for each group, the cover charge should equal the consumer surplus received at the given number of tokens. That is,
Cover charge for a student $=\frac{1}{2}(6-2)(12)+2(12)=48$
Cover charge for an adult $=\frac{1}{2}(5-2)(6)+2(6)=21$
Therefore, her profits $=$ Total revenue - total costs of drinks

$$
=\frac{1}{2}(6-2)(12)+2(12)+\frac{1}{2}(5-2)(6)+2(6)-2(12)-2(6)=33
$$

## Problem 6

Suppose the marginal values of the first minute for three groups are $V_{1}, V_{2}$, and $V_{3}$ respectively. Since the value of a marginal minute for each group declines at the rate of $\$ 0.0004$ per minute used, then the demand curves consistent with this pricing are:

$$
\begin{aligned}
& V_{1}-0.0004 \times 500=50 / 500 \Rightarrow V_{1}=0.3 \Rightarrow P_{1}=0.3-0.0004 Q_{1} \\
& V_{1}-0.0004 \times 750=62.5 / 750 \Rightarrow V_{1}=0.383 \Rightarrow P_{2}=0.383-0.0004 Q_{2} \\
& V_{1}-0.0004 \times 1000=75 / 1000 \Rightarrow V_{1}=0.475 \Rightarrow P_{3}=0.475-0.0004 Q_{3}
\end{aligned}
$$

The surplus each group can enjoy is:

$$
\begin{aligned}
& C S_{1}=\frac{1}{2}\left(0.3-\frac{50}{500}\right)(500)=50 \\
& C S_{2}=\frac{1}{2}\left(0.383-\frac{62.5}{750}\right)(750)=112.5 \\
& C S_{3}=\frac{1}{2}\left(0.475-\frac{75}{1000}\right)(1000)=200
\end{aligned}
$$

## Problem 7

Suppose the monopolist offers two entry fees, unit price combinations. One is targeted towards the low demanders and the other is targeted towards the high demanders. Let $\left(F_{1}, p_{1}\right)$ be the entry fee and unit price combination paid by the low demanders and $\left(F_{2}, p_{2}\right)$ be the entry fee and unit price combinations paid by the high demanders.

Now, from the discussions in the text, it follows that $F_{1}=\frac{1}{2}\left(12-p_{1}\right)^{2}$. Also, the monopolist needs to adjust $p_{2}$ so that the high demanders do not buy the combination intended for the low demanders. Therefore, it follows that

$$
\begin{aligned}
& \frac{1}{2}\left(16-p_{2}\right)^{2}-F_{2}=\frac{1}{2}\left(16-p_{1}\right)^{2}-\frac{1}{2}\left(12-p_{1}\right)^{2} \\
& \Rightarrow F_{2}=\frac{1}{2}\left(16-p_{2}\right)^{2}-\frac{1}{2}\left(16-p_{1}\right)^{2}+\frac{1}{2}\left(12-p_{1}\right)^{2}
\end{aligned}
$$

Thus, the profit of the monopolist is given by

$$
\pi_{1}=N_{h}\left[\left(p_{2}-4\right)\left(16-p_{2}\right)+F_{2}\right]+N_{l}\left[\left(p_{1}-4\right)\left(12-p_{1}\right)+\frac{1}{2}\left(12-p_{1}\right)^{2}\right]
$$

Differentiate $\pi$ with respect to $p_{1}$ and $p_{2}$, and equate each expression to zero to obtain

$$
p_{1}=4\left[1+\frac{N_{h}}{N_{l}}\right], p_{2}=4
$$

Substituting the optimal prices in to the profit expression, we obtain the maximum profit the monopolist can earn by serving both groups. Observe that
$\pi_{1}=\left[72-\frac{1}{2}\left(12-4 \frac{N_{h}}{N_{l}}\right)^{2}+\frac{1}{2}\left(8-4 \frac{N_{h}}{N_{l}}\right)^{2}\right] N_{h}+\left[4 \frac{N_{h}}{N_{l}}\left(8-4 \frac{N_{h}}{N_{l}}\right)+\frac{1}{2}\left(8-4 \frac{N_{h}}{N_{l}}\right)^{2}\right] N_{l}$
On the other hand, if the monopolist serves the high demanders only, its profit will be $\pi_{2}=N_{h} \frac{1}{2}(16-4)^{2}=72 N_{h}$
Therefore, the monopolist serves both groups if and only if $\frac{\pi_{1}}{\pi_{2}} \geq 1$.
It can be verified that (remark: use a program such as MAPLE)
$\frac{\pi_{1}}{\pi_{2}} \geq 1$ if and only if $\frac{N_{h}}{N_{l}} \geq 1$

