Chapter 4: Technology and Cost

Learning Objectives:

Students should learn to:

1. Understand the differences between the neoclassical (technological) view of the firm and other views related to agency theory, transactions costs, incomplete contracts, and residual control rights. The student will be able to discuss some of the limitations of the neoclassical approach in explaining the boundaries of the firm.
2. Explain the decomposition of the profit maximization problem for the single output firm under certainty into a two-step process involving cost minimization for a fixed output level and the selection of an optimal output level. The student will be able to explain the process of cost minimization and the idea of choosing inputs in an optimal fashion given their price and the overall relationship between output and the various production inputs.
3. Derive various cost relationships algebraically and graphically; given a cost function C(Q,w,z), where Q is output, w is a vector of input prices and z is a vector of fixed inputs, the student will be able to explain the intuition behind the various measures of cost and how they are related to one another. The concepts include:
    a. Total cost
    b. Fixed cost
    c. Variable cost
    d. Average fixed cost
    e. Average variable cost
    f. Average (total) cost
    g. Discrete marginal cost
    h. Marginal cost
    i. Minimum average variable and average total cost
    j. Economies of size (scale)
4. Explain economies of size and natural monopoly and how technology can affect industry structure.
5. State various reasons for the existence of scale economies including:
    a. Technical factors related to size and volume
    b. Specialization and the division of labor
    c. Economies of mass reserves
    d. Indivisibility of inputs and allocation of fixed costs
6. Explain and give examples of multiproduct technologies. The student will be able to differentiate truly joint products such as lamb and wool or soybean oil and soybean meal from outputs related by allocated fixed inputs such as products in a hardware store.
7. Explain and calculate various measures describing the structure of multiproduct technologies including:
    a. Marginal cost
    b. Elasticity of scale
    c. Economies of scope

8. Explain how flexible manufacturing technologies allow firms to capture economies of scope in offering related products to the market. The student will also be able to describe the basic differentiated product model of manufacturing due to Eaton and Schmitt and the tradeoffs between specialization and economies of scope.
9. Explain how a model based on distance can be used to describe product variety.
10. Present a reasoned argument on various non-cost determinants of market structure. The student will be able to give examples of some of these determinants. Items to be discussed include:
    a. Market size in relation to technological factors
    b. Network externalities
    c. Government regulatory policy
    d. Patents and copyrights
11. Understand how to derive cost functions based on profit-maximizing behavior
12. Gain an appreciation of the uses of econometric analysis to study, inter alia, cost functions

Suggested Lecture Outline:

Spend one fifty-minute long lecture on this chapter.

Lecture 1:

a. Cost measures - C(Q), AC(Q), MC(Q)

b. Economies of size and scale

c. Fixed, sunk, avoidable, and variable costs

d. Basic structure of multiproduct firms

e. Ray average cost and multiproduct economies of scale

f. Derivation of a cost function

g. Estimation of a cost function

Suggestions for the Instructor:

1. It is probably worth the time to discuss the relationship between cost minimization and profit maximization.
2. Numerical examples and tables are an excellent way to present and solidify the concepts related to cost, average costs, marginal cost, etc. It may be useful to have students develop a spreadsheet based on a cost equation. They can then compute various measures of cost, graph them, analyze them, etc.
3. The relationship between average and marginal curves is often facilitated by reference to cumulative and current semester GPAs. Another good example is total and average lifetimes earnings where the data is reported on a year-by-year basis.
4. The example of the pin factory due to Adam Smith is still a classic. Other examples of specialization include production of screws and bolts or horseshoes, use of robots in manufacturing, etc.
5. It is important to point why some inputs are indivisible. A good example is a caterpillar

tractor used for excavation or road construction. Another is an electrical generator. A table saw used by a furniture maker is yet another.

6. Point out that part of the distinction between fixed and variable costs has to do with timing. The costs of indivisible inputs that are not used up in a single production period are clearly fixed for that period. But the cost of something like seed (which is really expendable) is fixed once it is put in the ground. Most inputs become fixed over time as the production process evolves.
7. A discussion of avoidable costs is often a good way to introduce sunk costs.
8. The railroad example is an excellent one for introducing a multiproduct technology. The example of pancakes versus crepes versus waffles is another that makes some useful points. A wheat farmer produces both wheat and straw. A typical restaurant produces several different menu items, even a burger only place produces different types of burgers.
9. It may be useful to introduce multiproduct economies of scale before discussing ray average cost. The intuition is sometimes easier for students. It is helpful to write

$$S = \frac{AC(Q)}{MC(Q)} = \frac{C(Q)}{Q\dfrac{dC(Q)}{dQ}}$$

   Then the multiproduct definition is just a generalization of the single product one where a partial derivative, a vector and a gradient are used.

10. A graph of ray average cost for expansion along different rays is very helpful to students.
11. Be sure to emphasize the similarity between the single product and multiproduct measures. If the student can remember the multiproduct ones are generalizations, it helps a lot.
12. A good example of economies of scope is the student who mows lawns using a ride-on mower, trailer for hauling, fuel, and labor as inputs. If the student adds a snow removal business, the trailer, and to some extent the labor, are public inputs giving economies of scope.
13. There are lots of examples of economies of scope in retailing, sales, product promotion, accounting, and personnel management.
14. The section on non-cost determinants of structure might be a good place to start the discussion of barriers to entry and exit.

Solutions to the End of the Chapter Problems:

Problem 1

$$AC(q) = \frac{C(q)}{q} = \frac{100 + 4q + 4q^2}{q} = \frac{100}{q} + 4 + 4q$$

$$MC(q) = 4 + 8q$$

To find the range of production characterized by scale economies, equate AC(q) with MC(q).

$$AC(q) = MC(q) \Rightarrow \frac{100}{q} + 4 + 4q = 4 + 8q \Rightarrow q = 5$$

For $q \in [0,5]$, production is characterized by scale economies. At q = 5 production level scale economies exhausted.

Problem 2

The consultant has not distinguished between fixed and variable costs. Since the fixed costs will be incurred regardless of whether the train runs or not, there is no increase in fixed cost from making a trip during off-peak hours. What matters are the variable costs of making an off-peak hour trip? As long as they are less than the revenue from the sales of 10 tickets, the train should make the trip.

> Suppose that the train makes 20 total round trips per day and the total fixed cost per day is $800. The variable cost per trip is $10. The fixed cost per trip with 20 trips is $40. Suppose the train normally makes 5 peak load trips and 15 off-peak load trips. The variable cost per passenger for an off-peak hour trip is $1. If the fare exceeds $1, then the train should make the trip since the fixed costs will accrue whether the trip is made or not. In fact, if the number of off-peak trips is reduced by 10 to 5 trips so that the total trips per day is now 10, the total cost per trip is now ((800+100)/10) $90 instead of $50.

Problem 3

(a)

$$C(q \le 7) = 50 + 0.5q$$

$$AC = \frac{50 + 0.5q}{q} = \frac{50}{q} + 0.5$$

$$MC = 0.5$$

We can create a table with the values for various levels of q where average marginal cost is the average of the discrete changes to and away from q_i.

q	Cost	Average Cost	Discrete MC	Approx MC	MC
0	50		0.5		0.5
1	50.5	50.5000	0.5	0.5	0.5
2	51	25.5000	0.5	0.5	0.5
3	51.5	17.1667	0.5	0.5	0.5
4	52	13.0000	0.5	0.5	0.5
5	52.5	10.5000	0.5	0.5	0.5
6	53	8.8333	0.5	0.5	0.5
7	53.5	7.6429	2.5	1.5	0.5
8	56	7.0000	7	4.75	7
9	63	7.0000	7	7	7
10	70	7.0000	7	7	7
15	105	7.0000	7	7	7
20	140	7.0000			7

(b)

$$C(q > 7) = 7q$$

$$AC = \frac{7q}{q}$$

$$= 7$$

$$MC = 7$$

Problem 4

Yes, there is a minimum efficient scale of plant implied by these cost relationships. If we require integer values of q, then the minimum efficient scale is 8 units of output. Otherwise, it is any amount greater than 7.

Problem 5

Since the minimum average cost is $7.00 and this is also marginal cost we can assume that the price in market equilibrium is $7.00. Using the inverse demand curve we then obtain

$$\begin{aligned} P &= 84 - 0.5Q \\ \Rightarrow 7 &= 84 - 0.5Q \\ \Rightarrow 0.5Q &= 77 \\ \rightarrow Q &= 154 \end{aligned}$$

Since the minimum efficient scale is 8, the maximum number of firms producing 8 units is q* = 154/8 = 19.25. Each firm would produce 8 units given a total of 152. The firms would then need to allocate the remaining two units in some integer fashion among them if whole units of production are required. Otherwise we could have 21 firms each producing 7.333 units.

Problem 6

Demand has changed and so has the equilibrium quantity.

$$\begin{aligned} P &= 14 - 0.5Q \\ \Rightarrow 7 &= 14 - 0.5Q \\ \Rightarrow 0.5Q &= 7 \\ \rightarrow Q &= 14 \end{aligned}$$

Since the minimum efficient scale is 8, the maximum number of firms producing 8 units is q* = 14/8 = 1.75. One firm could produce 14 units at a total cost of $98. If there were two firms in the industry, one producing 8 units and the other one six units, the total cost of production would be $109 (56+53), which is larger than $98. If the industry price were $7, the second firm would not cover its average costs of $8.833 per unit. Thus there will be no second firm and the first firm will be a monopoly. There is not room in this industry for two firms.

If the first firm were a monopoly it would set marginal revenue equal to marginal cost and charge a price of

$$\begin{aligned} \pi &= PQ - C(Q) \\ &= (14 - 0.5Q)Q - 7Q \\ &= 14Q - 0.5Q^2 - 7Q \\ &= 7Q - 0.5Q^2 \\ \frac{d\pi}{dQ} &= 7 - Q = 0 \\ \Rightarrow Q &= 7 \\ \Rightarrow P &= 10.5 \end{aligned}$$

At this point a second firm will try to enter producing at least 6 units. But this will cause price to fall and the second firm will be forced out.

Problem 7

(a) It is clear that average costs start to rise once we move from 1,500 to 1,750 units of output. This can also be seen by computing the total cost at each output level and then computing a discrete measure of marginal cost as in the table below. Once we get beyond 1,500 units, marginal cost is higher than average cost.

(b) Find the MC first. Here is the answer for Q =1000. The answers for other values of Q can be found in a similar fashion.

For output level 1,000, it is computed as

$$S(1{,}000) = \frac{AC}{MC} = \frac{21.63}{18.48} = 1.17045$$

It may be more accurate here to compute the average marginal cost as opposed to the discrete one given the large changes in output.

Problem 8

If the main product is meat, then the additional costs of supplying the byproducts (offal) are quite small. For example the cost of supplying the hide is the cost of removing the hide in a fashion that preserves its usefulness for leather as opposed to a technique that might be cheaper but reduces it to a pile of scrap. These economies exist because the process of feeding and slaughtering a steer or heifer produces a whole animal (hide, horns, meat, viscera, etc.) and since these come in more or less fixed proportions, the cost of obtaining whatever is considered a byproduct is close to zero for all amounts less than that implied by the fixed proportion technology.

The supply of leather will then depend on the price of steak. If the demand for steak is very high, then the supply of cattle will be high, which will increase the supply of hides and lower the price of leather. Similarly, a very high price of gelatin may lead to a different process in removing the horns and hoofs so that more is preserved for the making of gelatin. In a similar fashion, the percentage of the animal that goes to make ribs as opposed to hamburger depends on the demand for ribs in a given area.