## Part V

## Contractual Relations Between Firms

In this part we examine the various ways in which firms may interact that involve formal and legally enforceable contracts. Such formal relationships employ strategic considerations just as much as do the pricing and production decisions considered in the last several chapters. However, the manifestation of these tactical issues is more subtle because, by its very nature, a formal contract involves some elements of cooperation as well as the usual ingredient of self-interest.

Chapters 16 and 17 explore the implications of the most binding of all contracts, the marriage agreement. The corporate term for marriage, however, is merger (or acquisition). Chapter 16 explores the issues surrounding the merger of two firms that formerly competed against each other in the same product market, a horizontal merger. Mergers happen with a frequency that is difficult to justify with formal economic analysis. We examine models that aim to resolve this difficulty and, in particular, that help to explain why one merger in an industry often leads to others so that mergers often come in waves. Of course, a merger between two former rivals runs the risk of weakening competition. Therefore, the antitrust authorities and the courts must often evaluate such mergers and try to forecast the post-merger market outcome. Increasingly, such evaluations have made use of econometric data on market demand conditions to build computer models that are then used to simulate the post-merger equilibrium. Chapter 16 includes an extensive description of this process and the difficulties it can involve as illustrated by the well-known proposed merger of Staples and Office Depot In Chapter 17 we then turn to consider a merger not between two former competitors but, instead, between an upstream firm, such as a manufacturer, and a downstream firm, such as a retailer. Here again, an important goal is to explain why such mergers occur and what anticompetitive issues they may raise. Vertical mergers can increase efficiency and so raise both firm profit and consumer welfare. However, they can also disadvantage non-merged firms and reduce competitive pressures. We present some evidence on these issues based on a recent empirical investigation of vertical integration in the ready-mixed concrete industry.

Firms may use formal contracts that stop short of a merger in order to harmonize their interests. Chapter 18 focuses on contracts between an upstream and a downstream firm that are primarily concerned with the price charged to the final consumer. Chapter 19 focuses on other vertical contracts, such as those that grant a franchise or exclusive dealing rights. We also examine the public policy concerns raised by all vertical constraints. An empirical study of exclusive dealing in the U.S. beer industry illustrates how one might obtain evidence on these important questions.

## 16

## Horizontal Mergers

The merger mania that transformed much of corporate America through the 1990s largely disappeared in the wake of the terrorist attack of September 11, 2001, the corporate scandals at Enron, Tyco, HealthSouth, and WorldCom, and the bursting of the dot.com bubble. However, after quieting down for a few years, merger activity bounced back sharply in 2004, when over 10,000 deals were transacted in the U.S. for a total value of $\$ 746.3$ billion. 2005 followed with another 10,000-plus deals with a total value of over $\$ 1$ trillion. In 2006, the number of mergers rose still further to over 11,000 and the value increased to $\$ 1.23$ trillion. ${ }^{1}$ The urge to merge is back.

The organization and reorganization of firms brought about by mergers and acquisitions raises several issues. Perhaps the most central of these is the question, why? What is the motivation behind the marriage of two (or more) firms? One possible answer is that a merger creates cost savings by eliminating wasteful duplication or by improving information flows within the merged organization. Similarly, a merger may lead to more efficient pricing and/or improved services to customers. This is the case when two firms producing complementary goods such as nuts and bolts merge ${ }^{2}$.

To the extent that reducing cost or rationalizing complementary production is the primary motivation for mergers, the mergers are likely to be beneficial to society as well as to the merging firms and ought not to be discouraged. However, mergers can also be an attempt to create legal cartels. The merged firms come under common ownership and control. Hence, the new corporate entity will coordinate what were formerly separate actions with a view to achieving the joint profit-maximizing outcome. By placing such coordination within the boundaries of one firm a merger legitimizes precisely the kind of behavior that would have been illegal had the two firms remained separate. Viewed in this light, mergers are an undesirable attempt to create and exploit monopoly power in a market.

Mergers pose a difficult challenge for antitrust policy because policy makers need to be able to distinguish between anticompetitive mergers, on the one hand, and those that are not injurious to competition, on the other. This tension is openly acknowledged in the overview to the Merger Guidelines. "While challenging competitively harmful mergers, the Agency

[^0]seeks to avoid unnecessary interference within the larger universe of mergers that are either competitively beneficial or neutral. ${ }^{\prime 3}$

We explore these issues in this chapter and the next. We examine what economic theory can tell us about the profit rationale for mergers and whether the enhanced profit stems from greater efficiency or from enhanced monopoly power. While the relevant theory is mainly an extension of the Cournot and Bertrand models, we warn the reader in advance that it is nevertheless somewhat difficult. The rewards of a deeper understanding of mergers and merger policy justify, we hope, the necessary extra work.

Before proceeding further, it is useful to classify merger types because all mergers are not alike. An important source of distinction is the nature of the relationship that existed between the merging firms prior to their combination. This gives rise to three different kinds of mergers. First, there are horizontal mergers. These occur when the firms joining together in the merger were formerly competitors in the same product market. A horizontal merger involves two or more firms that, so far as their buyers are concerned, market substitute products. The 2006 merger of the telecommunications software firms, Alcatel and Lucent, is one example of a horizontal merger.

Vertical mergers are the second type. These typically involve firms at different stages in the vertical production chain. Consider the purchase by the Disney Company of Capital Cities/ABC. Here, a major producer of films and television programs has acquired a major distributor and network that airs this material. The 2006 purchase of Murphy Farms, a major hog farming enterprise by Smithfield Foods, the largest pork company in the world is a similar vertical combination. Vertical mergers include more than mergers between upstream-downstream firms. They include as well any combination of firms that, prior to the merger, produced complementary goods. The merger between Hewlett-Packard, primarily a producer of software, printers, and scanners, and Compaq, a major personal computer firm also would fall in the vertical category. The merger of CSX and Conrail, two large freight rail companies in the eastern United States provides another example because, for the most part, the rail lines of the two firms did not overlap but instead served adjacent regions, i.e., the southeast and the northeast United States. Thus, the two firms together offered complementary services to customers who wish to transport goods across both regions.

Finally, conglomerate mergers involve the combination of firms without either a clear substitute or a clear complementary relationship. General Electric, a firm that produces aircraft engines, electric products, financial services, and through its subsidiary NBC television programming, is one of the world's most successful conglomerate firms. Recent examples of conglomerate mergers include: (1) the purchase of Duracell Batteries by Gillette, (2) the purchase of Snapple (iced tea) and Gatorade (a sports drink) by Quaker Oats, and (3) the merger of CUC International, a health and home-shopping company with HFS, a major hotel firm.

In this chapter we focus on horizontal mergers. Since these reflect combinations of two or more firms in the same industry, they raise the most obvious antitrust concerns. Vertical and conglomerate mergers are discussed in Chapter 17.

[^1]
### 16.1 HORIZONTAL MERGERS AND THE MERGER PARADOX

Horizontal mergers replace two or more former competitors with a single firm. The merger of two firms in a three-firm market changes the industry to a duopoly. The merger of two duopolists creates a monopolist. The potential for a merger to create monopoly power is therefore clearly an issue in the horizontal case. Our first order of business is therefore rather surprising. It is to discuss a phenomenon known as the merger paradox. The paradox is that it is, in fact, quite difficult to construct a simple economic model in which there are sizable gains for firms participating in a horizontal merger that is not a merger to monopoly. ${ }^{4} \mathrm{We}$ illustrate the paradox using the Cournot model of section 9.4. ${ }^{5}$

Let's start with a simple example. Suppose we have three firms, each with a constant marginal cost of $c=\$ 30$ and jointly facing an industry demand curve described by: $P=150-Q$. The Cournot equilibrium results in each firm producing one-fourth of the competitive output or 30 so that total output is 90 . The price therefore is $P=\$ 60$ and each firm earns a profit of $30(\$ 60-\$ 30)=\$ 900$.

What happens if two of these firms merge? In the wake of a two-firm merger, the industry will become one with two firms, each of which will produce one-third of the competitive output or 40 so that total output now falls to 80 . The price will then rise to $\$ 70$ and each of the two remaining firms will earn a profit of $\$ 1,600$.

We may now evaluate the impact of the merger. First, note that the merger is bad for consumers. Output falls and the price rises. Second, the merger is good news for the firm that did not merge. It now expands its output to 40 units and sells these at a higher price than previously so that it enjoys a profit increase of $\$ 1,600-\$ 900=\$ 700$. Finally, we come to the central element in the merger paradox. For the two firms that merged, the merger did not pay off. Previously, each firm produced 30 units and earned a profit of $\$ 900$ for a combined pre-merger output and profit of 60 units and $\$ 1,800$, respectively. In the post-merger market, however, these two firms have a combined output of only 40 and a total profit of $\$ 1,600$. The merger has hurt the firms that merged and brought benefits to their rival. If this example is reflective of a more general result, then we ought not to observe many mergers. Of course, the paradox is that we do observe mergers all the time.

The fact of the matter is that the foregoing example is not a special case. It is in fact easy to show that a merger will almost certainly be unprofitable in the basic Cournot model whether it is between two firms or even more so long as the merger does not create a monopoly. To see this more general result, start by assuming a market of $\mathrm{N}>2$ firms, each of which produces a homogeneous product and acts as a Cournot competitor. The firms have identical costs given by the total cost function

$$
\begin{equation*}
C\left(q_{i}\right)=c . q_{i} \text { for } i=1, \ldots, N, \tag{16.1}
\end{equation*}
$$

where $q_{i}$ is output of firm $i$. Market demand is linear and, in inverse form, is given by the equation

$$
\begin{equation*}
P=A-B Q=A-B\left(q_{i}+Q_{-i}\right), \tag{16.2}
\end{equation*}
$$

[^2]where $Q$ is aggregate output produced by the $N$ firms and $Q_{-i}$ is the aggregate output of all firms except firm $i$; that is,
$$
Q_{-i}=Q-q_{i}
$$

The profit function for firm $i$ can then be written as

$$
\begin{equation*}
\pi_{i}\left(q_{i}, Q_{-i}\right)=q_{i}\left[A-B\left(q_{i}+Q_{-i}\right)-c\right] . \tag{16.3}
\end{equation*}
$$

In a Cournot game, firms choose their output levels simultaneously to maximize profit. The resulting profit to each firm in a Cournot equilibrium is

$$
\begin{equation*}
\pi_{i}^{C}=\frac{(A-c)^{2}}{B(N+1)^{2}} \tag{16.4}
\end{equation*}
$$

Suppose now that $M \geq 2$ of these firms decide to merge. In order to exclude the case of merger to monopoly we assume that $M<N$. Such a merger leads to an industry in which there are now $N-M+1$ firms competing in the industry. Since all firms are the same, we can think of the merged firm as comprised of firms 1 through $M$.

The new merged firm picks its output $q_{m}$ to maximize profit, which is given by

$$
\begin{equation*}
\pi_{m}\left(q_{m}, Q_{-m}\right)=q_{m}\left[A-B\left(q_{m}+Q_{-m}\right)-c\right] \tag{16.5}
\end{equation*}
$$

where $Q_{-m}=q_{m+1}+q_{m+2}+\ldots+q_{N}$ denotes the aggregate output of the $N-M$ firms that have not merged.

Each of the nonmerged firms chooses its output to maximize profit given, as before, by

$$
\begin{equation*}
\pi_{i}\left(q_{i}, Q_{-i}\right)=q_{i}\left[A-B\left(q_{i}+Q_{-i}\right)-c\right] . \tag{16.6}
\end{equation*}
$$

In this case the term $Q_{-i}$ now denotes the sum of the outputs $q_{j}$ of each of the $N-M$ nonmerging firms excluding firm $i$, plus the output of the merged firm $q_{m}$.

The only difference between equations (16.5) and (16.6) is that in the former we have a subscript $m$ while in the latter we have a subscript $i$. In other words, a crucial implication of equations (16.5) and (16.6) is that, after the merger, the merged firm becomes just like any one of the other firms in the industry. This means that all of these $N-M+1$ firms, each having identical costs and producing the same product, must in equilibrium produce the same amount of output and therefore earn the same profit. In other words, in the post merger Cournot equilibrium, it must be the case that the output and profit of the merged firm, $q_{m}^{C}$ and $\pi_{m}^{C}$, are the same as the output and profit of each nonmerged firm. Using the equations from the inset for a market with $N-M+1$ firms, these are, respectively:

$$
\begin{equation*}
q_{m}^{C}=q_{n m}^{C}=\frac{A-c}{B(N-M+2)} \text { and } \pi_{m}^{C}=\pi_{n m}^{C}=\frac{(A-c)^{2}}{B(N-M+2)^{2}} \tag{16.7}
\end{equation*}
$$

where the subscript $m$ denotes the merged firm and $n m$ a nonmerged firm.
Equations (16.4) and (16.7) allow us to compare the profit of the nonmerging firms before and after the merger. The first point to note is the free-riding opportunity afforded to the
non-merging firms when other firms merge. We know that in the Cournot model as the number of firms decreases industry output falls (and price rises). Of course, a merger does just that. It reduces the number of firms. So the price rises for all firms, including those that did not merge. Moreover the merger allows those firms to gain market share while also benefiting from an increase in the market price.

What about the merging firms? There are $M$ of these and, prior to the merger, each one earned the profit shown in equation (16.4). Hence, the aggregate profit of these firms taken together is $M$ times that amount. After the merger, the profit of the merged firm is the profit shown in equation (16.7). Is the profit of the merged firm greater than the aggregate profit earned by the $M$ firms before the merger? In order for the answer to be yes, it must be the case that

$$
\begin{equation*}
\frac{(A-c)^{2}}{B(N-M+2)^{2}} \geq M \frac{(A-c)^{2}}{B(N+1)^{2}} \tag{16.8}
\end{equation*}
$$

This requires

$$
\begin{equation*}
(N+1)^{2} \geq M(N-M+2)^{2} \tag{16.9}
\end{equation*}
$$

Note that equation (16.9) does not include any of the demand parameters or the firms' marginal costs. In other words, equation (16.9) tells us about the profitability of any $M$-firm merger. All that is required is that demand is linear and that the firms each have the same, constant marginal costs.

In our example in which the number of firms is $N=3$, and the number of firms merging is $M=2$, it's easy to see then that the inequality in (16.9) is not satisfied. In other words, in a 3-firm market satisfying our demand and cost assumptions no two-firm merger is profitable.

Condition (16.9) is a general condition that turns out to be very difficult to satisfy even when more than two-firms merge as long as the merger does not result in a monopoly. To see this, suppose that we substitute $M=a N$ in equation (16.9), with $0<a<1$. That is, $a$ is the fraction of firms in the industry that merge. We can then work out how large $a$ has to be for the merger to be profitable. A little manipulation of condition (16.9) shows that for a merger to be profitable, we must have $a>a(N)$ where: ${ }^{6}$

$$
\begin{equation*}
a(N)=\frac{3+2 N-\sqrt{5+4 N}}{2 N} \tag{16.10}
\end{equation*}
$$

Table 16.1 gives $a(N)$ and the associated minimum number of firms $\underline{M}$ that have to merge for the merger to be profitable for a range of values of $N$, the number of firms in the industry.

Table 16.1 Necessary condition for profitable merger

| $N$ | 5 | 10 | 15 | 20 | 25 |
| :--- | :---: | :---: | :--- | :--- | :--- |
| $a(N)$ | $80 \%$ | $81.5 \%$ | $83.1 \%$ | $84.5 \%$ | $85.5 \%$ |
| $\underline{M}$ | 4 | 9 | 13 | 17 | 22 |

[^3]
## 392 Contractual Relations Between Firms

Equation (16.10) and Table 16.1 illustrate what has come to be termed the 80 percent rule. For a merger to be profitable in our simple Cournot world of linear demand and identical constant costs, it is necessary that at least 80 percent of the firms in the market merge. The problem is that a merger of this magnitude would almost never be allowed by the antitrust authorities.

Suppose that demand for carpet-cleaning services in Dirtville is described by $P=130-Q$. There are currently 20 identical firms that clean carpets in the area. The unit cost of cleaning a carpet is constant and equal to $\$ 30$. Firms in this industry compete in quantities.
a. Show that in a Cournot-Nash equilibrium the profit of each firm is $\pi=22.67$.
b. Now suppose that six firms in the industry merge. Show that the profit of each firm in the post-merger Cournot game is $\pi=39.06$. Show that the profit earned by the merged firm is insufficient to compensate all the shareholders/owners who owned the 6 original firms and earned profit from them in the pre-merger market game.
c. Show that if fewer than 17 firms merge, the profit of the merged firm is not great enough to buy out the shareholders/owners of the firms who merge.

The merger paradox is that many, if not most, horizontal mergers are unprofitable when viewed through the lens of our standard Cournot model. Yet, as the events of the 1990s and more recent years tell us, horizontal mergers appear to happen all the time. What aspect of real-world mergers has the simple Cournot model failed to capture? Alternatively, what aspect of the Cournot model is responsible for this prediction that seems at odds with reality?
The critical aspect of the Cournot model that gives rise to the merger paradox is not difficult to find. When firms merge in the Cournot model, the new combined firm behaves after the merger just like any of the remaining firms that did not merge. Thus, if two firms in a threefirm industry merge, the new firm competes as a duopolist. The nonmerging firm in this case has, after the merger, equal status to the merged firm even though it now faces the combined strength of both of its previous rivals.

One cannot help but suspect that, for a merger of any substantial size, either the newly merged firm is different in some material sense from its unmerged rivals, or the overall market has changed in a way that alters rivals' behavior. In the next three sections, we explore such possible modifications while staying within the basic Cournot framework. Subsequently, we consider mergers in a market with differentiated products.

### 16.2 MERGERS AND COST SYNERGIES

In developing the merger paradox we assumed that all firms in the market have identical costs and that there are no fixed costs. What happens if we relax these assumptions? It seems reasonable to suppose that if a merger creates sufficiently large cost savings it should be profitable. In this section we develop an example to show that this can indeed be the case. ${ }^{7}$

[^4]Suppose that the market contains three Cournot firms. Consumer demand is given by

$$
\begin{equation*}
P=150-Q \tag{16.11}
\end{equation*}
$$

where $Q$ is aggregate output, which pre-merger is $q_{1}+q_{2}+q_{3}$. Two of these firms are lowcost firms with a marginal cost of 30 so that total costs at each are given by

$$
\begin{equation*}
C_{1}\left(q_{1}\right)=f+30 q_{1} ; C_{2}\left(q_{2}\right)=f+30 q_{2} \tag{16.12}
\end{equation*}
$$

The third firm is potentially high-cost with total costs given by

$$
\begin{equation*}
C_{3}\left(q_{3}\right)=f+30 b q_{3} \tag{16.13}
\end{equation*}
$$

where $b \geq 1$ is a measure of the cost disadvantage from which firm 3 suffers. In these cost functions $f$ represents fixed costs associated with overhead expenses such as those for marketing or for maintaining corporate headquarters. We now consider the effect of a merger of firms 2 and 3.

### 16.2.1 The Merger Reduces Fixed Costs

Consider first the case in which $b=1$ so that all firms have the same marginal cost of 30 . Suppose, however, that after the merger the merged firm has fixed costs of with $1 \leq a \leq 2$. What this means is that the merger allows the merged firms to economize on overhead costs, for example by combining the headquarters of the two firms, eliminating unnecessary overlap, combining R\&D functions and economizing on duplicated marketing efforts. These are, in fact, typical cost savings that most firms state that they expect to result from a merger.

Because the merger leaves marginal costs unaffected, this is similar to our first example only now firms also have fixed costs. Accordingly, we know that in the pre-merger market each firm earns a profit of $\$ 900-f$. In the post-merger market with just two firms, one earns a profit of $\$ 1,600-f$ while the merged firm earns $\$ 1,600-a f$. Hence, for this merger to be profitable, it must be the case that $\$ 1,600-a f>\$ 1,800-2 f$ which requires that $a<2-200 / f$. What this says is that a merger is more likely to be profitable when fixed costs are relatively high and the merger gives the merged firm the ability to make "substantial" savings in these costs. Note, however, that even if the merger is profitable for the merging firms, consumers are actually worse off as a result of the higher equilibrium price. That same higher price also raises the profit of the nonmerged firm. Moreover, it is still the case that the merged firm loses market share post-merger.

### 16.2.2 The Merger Reduces Variable Costs

Now consider the case in which the source of the cost savings is not a reduction in fixed costs but instead a reduction in variable costs which we capture by assuming that $b>1$. In other words, firm 3 is a high variable cost firm. It follows that after a merger of firms 2 and 3 , production will be rationalized and the high cost operations will be shut down (or redesigned to have the low cost technology). To make matters as simple as possible we will now assume that there are no fixed costs $(f=0)$.

## 394 Contractual Relations Between Firms

Once again we assume a Cournot framework. The outputs and profits of the three firms prior to the merger are:

$$
\begin{equation*}
q_{1}^{C}=q_{2}^{C}=\frac{90+30 b}{4} ; q_{3}^{C}=\frac{210-90 b}{4} \text { and } \pi_{1}^{C}=\pi_{2}^{C}=\frac{(90+30 b)^{2}}{16} ; \pi_{3}^{C}=\frac{(210-90 b)^{2}}{16} \tag{16.14}
\end{equation*}
$$

The equilibrium pre-merger price is ${ }^{8} P^{C}=\frac{210+30 b}{4}$. Total output is $Q=\frac{390-30 b}{4}$ with each of the low cost firms, 1 and 2, producing a greater amount than their high cost rival, firm 3.

Now, as before, suppose that firms 2 and 3 merge. Since for any $b>1$, it is always more expensive to produce a unit of output at firm 3 than it is at firm 2, all production will be transferred to firm 2's technology. The result is that the market now contains two identical firms, 1 and 2, each with marginal costs of $\$ 30$. Accordingly, in the post-merger industry, each firm produces 40 units, the product price is $\$ 70$ and each firm earns $\$ 1,600$.

Is this a profitable merger? For the merger to increase aggregate profit of the merged firms it must be the case that

$$
\begin{equation*}
1600-\left(\frac{(90+30 b)^{2}}{16}+\frac{(210-90 b)^{2}}{16}\right)>0 \tag{16.15}
\end{equation*}
$$

You can check that this simplifies to

$$
\begin{equation*}
\frac{25}{2}(7-3 b)(15 b-19)>0 \tag{16.16}
\end{equation*}
$$

The first bracketed term in equation (16.16) has to be positive for firm 3 to have been in the market in the first place (see footnote 8 ). So the merger is profitable provided that the second bracket is also positive, which requires that $b>19 / 15$. In other words, a merger between a high-cost and a low-cost firm will be profitable provided that the cost disadvantage of the high-cost firm prior to the merger is "large enough." In the case at hand, large enough means that firm 3's unit cost is at least 25 percent greater than firm 2's unit cost. However, as we have already demonstrated, whether the merger is profitable or not, the price rises and consumers are made worse off.

Together, our analysis of a merger that generates fixed cost savings and one that generates variable cost savings makes clear that mergers can be profitable when the cost savings are great enough. However, there is no guarantee that consumers gain from such a merger. Admittedly, the merger removes a relatively inefficient firm technology but it also reduces competitive pressures between the remaining firms. Farrell and Shapiro (1990) demonstrate that in the Cournot setting used here, the cost savings necessary to generate a gain for consumers are much larger than those needed simply to make the merger profitable. In turn, this suggests that we should be skeptical of cost savings as a justification of the benefits to consumers of horizontal mergers.

[^5]Research by both Lichtenberg and Siegel (1992) and Maksimovic and Phillips (2001) finds that merger related productivity gains and therefore marginal cost savings, while real, are typically no more than 1 to 2 percent. Salinger (2005) expresses even more doubt that fixed cost savings are substantial. Beyond all this it is also worth noting that even with cost savings, part of our initial paradox still remains since large profit gains continue to accrue to the firms that do not merge. Why should a firm incur the headaches of merging if it can enjoy many of the same benefits by free-riding on other mergers? ${ }^{9}$

Return to the market for carpet-cleaning services in Dirtville, now described by the demand function $P=180-Q$. Suppose that there are currently 3 firms that clean carpets in the area. The unit cost of cleaning a carpet is constant and equal to $\$ 30$ for 2 firms and is $\$ 30 b$ for the third firm, where $b \geq 1$. In addition, all firms have fixed overhead costs of $\$ 900$. Firms in this industry compete in quantities.
a. What is the Cournot-Nash equilibrium price and what are the outputs and profits of each firm? What is the upper limit on $b$ for the third firm to be able to survive?
b. Now suppose that a low-cost firm merges with the high-cost firm. In doing so, the fixed costs of the merged firm become $\$ 900 a$ with $1 \leq a \leq 2$. What is the post-merger equilibrium price? What are the outputs of the non-merged and the merged firms?
c. Derive a relationship between $a$ and $b$ that is necessary to guarantee that the profit earned by the merged firm is sufficient to compensate all the shareholders/owners who owned the two original firms and earned profit from them in the pre-merger market game. Graph this relationship and comment on it.

### 16.3 THE MERGED FIRM AS A STACKELBERG LEADER

If cost efficiencies are not a likely way to resolve the merger paradox, then perhaps a resolution can be found in some other change that gives the merged firm an advantage. One possibility is that merged firms become Stackelberg leaders in the post-merger market. ${ }^{10}$ Recall from our discussion in section 11.1 that the source of a Stackelberg leader firm's advantage is its ability to commit to an output before output decisions are taken by the follower firms. This permits a leader to choose an output that takes into account the reactions of the followers.

Let us assume that a merged firm acquires a leadership role and see whether this assumption can help resolve the merger paradox. Certainly, such a role seems plausible. After all, the new firm has a combined capacity twice that of any of its non-merged rivals, and so might well be able to act as a Stackelberg leader. Will this be enough to make a merger profitable? If so, what will be the response of other firms? Will they also have an incentive to merge? If they do, will their merging undo the profitability of the first merger and thereby, if firms are foresighted, discourage them from merging in the first place?

[^6]Suppose that demand is of the usual linear form: $P=A-B Q$. There are $N+1$ firms in the industry and each of the $N+1$ firms has a constant marginal cost of $c$. Again from section 9.4 we know that the equilibrium is described by the following equations:

$$
\begin{equation*}
q_{i}=\frac{A-c}{(N+2) B} \Rightarrow Q=\frac{(N+1)(A-c)}{(N+2) B} \text { and } P=\frac{A+(N+1) c}{N+2} \tag{16.17}
\end{equation*}
$$

The profit of each firm, $(P-c) q_{i}$ is therefore:

$$
\begin{equation*}
\pi_{i}=\frac{(A-c)^{2}}{B(N+2)^{2}} \tag{16.18}
\end{equation*}
$$

Suppose now that two of these firms merge and, as a result, become a Stackelberg leader. There will then be $F$, which is equal to $N-1$, follower firms and one leader firm so that we now have $N$ firms in total. Of course, the Stackelberg leader is able to choose its output first in a two-stage game. In stage one, the leader chooses its output $Q^{L}$. In the second stage, the follower firms independently choose their outputs in response to that chosen by the leader.

To find the equilibrium, we work through the game backwards. Accordingly, we consider the second stage of the game in which the follower nonmerged firms make their output decisions in response to the output choice $Q^{L}$ of the leader or merged firm. We use the notation $Q_{F-f}$ to denote the aggregate output of the follower firms other than $f$, and denote the output of follower firm $f$ by $q_{f}$. Aggregate output of all firms is $Q=Q^{L}+Q_{F-f}+q_{f}$. Moreover, the residual demand for firm $f$, which is the demand left after taking into account the outputs of the leader and the followers other than $\operatorname{firm} f$ is:

$$
\begin{equation*}
P=\left[A-B\left(Q^{L}+Q_{F-f}\right)\right]-B q_{f} \tag{16.19}
\end{equation*}
$$

Marginal revenue for firm $f$ is, therefore,

$$
\begin{equation*}
M R_{f}=\left[A-B\left(Q^{L}+Q_{F-f}\right)\right]-2 B q_{f} . \tag{16.20}
\end{equation*}
$$

Equating this with marginal cost gives the best response function for firm $f$ :

$$
\begin{equation*}
A-2 B q_{f}-B Q^{L}-B Q_{F-f}=c \Rightarrow q_{f}^{*}=\frac{A-c}{2 B}-\frac{Q^{L}}{2}-\frac{Q_{F-f}}{2} \tag{16.21}
\end{equation*}
$$

Equation (16.21) is the best response of a follower firm to both the output of the leader and the output of all the other follower firms. Since all follower firms are identical, symmetry demands that in equilibrium the output of each of the follower firms must be identical. The group of followers excluding the firm $f$ has $F-1=N-2$ firms. Therefore, $Q_{F-f}^{*}=(N-2) q_{f}^{*}$. Substituting this into equation (16.21) and simplifying gives the optimal output for each non-merged follower firm as a function of the aggregate output of the leader group of merged firms:

$$
\begin{equation*}
q_{f}^{*}=\frac{A-c}{B N}-\frac{Q^{L}}{N} \tag{16.22}
\end{equation*}
$$

The aggregate output of all followers as a function of the output of the leader is then

$$
\begin{equation*}
Q^{F}=(N-1) q_{f}^{*}=\frac{(N-1)(A-c)}{B N}-\frac{(N-1) Q^{L}}{N} \tag{16.23}
\end{equation*}
$$

We can use the same basic technique to determine the output for the leader firm in stage one of the game. The residual inverse demand function for the leader firm is dependent upon the output of all the other firms, which is given in equation (16.23). So, the demand function facing leader firm $l$ :

$$
\begin{align*}
P=A-B\left(Q^{F}+Q^{L}\right) & =A-B\left[\frac{(N-1)(A-c)}{B N}-\frac{(N-1) Q^{L}}{N}\right]-B Q^{L}  \tag{16.24}\\
& =A-\frac{(N-1)(A-c)}{N}-\frac{B}{N} Q^{L}
\end{align*}
$$

Its associated marginal revenue function is

$$
\begin{equation*}
M R_{L}=A-\frac{(N-1)(A-c)}{N}-\frac{2 B}{N} Q^{L} \tag{16.25}
\end{equation*}
$$

Equating this marginal revenue with marginal cost allows us to solve for the leader firm's optimal output:

$$
\begin{equation*}
M R_{l}=c \Rightarrow Q^{L}=\frac{A-c}{2 B} \tag{16.26}
\end{equation*}
$$

You will by now recognize that the output level in equation (16.26) is just the output level chosen by a uniform-pricing monopolist. This is, of course, a standard result for a single leader model with linear demand and constant costs. In turn, this implies the following industry equilibrium values:

$$
\begin{align*}
& q_{f}^{*}=\frac{A-c}{2 B N} ; Q^{F}=\frac{(N-1)(A-c)}{2 B N} ; Q=Q^{L}+Q^{F}=\frac{(2 N-1)(A-c)}{2 B N} ; \text { and } \\
& P=\frac{A+(2 N-1) c}{2 N} \tag{16.27}
\end{align*}
$$

Profits for the leader and for each follower firm are then:

$$
\begin{equation*}
\pi^{L}=\frac{(A-c)^{2}}{4 B N} \text { and } \pi^{F}=\frac{(A-c)^{2}}{4 B N^{2}} \tag{16.28}
\end{equation*}
$$

A comparison of equation (16.28) with (16.18) reveals that for any industry initially comprised of three or more firms and characterized by symmetric Cournot competition, a twofirm merger that creates a Stackelberg leader will be profitable. This seems to resolve the merger paradox. However, equations (16.28) and (16.18) also show that the unmerged firms
who have become followers are definitely worse off as a result of the merger. We may therefore expect some response from these firms.

Furthermore, if we compare the market price and output in (16.17) with that in (16.27), we find that while the merger has raised the profit of the merging parties, it also has lowered price. Hence, the merger is good for consumers. Hence, we may have replaced one paradox with another. We now have a model in which a merger is profitable, but that model also removes a principal reason why the antitrust authorities should object to such a merger.

However, we also need to consider the response of other firms to the merger. Since leadership confers additional profit, they, too, will have an incentive to merge and try to become a leader. This raises the question as to what happens if there is a second or third two-firm merger. Daughety's (1990) model answers this question by assuming that there can be more than one leader firm and that merging is the ticket to entry into the club of such leaders. That is, imagine a market that may be divided into two groups of firms: followers and leaders. The first of these groups acts just as the followers did in the preceding analysis. They compete as Cournot rivals over the demand remaining after the leaders make their output decisions. The group of leaders understands this reaction. They compete as Cournot rivals with the knowledge that they act first and the followers will take their production decisions as given.

To analyze this two-stage competition, we can use the model derived above. In particular, instead of assuming $N$ firms with one leader and $N-1$ followers, let us assume $N$ firms with $L$ leaders and $N-L=F$ followers. Since followers simply take their cue from the total leader output $Q^{L}$ regardless of whether it is produced by one firm or many, equation (16.21) still describes the best response of the typical follower firm. Since there are $N-L$ such firms, a little algebra quickly reveals that total follower output $Q^{F}$ is then:

$$
\begin{equation*}
Q^{F}=(N-L) q_{f}^{*}=\frac{(N-L)(A-c)}{B(N-L+1)}-\frac{(N-L) Q^{L}}{(N-L+1)} \tag{16.29}
\end{equation*}
$$

Let us denote the output of any one leader firm as $q^{l}$ and that of all the leaders other than firm $l, Q_{L-l}$. The residual demand function for firm $l$ is then:

$$
\begin{equation*}
P=\left[A-B\left(Q^{F}+Q_{L-l}\right)\right]-B q_{l} \tag{16.30}
\end{equation*}
$$

Substituting for total follower output $Q^{F}$ from equation (16.29) and re-arranging yields the typical leader's demand:

$$
\begin{equation*}
P=\frac{A+(N-L) c-B Q_{L-l}}{(N-L+1)}-\frac{B}{(N-L+1)} q_{l} \tag{16.31}
\end{equation*}
$$

Hence, the associated marginal revenue function is

$$
\begin{equation*}
M R_{l}=\frac{A+(N-L) c-B Q_{L-l}}{(N-L+1)}-\frac{2 B}{(N-L+1)} q_{l} \tag{16.32}
\end{equation*}
$$

Equating this marginal revenue with marginal cost gives the leader firm $l$ 's best output response to the output produced by all the other leader firms, $Q_{L-l}$ :

$$
\begin{equation*}
M R_{l}=\frac{A+(N-L) c-B Q_{L-l}}{(N-L+1)}-\frac{2 B}{(N-L+1)} q_{l}=c \Rightarrow q_{l}^{*}=\frac{A-c}{2 B}-\frac{Q_{L-l}}{2} \tag{16.33}
\end{equation*}
$$

Once again, we can take advantage of the fact that since all of the leader firms have the same costs they will each produce the same level of output in equilibrium. Because there are $L-1$ leaders other than firm $l$, this gives the symmetry condition $Q_{L-l}^{*}=(L-1) q_{i}^{*}$ which when substituted into equation (16.33) allows us to solve for the output chosen in stage one by each merged firm in the leader group:

$$
\begin{equation*}
q_{l}^{*}=\frac{A-c}{2 B}-\frac{(L-1)}{2} q_{l}^{*} \Rightarrow q_{l}^{*}=\frac{A-c}{B(L+1)} \text { and } Q^{L}=L q_{l}^{*}=\frac{L(A-c)}{B(L+1)} \tag{16.34}
\end{equation*}
$$

Substituting the value for $Q^{L}$ into equation (16.29) we can then find total follower output $Q^{F}$ and individual output for each follower $q_{f}^{*}=Q^{F} /(N-L)$. These are:

$$
\begin{equation*}
q_{f}^{*}=\frac{A-c}{B(L+1)(N-L+1)} \text { and } Q^{F}=\frac{(N-L)(A-c)}{B(L+1)(N-L+1)} \tag{16.35}
\end{equation*}
$$

Finally, summing $Q^{L}$ and $Q^{F}$ together yields total market output $Q$ and, via the demand curve, the equilibrium price $P$ :

$$
\begin{equation*}
Q^{T}=Q^{L}+Q^{F}=\frac{\left(N+N L-L^{2}\right)(A-c)}{B(L+1)(N-L+1)} \text { and } P=\frac{A+\left(N+N L-L^{2}\right) c}{(L+1)(N-L+1)} \tag{16.36}
\end{equation*}
$$

In turn, the price and output equations imply that in an industry comprised of $N$ firms in total $L$ of which are leaders, the price-cost margin $(P-c)$ and the profits for the typical leader firm $(P-c) q_{l}^{*}$ and typical follower firm $(P-c) q_{f}^{*}$ are:

$$
\begin{align*}
& P-c=\frac{A-c}{(L+1)(N-L+1)} ; \pi^{L}(N, L)=\frac{(A-c)^{2}}{B(L+1)^{2}(N-L+1)} ; \text { and }  \tag{16.37}\\
& \pi^{F}(N, L)=\frac{(A-c)^{2}}{B(L+1)^{2}(N-L+1)^{2}}
\end{align*}
$$

You can readily confirm that the profit values shown in equation (16.37) for the general case of $N$ total firms with $L$ leaders, yields the same profits as those given in equation (16.28) for the special case of $N$ firms and $L=1$ leader.

It is clear from the profit equations in (16.37) that the leader firms are more profitable than the nonmerged followers. Yet that is not the real issue facing two firms that are contemplating merger. The question is whether one more merger is profitable, given that there will then be one more leader, two fewer followers, and one less firm in total. This is why we have written the profit expressions as functions of $N$ and $L$. The point is that an additional merger creates two countervailing forces. On the one hand there are fewer firms in total, which ought to increase profits, but there are also more leaders, which ought to decrease the profits of the leaders. Which force is greater?

Suppose there is an additional merger of two followers, so that the newly merged firm and all other leaders earn profit given by equation (16.37) with $N$ replaced by $N-1$ and $L$ replaced by $L+1$ to give $\pi_{l}^{L}(N-1, L+1)$. For there to be an incentive to merge, this profit must exceed the combined profit earned by the two follower firms prior to the merger.

400 Contractual Relations Between Firms

Table 16.2 Profit effect of two follower firms merging to become a leader, given $N$ followers and $L$ eaders prior to the merger

| Original <br> number of <br> leaders $L$ | Original number of firms $N$ |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | :---: |

This latter profit is $2 \pi_{f}^{F}(N, L)$. So, the merger will be profitable if the following condition is satisfied:

$$
\begin{equation*}
\pi^{L}(N-1, L+1)=\frac{(A-c)^{2}}{B(L+2)^{2}(N-L-1)}>2 \pi^{F}(N, L)=\frac{2(A-c)^{2}}{B(L+1)^{2}(N-L+1)^{2}} \tag{16.38}
\end{equation*}
$$

This simplifies to the condition

$$
\begin{equation*}
(L+1)^{2}(N-L+1)^{2}-2(L+2)^{2}(N-L-1)>0 \tag{16.39}
\end{equation*}
$$

Note that this condition does not include the demand parameters $A$ and $B$ or the marginal cost $c$. In other words, the profitability or otherwise of this type of merger depends only on the number of leaders and followers, not on the precise demand and cost conditions.

As turns out, the condition in (16.39) is always met. This is shown in Table 16.2 where we have calculated the left-hand side of equation (16.39) for any two-firm merger for a range of values of $N$ and $L$. In other words, starting from any configuration of leaders and followers, an additional two follower firms always wish to merge.

This result is encouraging. It says that our model offers one way to resolve the merger paradox. A merger raises the profit of the two merging firms by allowing them to take a position as one of, perhaps several, industry leaders. Moreover, the fact that such a merger is always profitable also helps us to understand better the domino effect so often observed within an industry. Once one firm merges and becomes a leader, the remaining firms will wish to do the same rather than watch their output and their profits be squeezed.

Return again to the town of Dirtville where the inverse demand for carpet-cleaning services is described by $P=130-Q$. Once again assume that there are 20 identical firms that clean carpets in the area, and the unit cost of cleaning a carpet is constant and equal to $\$ 30$. Firms in this industry compete in quantities.
a. Show that in a Cournot equilibrium the aggregate number of carpets cleaned is $Q=95.24$. What is the equilibrium price?
b. Suppose that five two-firm mergers occur, that these five merged firms become leader firms, and the remaining ten nonmerged firms are followers. Now there are 15 firms in the industry. Work through the model just described and show that in the two-stage game a leader firm cleans 16.67 carpets and each follower firm cleans 1.51 carpets. Leadership certainly has its benefits! Show that the total industry output in this case will be $Q=98.45$. What is the equilibrium price now?
c. If after the five two-firm mergers took place there were no leadership advantage conferred to the merged firms, then we would have 15 firms competing like Cournot firms in the market. Show that in this case aggregate output is $Q=93.75$.

While the Daugherty model can resolve the merger paradox it does leave unanswered the question as to whether such mergers are in the public interest. Is there some point at which further mergers are harmful to consumers? The answer to this question can be most easily derived from the price-cost relation $P-c$ shown in equation (16.36). Since marginal cost $c$ is constant, any rise or fall in $P$ will be reflected in a rise or fall of $P-c$.

With $L$ leader merged firms and $N-L$ follower non-merged firms the price-cost differential is $\frac{A-c}{(L+1)(N-L+1)}$. An additional two-firm merger increases $L$ to $L+1$ and decreases $N$ to $N-1$, so that the price-cost margin is now $\frac{A-c}{(L+2)(N-L-1)}$. So for this additional merger to benefit consumers it must be the case that:

$$
\begin{align*}
& \frac{A-c}{(L+2)(N-L-1)}<\frac{A-c}{(L+1)(N-L+1)}  \tag{16.40}\\
& \Rightarrow(L+1)(N-L+1)<(L+2)(N-L-1) \Rightarrow N-3(L+1)>0
\end{align*}
$$

What this tells us is that an additional two-firm merger benefits consumers only if $N>3(L+1)$ or, equivalently, $L<N / 3-1$. In other words, a two-firm merger that increases the number of leaders benefits consumers only if the current group of leaders contains fewer than a third of the total number of firms in the industry. We know from Table 16.2 that a two-firm merger that creates a leader will always be profitable. Yet, as also just shown, such a merger will be harmful to consumers once the leader group includes one-third or more of the industry's firms. In other words, some mergers are bad-at least for consumers. Accordingly, we now have a model that both resolves the merger paradox and explains why the antitrust authorities are correct to worry about anticompetitive mergers.

For example, return to Practice Problem 16.3 in which we had five leader firms and ten follower firms cleaning carpets in Dirtville. In that scenario we know that the equilibrium

## Reality Checkpoint

## It's a Gusher! Merger Mania in the Oil Industry

In August of 1998, British Petroleum or BP announced plans to merge with Amoco another large oil firm although not quite as large as BP. The price tag was $\$ 48.2$ billion making it, at the time, the biggest industrial merger ever. The new BP-Amoco would control more oil and gas production within North America than any other firm. It would also be the thirdlargest publicly traded oil firm in the world. (The largest firm of all, Saudi Aramco, is not publicly traded.)

Reaction from the rest of the oil industry came swiftly. Within a year, Exxon and Mobil merged in a deal worth $\$ 73.7$ billion to become the largest publicly traded firm on earth. That was quickly followed by the merger of Phillips Petroleum and Conoco. Almost simultaneously, Paris-based Total acquired both PetroFina and Elf to create TotalFinaElf. Soon after, Chevron acquired Texaco for $\$ 36$ billion. BP then went a step further and acquired Arco for $\$ 27$ billion. The oil merger wave subsided with the economic decline of 2000-1 but even then, did not die altogether. Chevron acquired Unocal in 2005.

This wave of merger activity concentrated oil and gas refining and marketing into the hands of a noticeably smaller number of firms relative to the situation prior to BP's purchase of Amoco. The BP-Amoco merger was then the catalyst for a major wave of mergers and consolidations. In turn, this suggests that a common motive must be behind all these mergers.

Yet whether this common factor was the naked pursuit of market power or simply the profit-maximizing response of firms to similar problems is difficult to say. The mergers were taken at a time when energy prices were quite low. Oil, for example, was selling at less than $\$ 12$ per barrel in 1998. Indeed, this low price -and the low energy company profits that went with it-is probably the reason that none of the mergers was seriously challenged by antitrust authorities. However, oil prices and profits have risen dramatically since that time. This could reflect the demand growth driven by a rising world economy coupled with improved cost efficiency that the mergers generated. However, it could also reflect the exploitation of newly enhanced market power. In this connection, a recent study by the Government Accounting Office (GAO) found that increased concentration could account for only a few cents of the large rise in wholesale gasoline prices. The rest appeared to be demand and cost pressures. If this view is correct, lower oil and gas prices are only likely if conservation measures reduce energy demand.

Sources: Jim Wells, "Energy Markets: Factors Contributing to Higher Gasoline Prices," Statement of Director of Natural Resources and Environment, General Accounting Office, to U. S. Senate Judiciary Committee, February 1, 2006; and B. Bahree, C. Cooper, and S. Liesman, "BP to Buy Amoco in Biggest Industrial Merger Ever," Wall Street Journal, August 12, 1998, p. A1.
price for cleaning a carpet is $\$ 31.55$. Now suppose that two additional firms merge to join the leadership group. We then have a market structure of six leaders and eight followers. In this case, the equilibrium price for cleaning a carpet is $\$ 31.60$. This merger harms the consumers in Dirtville.

Daughety's (1990) model solves the merger paradox and gives rise to a merger wave by assuming an asymmetry between newly merged firms and their remaining unmerged rivals. The former gain membership in the club of industry leaders. However, this is a rather strong assumption. While some mergers may create corporate giants with an ability to commit to large production levels, it is far from obvious that every two-firm merger should have this leadership role regardless of which two firms are joined and irrespective of the number of
leaders already present. In principle, Daughety's (1990) model implies that in an industry of ten firms there could be, say, eight leaders. It seems odd though to imagine a configuration with so many leaders and so few followers. Moreover, it leaves unanswered the question as to what happens if two leaders merge. Does this merger create a super-leader?

It is also worth noting that while production is sequential in Daughety's (1990) model, merging is not. While leader firms choose production first, it is not accurate to describe the decision to merge in a sequential way. The model simply says that for any market configuration, if a two-firm merger creates an industry leader, all follower firm pairs will wish to merge as well. One pair does not merge only after it sees another pair merge. Instead, at any single point in time, merging is a dominant strategy and, absent any antitrust intervention, all follower firms will pursue it. Again, this is not because of any new cost savings or product development. It is simply because merging confers leadership status. Thus, Daughety's (1990) model does not give rise to the sporadic merger waves that we often see as much as it suggests an ever-present tendency for the industry to become more concentrated.

To capture the idea that merger decisions may be explicitly sequential, i.e., that the decision of one firm pair to merge is a catalyst for another pair to do the same, a number of papers including Nilssen and Sørgaard (1998), Fauli-Oller (2000) and Salvo (2006) have recently presented models in which a sudden change in cost or product qualities give rise to merger opportunities that are only profitable if other mergers also occur. It is difficult for this to happen in a simultaneous game because each potential merger pair cannot be sure if others will also merge. However, in a sequential game, some firms get to make their merger decision knowing for certain that others have already merged. This greatly enhances the likelihood of a successful merger.

We illustrate the sequential merger model with a simplification of the Fauli-Oller (2000) model. Consider a four-firm industry characterized by Cournot competition. Initially, all of these firms are high-cost firms with constant unit cost $c^{h}=c$. Suppose that two firms have had a technical breakthrough that allows them to become low-cost firms with low constant unit cost $c^{l}=0$. Industry demand is described by: $P=A-Q$.

Using the Cournot model in section 9.4, it is easy to show that the initial, post-innovation equilibrium is described as follows:

$$
\begin{equation*}
q_{i}^{h}=\frac{A-3 c}{5} ; i=1,2 \quad q_{j}^{l}=\frac{A+2 c}{5} ; j=1,2 \quad \Rightarrow \quad Q=\frac{4 A-2 c}{5} ; P=\frac{A+2 c}{5} \tag{16.41}
\end{equation*}
$$

As a result, each firm earns a profit $\pi$ of

$$
\begin{equation*}
\pi_{i}^{h}=\left(\frac{A-3 c}{5}\right)^{2} ; i=1,2 \quad \text { and } \quad \pi_{j}^{l}=\left(\frac{A+2 c}{5}\right)^{2} ; j=1,2 \tag{16.42}
\end{equation*}
$$

Now consider two possible mergers. In each merger, a low-cost firm buys a high-cost firm and the two then operate as a single low-cost producer. The merger permits the transfer of production from the high-cost plant to the low-cost one. If both of these mergers happen, there will be just two low-cost firms and the industry equilibrium will have the following features:

$$
\begin{equation*}
q_{j}^{l}=\frac{A}{3} ; j=1,2 \Rightarrow Q=\frac{2 A}{3} ; P=\frac{A}{3} ; \quad \text { and } \quad \pi_{j}^{l}=\left(\frac{A}{3}\right)^{2} ; j=1,2 \tag{16.43}
\end{equation*}
$$

## 404 Contractual Relations Between Firms

Observe that we must have $A>3 c$ for the pre-merger equilibrium to involve any positive output for the high-cost firms. In turn, this means that the expression for $\pi_{j}^{l}$ in (16.43) will always exceed the sum of $\pi_{i}^{h}$ and $\pi_{j}^{l}$ from (16.42). That is, a merger between one pair of high- and low-cost firms will be profitable so long as the other pair also merges.

For example, suppose that $A=100$ and $c=10$. Then in the pre-merger equilibrium, highcost firms each earn $\$ 196$ in profit and low-cost firms each earn $\$ 576$ in profit. So, the premerger profit of any low-cost and high-cost pair is $\$ 772$. If both such pairs merge, however, the profit of the merged firm rises to $\$ 1,111.11$. Clearly, this is the preferable outcome from the viewpoint of the two firms.

A potential problem is that one merger will be unprofitable just as the merger paradox suggests. If only one low-cost and high-cost pair merges, the new equilibrium will be:

$$
\begin{equation*}
q_{1}^{h}=\frac{A-3 c}{4} ; q_{j}^{l}=\frac{A+c}{4} ; j=1,2 \Rightarrow Q=\frac{3 A-c}{4} ; P=\frac{A+c}{4} \tag{16.44}
\end{equation*}
$$

The profit of the remaining high-cost firm and of each of the two low-cost firms, respectively, will be:

$$
\begin{equation*}
\pi_{i}^{h}=\left(\frac{A-3 c}{4}\right)^{2} ; i=1,2 \quad \text { and } \quad \pi_{j}^{l}=\left(\frac{A+c}{4}\right)^{2} ; j=1,2 \tag{16.45}
\end{equation*}
$$

Thus, in our numerical example, the remaining high-cost firm will benefit from the price increase the merger causes and see its profit rise to $\$ 306.25$. The same is true for the unmerged low-cost firm whose profit will rise to $\$ 756.25$. The merged firm will also now earn $\$ 756.25$ as it transfers production from the acquired high-cost plants to the more efficient low-cost ones. However, this is less than the $\$ 772$ earned as two separate companies in the pre-merger equilibrium. Therefore, no one pair has an incentive to merge on its own. As a result, it seems difficult to reach the two-merger equilibrium outcome even though this would raise profits for all involved.

Let us now introduce a sequential structure to the game, where firms do not make the merger decision simultaneously but, instead, sequentially. Thus, the second merger pair gets to make its decision after the first pair. Moreover, the first pair knows this. The rules of the game-in this case, simultaneous versus sequential play-matter a lot for the outcome.

Consider again the outcome when only one firm-pair merges from equation (16.45). In our numerical example, this results in the remaining high-cost firm earning $\$ 306.25$ in profit while the merged firm and its low-cost rival each earn $\$ 756.25$. Knowing that one merger has already taken place, the second merger pair now has a choice of either staying in this equilibrium or merging themselves in which case the market outcome would have two, lowcost firms as described by equation (16.43). If they merge, this second merged new firm will earn a profit of $\$ 1,111.11$, which is an increase over the $\$ 1,062.50$ in combined profit that the two firms will earn if they do not merge. Conditional on the first merger taking place then, the second merger is profitable. In effect, the sequential nature of the game allows the first pair to commit credibly to merging. In turn, that means that the second merger pair does not have to worry that in merging they may be acting alone.

The first pair of merging firms can work through the foregoing logic as well as we can. These firms will therefore understand that their merger will also not be the only one but, instead, be followed by another in which case they too will see their combined profits rise
to $\$ 1,111.11$. As a result, we can expect a merger wave in which first one pair merges and then the second pair merges. Suppose that in this wave, mergers are motivated by a lowcost firm buying a high-cost firm. Since the profit foregone by a high-cost firm is $\$ 306.25$ and that foregone by a low-cost firm is $\$ 756.25$ when each is part of the second merger, and since the order of mergers is arbitrary, there will be a sequence of mergers in which the acquisition price is somewhere between $\$ 306.25$ and $\$ 354.86$ ( $=\$ 1,111.11-\$ 756.25)$, and the industry will end up with just two, low-cost firms. The merger wave is not, however, desirable for consumers. The industry price rises from $\$ 24$ to $\$ 33.33$.

The foregoing story is not limited to just two mergers or to models of Cournot competition. Once cost asymmetries or product quality differences are introduced, we can construct sequential merger models that lead to merger waves for a large number of firms in a variety of settings, e.g., Nilssen and Sørgaard (1998) and Salvo (2006), and these mergers are also anticompetitive. This approach offers another resolution to the merger paradox not simply because it demonstrates why mergers may happen but, in addition, why they often happen in sequential waves. As with Daughety's (1990) model, these models also justify concern over the impact that mergers may have on consumer prices.

### 16.4 HORIZONTAL MERGERS AND PRODUCT DIFFERENTIATION

Our analysis of mergers has so far been set in the Cournot framework of identical products and quantity competition. However, many firms expend considerable effort differentiating their products and this differentiation gives them some latitude in setting their price. Accordingly, we also need to consider the incentives for and the impact of mergers in industries in which firms produce and market differentiated products.

It is particularly important to explore the merger phenomenon in differentiated product markets because often firms are price setters in such markets and the nature of competition is different with price competition than with quantity competition. In quantity competition firms' best response functions are downward sloping, i.e., quantities are strategic substitutes. Thus, when merging occurs, the non-merged firms want to increase their outputs in response to the lower output produced by the merger. This response undermines the effectiveness of the merger. By contrast, with price competition best response functions are upward sloping: prices are strategic complements. A merger leading to an increase in the merged firms' price(s) will encourage the non-merged firms also to increase their prices, potentially strengthening the effectiveness of the merger.

We develop this intuition more explicitly using two different approaches to product differentiation. The first approach is to extend our standard linear demand representation of consumer preferences to incorporate product differentiation. The second is to adopt the spatial model of horizontal differentiation, which we first introduced in Chapter 4 and then revisited in Chapter 10. ${ }^{11}$

[^7]
### 16.4.1 Bertrand Competition and Merger with Linear Demand Systems

Suppose that there are three firms in the market, each producing a single differentiated product. ${ }^{12}$ Inverse demand for each of the three products is assumed to be given by:

$$
\begin{align*}
& p_{1}=A-B q_{1}-s\left(q_{2}+q_{3}\right) \\
& p_{2}=A-B q_{2}-s\left(q_{1}+q_{3}\right) \quad(s \in[0, B))  \tag{16.46}\\
& p_{3}=A-B q_{3}-s\left(q_{1}+q_{2}\right)
\end{align*}
$$

In these inverse demand functions the parameter $s$, where $0 \leq s \leq B$, measures how similar the three products are to each other. If $s=0$ the products are totally differentiated. In this case, each firm is effectively a monopolist. By contrast, as $s$ approaches $B$ the three products become increasingly identical, moving us closer to the homogeneous product case. We will also assume that the three firms have identical marginal costs of $c$ per unit. Finally, assume that the three firms are Bertrand competitors, i.e., they compete in prices and set their prices simultaneously.
We show in Appendix A to this chapter that when these firms compete they each set a price of $p_{n m}^{*}=\frac{A(B-s)+c(B+s)}{2 B}$ and each sell quantity $q_{n m}^{*}=\frac{(A-c)(B+s)}{2 B(B+2 s)}$. Profit of each firm is then

$$
\begin{equation*}
\pi_{n m}^{*}=\frac{(A-c)^{2}(B-s)(B+s)}{4 B^{2}(B+2 s)} \tag{16.47}
\end{equation*}
$$

Now suppose that firms 1 and 2 merge but that the merged and non-merged firms continue to set their prices simultaneously. The two previously independent, single-product firms are now product divisions of a two-product merged firm, coordinating their prices to maximize the joint profit of the two divisions. The result is that the merged firm sets its product prices to $p_{1}^{m}=p_{2}^{m}=\frac{A(2 B+3 s)(B-s)+c(2 B+s)(B+s)}{2\left(2 B^{2}+2 B s-s^{2}\right)}$ while the remaining non-merged firm 3 sets its product price as $p_{3}^{n m}=\frac{A(B+s)(B-s)+c B(B+2 s)}{\left(2 B^{2}+2 B s-s^{2}\right)}$.

It is straightforward to confirm that the merger increase the prices of all three products, as we might have expected since the merger reduces competitive pressures in the market. However, there remains the question of the merger's profitability. The profits of each product division of the merged firm, and of the independent non-merged firm are:

$$
\begin{equation*}
\pi_{1}^{m}=\pi_{2}^{m}=\frac{(A-c)^{2} B(B-s)(2 B+3 s)^{2}}{4(B+2 s)\left(2 B^{2}+2 B s-s^{2}\right)^{2}} ; \pi_{3}^{m}=\frac{(A-c)^{2}(B-s)(B+s)^{3}}{(B+2 s)\left(2 B^{2}+2 B s-s^{2}\right)^{2}} \tag{16.48}
\end{equation*}
$$

In comparing equations (16.47) and (16.48) we can simplify matters by normalizing $A-c=1$ and $B=1$, so that profits are functions solely of the degree of product differentiation $s$. It is then easy to confirm that this two-firm merger is profitable for the merged firm

[^8]and for the non-merged firm. More generally, Davidson and Deneckere (1986) show that in a market containing $N$ firms any merger of $M \geq 2$ firms is profitable for the merged firms and for the non-merged firms. This simple framework of price setting in a product differentiated market avoids the merger paradox, suggesting that mergers are both profitable and of potential concern to antitrust authorities unless accompanied by cost efficiencies.

### 16.4.2 Mergers in a Spatial Market

In the spatial model, a merger between two firms may well bring increased profit for reasons similar to those in the previous section. Although merging means that the firms lose their separate identity, they do not lose the ownership or control of the product varieties they can offer. For example, the merger of two major banks, Bank of America and Fleet Bank, results in a single new corporate entity. Yet it does not require that the new firm give up any of the locations at which either Bank of America or Fleet currently operate-or that it lose control over the choice of moving some of those locations. Similarly, the acquisition some years ago of American Motors by Chrysler did not mean that the Jeep product line disappeared.

When we consider a firm's product lines, there is a second source of potential profit increase. The merged firms can now coordinate not just the prices but also the design of their product line, or in the context of the spatial model, their location choices. Chrysler can redesign the Jeep line to fit better in its overall range of models. Similarly, Bank of America and Fleet can change the locations of their branches in those areas where each formerly operated an outlet quite close to the other.

To investigate the impact of a merger in the spatial model we begin by recalling the basic setup of the model. ${ }^{13}$ There is a group of consumers who are uniformly distributed over a linear market of length $L$. Again, we can think of this as Main Street in Littlesville. However, one small problem with the Main Street analogy is that outlets at either end of the market can only reach consumers on one side. This restriction introduces an asymmetry in the model, which we would like to avoid. To make the product differentiated market symmetric we can bend the ends of the line around until they touch each other, and replace our straight line of length $L$ with a circle of circumference $L$. If we use the spatial model to represent departure times in the differentiated airline market, the circle represents the 24 hours of the day about which consumers differ in terms of their most preferred time of departure. In all other respects, the spatial model remains as before.

Each consumer has an "address" indicating her location on the circle and, hence, her most preferred product type. Each consumer is also willing to buy at most one unit of a particular good. The consumer's reservation price for her most preferred good is denoted by $V$. Different varieties of the good are offered by the firms that are also located on Main Streetor, more appropriately, Main Circle..$^{14}$ A consumer buys from the firm that offers the product to her at the lowest price, taking into account the costs of transporting the good from the firm's address to the consumer's. We assume that these transport costs are linear in distance. If the distance between a firm and a consumer is $d$, the transport costs from the firm to the consumer is $t d$, i.e., $t$ is the transport cost per unit distance. Recall that in the nongeographic interpretation of the model, transport costs become the consumer's valuation of

[^9]

Figure 16.1 Product differentiation: no price discrimination
the loss of utility incurred by consuming a product with characteristics that are not the consumer's most preferred characteristics.

Suppose that there are five firms selling to a group of $N$ consumers who are distributed evenly around the circle of circumference $L$. A firm is differentiated only by its location on the circle, and we assume that the distance between any two neighboring firms is the same and equal to $L / 5$. Each firm has identical costs given by $C(q)=F+c q$, where $F$ is fixed cost and $c$ is (constant) marginal cost. In contrast to our earlier merger analysis, we do not set $F$, the fixed cost, equal to zero, but instead set unit cost $c=0$. This simplifies the analysis without losing any generality. What it does do is make it easy to talk about the pricecost margin, which is now just price, denoted by $m$ for mill price. ${ }^{15}$

## No price discrimination

We start by considering the case in which firms do not engage in price discrimination. This means that each firm sets a single mill price $m$ that consumers pay at the firm's store or mill location. The consumer then pays the fee for transporting the product back to her location. The full price paid by a consumer who buys from firm $i$ is $m_{i}+t d_{i}$, where $m_{i}$ is firm $i$ 's mill price and $t d_{i}$ is the consumer's transport cost (or the utility lost by this consumer in buying a product that is not "ideal"). Since marginal cost is zero, the net revenue or profit margin earned by firm $i$ on every such sale is $m_{i}$. Consumers buy from the firm offering the product at the lowest full price. As a result, for any set of mill prices across our five hypothetical firms ( $m_{1}, m_{2}, m_{3}, m_{4}, m_{5}$ ) the market is divided between the firms as illustrated in Figure 16.1. The dotted lines indicate the market division between the firms. Firm 1, for example, supplies all consumers in the region $\left(r_{15}, r_{12}\right)$.

When the firms set their prices noncooperatively and the maximum willingness to pay $V$ is relatively large, the market is completely covered. That is, every consumer buys from some firm. Hence, the marginal consumer for any firm is the one who is just indifferent between

[^10]

Figure 16.2 Price equilibrium without a merger
buying from that firm and buying from one of the firm's neighbors. ${ }^{16}$ We show in Appendix B to this chapter that in equilibrium the mill price set by each firm is $m_{i}^{*}=t L / 5$. At this price, the profit earned by each firm is

$$
\begin{equation*}
\pi_{i}^{*}=\frac{N t L^{2}}{25}-F \tag{16.49}
\end{equation*}
$$

The market outcome is illustrated in Figure 16.2 in which we have "flattened out" the circular market to simplify the geometry; that is, firm 1 is to the right of firm 5 and firm 5 is to the left of firm 1. In Figure 16.2, the vertical distance is the effective price-mill price plus transport cost-that each buyer pays. The sloped lines show that this price rises as for consumers who live farther from a firm.

Now consider a merger between some subset of these firms. The first point to note is that, taking store locations or product choice as given, such a merger will have no effect unless it is made between neighboring firms. A merger, for example, between firms 2 and 4 leaves prices and market shares unaffected. More generally, this suggests that a merger has no effect on the market outcome unless the market areas of the merging firms have a common boundary. The reason is straightforward. The merging firms hope to gain by softening price competition between them. This will happen only if, prior to the merger, they actually competed for some of the same consumers. For example, the merger of the two investment firms Dean Witter and Morgan Stanley was not for the most part regarded as anticompetitive because the two firms market their services to different, or non neighboring customers of households and businesses.

Mergers between neighboring firms, however, do alter the market outcome. Consider a merger between firms 2 and 3 . Suppose that after the merger, the firms do not change either the locations of their existing products or the number of products they offer. Acting now as a single corporate firm with stores in two locations, the merged firm has an incentive to set prices to maximize the joint profits of both products 2 and 3, while the remaining firms continue to price noncooperatively. Of course, firms 1, 4, and 5 also take account of the fact that the merger has taken place. Since firms 2 and 3 are now cooperating divisions of the merged firm they no longer compete for the consumers located between them and so have

[^11]
## 410 Contractual Relations Between Firms

an incentive to raise the prices of products 2 and $3 .{ }^{17}$ This will likely lead to the loss of some consumers, namely, those just on the boundaries identified by the points $r_{12}$ and $r_{34}$. But provided that the merged firm does not raise prices too much the loss of market share will be more than offset by the increased profit margins on their "captive" consumers-the consumers between the two merging firms. Moreover, the increased prices set by the merged firm will induce a similar increase in prices set by firms 1,4 , and 5 . Such a response reduces the loss of market share that the merged firm actually suffers making the price increase all the more profitable.

Again we show in Appendix B that the merger leads to a new equilibrium with the following prices:

$$
\begin{equation*}
m_{2}^{*}=m_{3}^{*}=\frac{19 t L}{60} ; m_{1}^{*}=m_{4}^{*}=\frac{14 t L}{60} ; m_{5}^{*}=\frac{13 t L}{60} \tag{16.50}
\end{equation*}
$$

Profits to each product are

$$
\begin{equation*}
\pi_{2}^{*}=\pi_{3}^{*}=\frac{361 N t L^{2}}{7,200}-F ; \pi_{1}^{*}=\pi_{4}^{*}=\frac{49 N t L^{2}}{900}-F ; \pi_{5}^{*}=\frac{169 N t L^{2}}{3,600}-F \tag{16.51}
\end{equation*}
$$

This equilibrium is illustrated in Figure 16.3. Comparison with equation (16.37) confirms that this merger is profitable for the merging firms.

The equilibrium we have identified is based upon the assumption that the merged firms leave their product lines unchanged after the merger. What do we expect to happen if we relax this assumption? It turns out that the answer to this question depends upon the precise nature of transport costs. Consider the product location choice facing the newly merged firm 2 and firm 3. The firm faces a trade-off. On the one hand, relocating products 2 and 3 nearer to products 1 and 4 , respectively gives the merged firm two advantages. First, it softens the competition between the merged firm's own two product lines, so that when the firm tries to reach out to customers near the boundary with a lower price there is less of a fear of simply "robbing Peter to pay Paul." Second, the move also makes it easier for the firm to steal some customers away from its true rivals, firms 1 and $4 .{ }^{18}$ On the other hand, relocating

[^12]

Figure 16.3 Price equilibrium after merger of firms 2 and 3
products 2 and 3 further from products 1 and 4 offers potential advantages. Admittedly this gives up market share to the rivals but such a move also softens price competition, leading to increased prices by all firms. In our example with linear transport costs it turns out that the merged firm will wish to relocate its products closer to the rivals 2 and 3. On the other hand, if transport costs were quadratic, of the form $t d_{i}^{2}$ the merged firms would actually want to relocate further from their rivals. ${ }^{19}$

A merger between two firms in our spatial market is clearly advantageous to the merging firms but is disadvantageous to consumers because a merger tends to raise prices throughout the industry. Both merged and unmerged firms enjoy greater profit and consumers obtain less surplus. There is a possible gain that could benefit consumers: when the merger leads to cost savings that permit lower prices. Remember that the two products, while not identical, are close substitutes. They might be, for example, low-sugar and high-sugar versions of a soft drink. We might expect there to be some cost complementarities in the production of these products. If so, then production of both goods by one firm will be cheaper than production of both by two separate firms. In short, we should not be surprised if in a product-differentiated market, production of many closely related product lines exhibits economies of scope. ${ }^{20}$

Scope economies provide a strong incentive to merge. The merger allows the new firm to operate as a multiproduct company and thereby exploit the cost-savings opportunities this generates. These savings may be reflected in a reduction in fixed costs. For example, the firms can combine their headquarters, research and development, marketing, accounting, and distribution operations. If in addition the merger leads to a reduction in variable costs of production, then this change will be reflected in lower prices. Moreover, even if scope economies are not present, it is still possible that one of the merging firms has a more effective purchasing division or a superior production technology that, following the merger, will be extended to its new partners. The greater are such cost synergies, the more likely it is that consumers will benefit from the merger.

## Price discrimination

Firms that operate in a spatial or product-differentiated setting clearly have some monopoly power. Yet if firms have monopoly power, we might expect them to use discriminatory pricing strategies to exploit this power. In particular, we might expect these firms to adopt

[^13]

Figure 16.4 Price equilibrium with price discrimination
some of the price discrimination strategies that we developed in earlier chapters. We now turn to the analysis of how price discrimination affects the incentives for and the impact of mergers in a product differentiated market.

Suppose that firms adopt first-degree or personalized discriminatory pricing policies (section 6.1, Chapter 6) but maintain at the same time all the remaining assumptions of our spatial model in the no-price-discrimination case. The noncooperative price equilibrium is then easy to identify. Remember that firms compete in prices for customers. Accordingly, they set the price as low as need be-at the margin-to attract customers, so long as that price covers their marginal cost. As a result, the equilibrium must be characterized by the following condition. Suppose that firm $i$ is the firm that can supply consumer location $s$ at the lowest unit cost, say $c+t s$ (the marginal production cost plus transport fee), and that firm $j$ is the firm that can supply this location at the next cheapest unit cost, $c+t s+e$, where $e$ is a measure of how much closer the consumer is to firm $i$ than it is to firm $j$. The Bertrand-Nash equilibrium price to consumer $s$ will be for firm $i$ to charge one cent or epsilon less than the cost of firm $j$ to supply consumer $s$; that is, to charge just less than $c+t s+e$.

The heavy shaded line in Figure 16.4 illustrates this price equilibrium. Firm 2 is the lowest-cost supplier (including transport cost) for all consumers in the region $\left(r_{12}, r_{23}\right)$. Therefore, firm 2 supplies all consumers in this market region, charging its consumers on the left one cent less than firm 1's costs of supplying them, and its consumers to the right one cent less than firm 3's costs of supplying these consumers. By adopting this pricing strategy, each firm earns a gross profit (profit before deducting fixed cost) given by the shaded areas for their market regions in Figure 16.4.

An interesting feature of the set of discriminatory prices is that the highest price now paid by any consumer is $c+t L / 5$. This was the lowest price paid by any consumer when firms did not practice price discrimination! Price discrimination in this oligopolistic market unambiguously benefits consumers. Why is this? ${ }^{21}$ With nondiscriminatory pricing, when a firm reduces the price to one consumer, it has to reduce the price to every consumer-an expensive prospect. With discriminatory pricing, by contrast, a firm can lower price in one location

[^14]without having to lower its prices elsewhere. But this means that price discrimination weakens each firm's ability to commit to a set of higher prices, making price competition between the firms much fiercer and so leading to the lower prices that we have just identified.

Now consider the effect on this equilibrium of a merger between two of these firms, say firms 2 and 3, as before. Two points should be clear. First, as in the no-price-discrimination case, a merger of nonneighboring firms has no effect. Second, the merged firm's ability to coordinate the formerly separate pricing strategies is particularly valuable in this discriminatory setting. This is because prior to the merger these firms were engaged in what is nearly cutthroat price competition. By merging, the two firms can avoid this expensive conflict, at least with respect to each other. ${ }^{22}$ From the perspective of the merged firm, the nearest competitor for consumers in the region between firm 2's location and $r_{23}$ is now firm 1. Similarly, for consumers in the region between firm 3's location and $r_{23}$, the nearest competitor is now firm 4. As a result, the merged firm can raise prices to all consumers located between firms 2 and 3, as indicated by the line $a b c$ in Figure 16.4. A merger of firm 2 and firm 3 increases the profits of the merging firms by an amount given by the area $a b c d$. One further effect of this type of merger, which is not quite so intuitive, is that when firms practice price discrimination the merger only benefits the merging firms. Prices and profit increase only for those consumers who were served by the merged firm prior to the merger. All other prices are unaffected, and so the profits of the nonmerging firms are unaffected by the merger.

We could also consider issues regarding the merged firm's product location strategies, but the basic point has been made. Prices to consumers rise and the merging firms are more profitable. There is absolutely no paradox about merging in this price discrimination case. Our conclusions for the no-price discrimination case hold all the more strongly when firms engage in discriminatory pricing practices.

There is one final point to emphasize. Why is it that mergers with price competition in a product-differentiated market do not run into the merger paradox that so bedeviled our earlier analysis with homogenous products and quantity-setting firms? The first part of the answer has already been suggested. Prices are strategic complements whereas quantities are strategic substitutes. With price competition, therefore, the strategic responses of nonmerged firms are potentially beneficial to the merged firms whereas with quantity competition they are potentially harmful.

The second part of the answer is equally important and is related to the notion of credible commitment discussed in section 11.3, Chapter 11. The reason why mergers are profitable in the spatial or differentiated products context is that the merged firms can credibly commit to produce some particular range of products-that is, the commitment required in the spatial context is a commitment to particular locations or to continue marketing the products of the previously independent firms. By contrast, the commitment necessary with homogenous products and quantity competition must be in terms of production levels. The merging firms must be able to commit to a high volume of output following the merger. Generally, this is not credible because such a high volume of production is not the merged firm's best response to a Cournot output decision by the other firms. If however the merged firm becomes a Stackelberg leader then the commitment to a high level of post-merger output is credible.

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### 16.5 PUBLIC POLICY TOWARD HORIZONTAL MERGERS

U.S. public policy with respect to horizontal mergers has changed dramatically over the last forty years. To a large extent, this change is reflected in the differences between the first Merger Guidelines issued by the Justice Department in 1968 and the Merger Guidelines currently in force. While it is tempting to summarize these differences as a move from a very strict regime to a more permissive one, it is more accurate to describe the evolution of merger policy as one that has increasingly become more sophisticated and that gives greater recognition to the complexity of corporate organizations in the real world.

The 1968 Merger Guidelines relied heavily on market structure-particularly the four-firm concentration ratio ${ }^{23}$ - to determine the legality of a proposed merger. Mergers would be challenged in any industry in which the four-firm concentration ratio exceeded 75 percent and the merging firms each had a market share of as little as four percent. In markets with a four-firm ratio below 75 percent, mergers would be challenged if the two firms each had market shares of five percent or more. Thus, under the 1968 Guidelines, a combined share of as little as ten percent would be sufficient in many cases for the government to challenge a merger.

The approach taken in 1968 reflected many years of empirical work within the Structure-Conduct-Performance (SCP) framework. We described economists' increasing discomfort with that framework in Chapter 1. By the mid-1960s, economists were also increasingly concerned with the rigidity with which the courts seemed to apply the SCP learning. Perhaps nowhere was this more apparent than in the controversial merger case, U.S. v. Von's Grocery (1966), in which the Supreme Court upheld the government's prohibition of a merger between two grocery store chains in Los Angeles that, in combination, had less than 10 percent of the market.

Ironically, courts began to deviate from the rigid, structure-based Guidelines of 1968 almost as soon as they were adopted. One early such case was the acquisition by General Dynamics of another coal producer, which was ultimately allowed by the Supreme Court in 1974 despite the fact that the combined market shares of the two firms clearly exceeded the permissible levels set forth by the then 1968 Merger Guidelines. As the courts permitted a number of similar mergers it soon became clear that the 1968 Guidelines were no longer compelling. This eventually led to the Justice Department issuing of a new set of Merger Guidelines in 1982.

Under the new rules, reliance on the four-firm concentration ratio was abandoned in favor of the Herfindahl-Hirschman Index (HHI). ${ }^{24}$ The threshold for intervention now became an HHI of 1,800 (a little more concentrated than an industry comprised of six, equally large firms). Mergers in less concentrated industries would only be challenged if they raised the HHI by more than one hundred points and even then, only if the industry HHI already exceeded one thousand. Subsequent amendments to the Guidelines in 1984, 1992, and 1997 relaxed even more the constraints on mergers by specifying and enlarging the ability of mergergenerated cost efficiencies as a merger justification.

Underlying these developments was an increasing awareness of modern industrial organization theory as well as a growing body of empirical data that suggested many mergers did

[^16]
## Reality Checkpoint

## Baby, Baby, Where Did that Brand Go?

Cost savings have always been a possible justification for horizontal mergers. Such efficiencies took on increased importance after 1997 when the U.S. Federal Trade Commission (FTC) and Department of Justice amended their well-known merger guidelines to give greater weight to such cost efficiencies as a rationale for what otherwise might be a questionable merger. The intuition is that while there may be potential harm to consumers from the monopoly power that the merger creates, this will often be offset by the lower prices that result from the lower costs that the merger makes possible.

Evaluation of the cost-efficiency defense is therefore important. It is also tricky. Besides the question of how real the cost savings may be, there is the further question as to whether the cost savings will be passed on to consumers in the form of lower prices.

Consider the proposed acquisition of Beech-Nut Baby Food by Heinz in 2001. Along with Gerber, these two companies controlled the bulk of the jarred or prepared baby food market. Gerber was industry giant with a market share between 65 and 70 percent. The remaining 30 to 35 percent was split fairly evenly between Heinz and Beech-Nut.

The FTC sought to block the merger arguing that it would significantly decrease competition in the baby food industry. Heinz and BeechNut responded that the merger would actually increase competition. Their analysis relied heavily on cost savings. In brief, the merging parties argued that Beech-Nut has a superior brand image but very old and costly production techniques relative to Heinz. They further argued that the merger would permit the two firms to offer a single product of the higher Beech-Nut quality but at the lower Heinz cost. As a result, this product would enable the merged firm to really put pressure on the industry giant, Gerber. Given Gerber's large size, any market share, a fall in its price would bring large gains to consumers.

Heinz and Beech-Nut backed up their claims with statistical evidence. Using a model of the
baby food industry that is similar in spirit to the circular spatial model used here, they provided simulations of the post-merger market that implied a fall in baby food prices. These simulations took the assumptions of a 15 percent cost savings as given and suggested that between 50 and 100 percent of these savings would be passed through to consumers as lower prices.

The claim that much of the cost savings would be passed on to consumers depends critically on the nature of competition in the post-merger market. As noted, Heinz and Beech-Nut assumed that that market could be described as a spatial one of the type used in this chapter. Horizontal differentiation is not the only type of product differentiation that we observe. An alternative approach is to view the market as vertically differentiated (Chapter 7) with each brand representing a different level of quality and consumers differing in how much they are willing to pay for quality. Gerber would be the highest quality, Beech-Nut the next highest, and Heinz (well known as the discount brand) would be the least highest. In this set-up it is the Beech-Nut quality that directly competes with the Gerber premium brand. If this is the case, then Heinz and Beech-Nut have a strong incentive to discontinue the Beech-Nut brand after the merger. This would allow the firms post-merger to soften price competition in the market by producing the brand that is maximally differentiated (furthest) from Gerber. If so, consumers could be hurt in two ways. Not only would prices rise but consumers would also suffer a loss in choice as one brand was removed from the market. Moreover, removal of a brand in the post-merger market means that the demand estimates made for the pre-merger market (the ones relied on by Heinz and Beech-Nut in their simulations) would no long be relevant.

Norman, Pepall, and Richards (2002) show that the foregoing concern is very real. Indeed, they show that no matter what the cost savings, a merger of two lower quality brands will always lead to the removal of the higher
quality one and a rise in consumer prices on the remaining brands. They show that this is true even when there is potential competition from a later entrant.

Sources: Norman, G., L. Pepall, and D. Richards "Product Differentiation, Cost-Reducing Mergers, and

Consumer Welfare," Canadian Journal of Economics, 38 (November, 2005), 1204-23. See also, J. Baker "Efficiencies and High Concentration: Heinz Proposes to Acquire Beech-Nut (2001)," in J. Kwoka and L. White, eds., The Antitrust Revolution, Oxford University Press, Oxford, 2004, 150-69; and Gandhi, et al. (2007), "Post-Merger Product Repositioning," forthcoming, Journal of Industrial Economics.
not threaten competition as much as the structure-conduct-performance paradigm implied. Moreover the evidence on profitability of mergers was mixed as well. Quite a long list of studies including Mueller (1985), Ravenscraft and Scherer (1987), Lichtenberg and Siegel (1992), Loughran and A. Vijh (1997), Andrade, Mitchell, and Stafford (2001), and Maskimovic and Phillips (2001) have found that mergers are not terribly profitable-especially for the acquiring firm. Indeed, many acquisitions are later reversed by "spin-offs." ${ }^{25}$

The change in attitude reflected by the 1982 Guidelines has led to many more mergers being permitted. These have included such major consolidations as Union Pacific and Southern Pacific (railroads), AOL and Time Warner (telecommunications), Chase Manhattan and J. P. Morgan (finance), Exxon and Mobil and also British Petroleum and Amoco (both petroleum mergers), Westinghouse and Infinity Broadcasting (radio), Aetna and U.S. Healthcare (health services), MCI and WorldCom (telecommunications), Maytag and Whirlpool (laundry machines) among others. Many of these mergers were controversial and virtually all raised some competitive concerns. Yet these and other mergers were nevertheless approved.

Public policy on mergers has increasingly made use of sophisticated empirical techniques to estimate key market parameters and then to use these parameters to model the most likely post-merger scenario. We briefly describe this process of merger simulation in the next section. As developed by Werden and Froeb (1994, 2002) and extended by Epstein and Rubinfeld (2002), among others, merger simulation has become an important, albeit somewhat controversial tool in merger policy. (See Slade (2007).)

In addition to a greater reliance on econometric evidence and economic modeling, policy makers have taken two additional steps that permit horizontal mergers to be approved despite some clear antitrust concerns. First, the antitrust authorities have increasingly used a "fix-it-first" approach regarding proposed mergers. This procedure usually centers on divestiture of some of the assets of the merging parties to another, third firm so as to ensure that competitive pressures are maintained. If, for example, the two firms operate in several towns across the country, but in one town they are the only two such suppliers, then the government may permit the merger so long as one of the firms sells off its operations in the town in question to a new, rival entrant firm. This principle was applied in both of the petroleum mergers mentioned above and it is often used in the case of media mergers where

[^17]newspaper and broadcasting firms have been required to sell their operations in certain locations before being permitted to conclude a merger.

Divestiture does have some problems. Cabral (2003) notes that divestiture allows the merging firms to dictate the entry position for new rivals. If we think of the circle spatial model described above, if two firms merge but sell the location of some of their stores to a formerly excluded entrant, it means that entrant enters at the same location at which the initial stores existed rather than at other locations on the circle that would be better for consumers. Further, firms can act strategically to reduce the competitive threat presented by divested stores. This occurred in 1995 when Schnucks Markets, a supermarket chain, acquired National Food Markets, which was the major competitor of Schnucks in the St. Louis area. The merger was approved when Schnucks agreed to divest 24 supermarkets in the St. Louis area over the next year. However, no immediate buyer was named. Schnucks then took the stores to be divested and proceeded to run them into the ground. It closed departments. It kept the stores understaffed, and referred customers to the other Schnucks stores that were not being divested. Soon, sales at the divesting stores had declined by about one-third and, as a result, they posed less of a competitive threat to the stores that the new Schnucks/ National firm continued to operate. It was partly a response to this case that led the FTC to require now that the buyer of the divested plants be named in advance and that the firm be one that has the industry knowledge to be an effective competitor. This remedy does not, however, correct for the problems identified by Cabral (2003).

A second, alternative procedure has been to approve mergers subject to behavioral constraints on the merging firms, and then to follow this agreement with active monitoring by government agents. Typically, these consent agreements require the firms to take specific actions and to avoid engaging in certain practices. In monitoring these agreements, the regulatory agencies can always count on a reliable source of outside help, namely, the competitors of the merged firms and other parties who opposed the merger. They are always quick to report violations of the consent agreement. Since 1992, the number of consent decrees issued by the FTC and the Justice Department has dramatically increased.

In addition to these procedural changes, the FTC and the Justice Department have also continued to adjust the merger guidelines themselves. In this connection, an important recent modification is the 1997 expansion of section 4 of the Guidelines to permit greater reliance on documented cost savings as a justification of a merger. With this change, the antitrust authorities have indicated an increased willingness to judge a merger to be pro-competitive if it generates cost savings that are likely to translate into lower consumer prices. As we have already noted, however, most analysis finds that the cost savings necessary to generate lower prices are substantial. This may be why the proposed acquisition of Beech-Nut baby food by the Heinz Corporation (see inset) was ultimately denied. Hence, the full implication of the 1997 cost efficiencies amendment is yet to be seen.

We should also note that merger-generated cost efficiencies are not necessarily completely beneficial once entry possibilities are considered. If a merger generates lower marginal costs, then any potential entrant will know that if it enters price competition will be relatively fierce. If the entrant has fixed costs, this will mean that the market will need to be larger for entry to be profitable. In other words, for a given market size, merger-generated cost efficiencies make post-merger entry less likely. Thus, cost savings can have two price effects. One is the downward pressure on prices exerted by lower costs while the other is the upward pressure exerted by the reduced likelihood of rival entry. Cabral (2003) shows that it is possible that the former outweighs the latter.

## Reality Checkpoint <br> Whose Welfare Is It, Anyway?

In January of 2003, a Canadian federal court upheld the decision of that country's Competition Tribunal to permit the merger of Superior Propane and IGC Propane, the two major producers in Canada's propane gas market, both based in Calgary. This was the final act in a five-year case in which the Tribunal and, ultimately, the courts, rejected the argument of the Canadian Competition Bureau, Canada's major antitrust enforcement agency, that the merger should be blocked.

The issue that kept the Superior/IGC case alive through so many rounds of litigation was the question of the proper standard for evaluating mergers. Section 96 of the Canadian Competition Act says that mergers should be allowed if they result in cost efficiencies "that will be greater than, and will offset, the effects of any prevention or lessening of competition" that the merger might create. Effectively, this clause appears to state that mergers ought to be judged on the basis of their impact on the total, producer plus consumer, surplus. This stands in sharp contrast to the U.S. (and European) approaches that give primary emphasis to a merger's impact on consumer surplus, alone. In either setting, merger-generated cost efficiencies may help justify a merger but, in the U.S., those efficiencies must be passed on to consumers whereas the Canadian framework only requires that the cost savings be verified. Whether they result in greater profits or greater consumer surplus is irrelevant in this view.
The relevant surplus question was particularly crucial to the Superior-IGC case because there was no question that the merger conferred substantial market power on the two merging companies. Together, the two would control more than 70 percent of the propane market. Moreover, there was widespread agreement that this power would translate into price increases on the order of 9 percent. The estimated deadweight loss resulting from this price increase was about $\$ 3$ million. However, this loss was dwarfed by an estimated gain in
profit for the two firms of $\$ 29$ million. Thus, it was crucial to determine whether the act's apparent endorsement of a total surplus standard was, in fact, the proper interpretation of the law. As indicated above, the tribunal's decision, with the court's support, endorsed that view.

It is probably fair to say that many economists support the total surplus criterion. It is rooted in the economic definition of efficiency and can be presented with clarity and certainty to all parties involved in a merger case. What justification can there be then for the U.S. (and European) approaches that focus primarily on just consumer surplus?

Apart from distributional concerns, there are at least two arguments in support of a consumer surplus only standard. First, as noted in the text, merging firms may well exaggerate the cost efficiencies stemming from the merger. Focusing on consumer surplus alone will diminish the incentives to do this since the profits that result from such cost savings will not help to justify the merger. Since consumers are not well represented at these proceedings, there may be merit in giving less weight to the claims of producers. Second, even when total surplus is the real goal, focusing on consumer surplus may still prove a useful selection criterion. Suppose that a firm is considering two different mergers. Each will raise total surplus by $X$. However, the first will raise producer surplus by $X+e$ while reducing consumer surplus by $e$. In contrast, the second merger will raise both producer and consumer surplus by $X / 2$. Under a total surplus standard, the firm will choose the first merger. Under a consumer surplus alone standard, it will choose the second. In short, a consumer surplus criterion may serve as a useful instrument to guide firms' choice of merger possibilities even when total surplus is the real target.

Source: T. Ross and R. Winter, "Canadian Merger Policy following Superior Propane," Canadian Competition Record (2003).

### 16.6 EMPIRICAL APPLICATION <br> Evaluating the Impact of Mergers with Computer Simulation

In recent years the important new tool of merger simulation has emerged to assist with the evaluation of mergers. Merger simulation basically works in two steps. The first is to obtain relevant information on such variables as firms' costs, prices, and demand elasticities, among others. This is usually accomplished with the aid of modern econometric techniques. The second step is then to use this evidence to run computer-simulated models of the market in question both before and after a proposed merger. In effect then, merger simulation allows economists to conduct laboratory experiments to examine a merger's likely effects. While not necessarily conclusive, such experiments can be very helpful as an evaluative tool.

To understand merger simulation better, consider an industry with four firms each of which produces a differentiated product and which competes in prices against its rivals. For any one firm, the first-order condition for profit maximization is effectively the Lerner condition, as first identified in section 3.2, Chapter 3. That is,

$$
\begin{equation*}
\frac{p_{i}-c}{p_{i}}+\frac{1}{\eta_{i i}}=0 \quad i=1 \text { to } 4 \tag{16.52}
\end{equation*}
$$

Here, $\eta_{i i}$ is the (negative of) the elasticity of the firm $i$ 's demand with respect to its own price. If we denote the price-cost margin term as $\mu_{i}$; firm $i$ 's market share as $s_{i}$; and then multiply through by the elasticity of demand, equation (16.52) becomes:

$$
\begin{equation*}
s_{i}+s_{i} \eta_{i i} \mu_{i}=0 \tag{16.53}
\end{equation*}
$$

If two firms merge, however, the first-order condition will change as we saw in section 16.5. Now, the merged firm will coordinate the prices of its two separate products by taking account of the cross demand effects between the two products. Specifically, assume that firms 1 and 2 merge. Then it is straightforward to show that for the merged firm, the first-order condition is:

$$
\begin{align*}
& s_{1}+s_{1} \eta_{11} \mu_{1}+s_{2} \mu_{2} \eta_{21}=0  \tag{16.54}\\
& s_{2}+s_{2} \eta_{22} \mu_{2}+s_{1} \mu_{2} \eta_{12}=0
\end{align*}
$$

Where $\eta_{i j}$ is the cross-elasticity of good $i$ with respect to the price of good $j$. It is clear from equations (16.53) and (16.54) that measures of the own and cross-price elasticities for each good are critical to estimating the impact of a proposed merger. Indeed, once these elasticities are known, it is relatively straightforward to work out the implied post-merger equilibrium and, therefore, the post-merger prices.

In order to estimate the elasticities, one needs a model of market demand. One commonly used such model is derived from what is referred to as the Almost Ideal Demand System (AIDS) as first described by Deaton and Mulbauer (1980). Essentially, such a system describes the demand facing each firm as a function of its own price and the prices charged by other firms, similar to the linear demand that we used to describe our initial model of Bertrand competition with differentiated products. In the case of our four-firm example above,
a conventional approach would be to describe market demand with a system of equations something like the following:

$$
\begin{align*}
& s_{1}=a_{1}+b_{11} \ln p_{1}+b_{12} \ln p_{2}+b_{13} \ln p_{3}+b_{14} \ln p_{4} \\
& s_{2}=a_{2}+b_{21} \ln p_{1}+b_{22} \ln p_{2}+b_{23} \ln p_{3}+b_{24} \ln p_{4}  \tag{16.55}\\
& s_{3}=a_{3}+b_{31} \ln p_{1}+b_{32} \ln p_{2}+b_{33} \ln p_{3}+b_{34} \ln p_{4} \\
& s_{4}=a_{4}+b_{41} \ln p_{1}+b_{42} \ln p_{2}+b_{43} \ln p_{3}+b_{44} \ln p_{4}
\end{align*}
$$

The $b_{i j}$ coefficients in the above system are directly linked to the demand elasticities needed to run the merger simulation. Thus, econometric estimation of those coefficients is the first step in obtaining a simulated outcome.
Not counting the $a_{i}$ coefficients or intercepts, this still leaves $16 b_{i j}$ coefficients to estimate even in our small four-product example. In general, unless some restrictions are imposed on the nature of the own and cross-price effects, there will be on the order of $n^{2}$ coefficients to estimate in a general $n$-product demand system of the type illustrated above. This is a rather large number of estimates to make with any degree of precision. To simplify matters, it is common to impose restrictions that reduce the number of parameters to be estimated directly.

For example, suppose that our four-firm example is characterized by $q_{1}=250 ; q_{2}=100$; $q_{3}=100$; and $q_{4}=50$, or $s_{1}=50$ percent; $s_{2}=s_{3}=20$ percent; and $s_{4}=10$ percent. One way to proceed is to calibrate the model under the assumption of proportionality. As developed by Epstein and Rubinfeld (2002) Proportionally Calibrated AIDS (PCAIDS) assumes that the output loss for good 1 caused by an increase in $p_{1}$ will be allocated to the other products in proportion to their market shares. Suppose that the overall elasticity of market demand $\eta=-2$ and the own price elasticity of good 1 is $\eta_{11}=-4$. If we think of the overall industry price as the share-weighted average price across the four firms, then a 1 percent increase in firm 1's price $p_{1}$ translates into a 0.5 percent increase in the industry price, all else equal. Firm 1's price increase will then reduce industry output by one-half of 2 percent, or by 1 percent, which in this case is five units. Firm 1's own output will fall by 4 percent, or ten units. Thus, five of these ten units will be picked up in the demand for the other firms if the net industry demand decline is to be just five units. The proportionality assumption is that $[0.2 /(0.2+0.2+0.1)] \times 5$ or 2 units will be diverted to each of firms 2 and 3 , while the remaining one unit will be diverted to firm 4 . Note that this implies that a one-percent increase in firm 1's price will raise the demand at each of the other firms by two percent, i.e., the cross elasticities $\eta_{21}, \eta_{31}$, and $\eta_{41}$ are -2 in each case.

What we have just shown is that with the proportionality restriction, the knowledge of just the market demand elasticity and firm 1's own price elasticity has permitted us to deduce three other of the elasticity measures needed for simulation. As it turns out, we can go much farther. In fact, the proportionality assumption permits the complete derivation of all the relevant elasticities once the elasticity is known for the market and for one firm. To put it slightly differently, knowing the market elasticity and own-price elasticity of one firm permits complete calculation of all the $b_{i j}$ coefficients in equation (16.55). The proportionality assumption reduces the number of parameters to be estimated from $n^{2}$ to just 2 . Once that estimation is complete, we may use the resulting elasticity and market share data to solve the first-order conditions in (16.53) and (16.54) for both the pre-merger and post-merger market. We can then evaluate the price effects of the merger.

Of course, proportionality is a strong assumption. Other techniques for simplifying the estimation procedure also exist. Unfortunately, which technique is chosen can affect the
predicted post-merger price change by a very large amount, as Slade (2007) in particular, has emphasized. Moreover, even if proportionality is assumed there still remain two elasticity parameters to be estimated. There is ultimately no way to avoid the use of some econometric analysis in the merger evaluation process.

Efforts to estimate the relevant parameters from a demand system such as the one in (16.55) are tricky at best. Even if only a few parameters are required, there remain difficult measurement questions. And while the specification in (16.55) is common it is not the only way to structure market demand. Alternative specifications will imply different functional forms and cross-product elasticity restrictions that in turn will have different effects on the postmerger equilibrium. For example, the linear demand function that we use in most of the examples in this text implies that demand becomes more elastic as prices rise. This imposes a constraint on post-merger prices even if a merger raises market power because it means that consumers become increasingly sensitive to such price increases. In contrast, a log-linear demand function implies a constant price elasticity of demand that will yield a notably higher price rise for the same market power increase. Yet it is often far from clear what precise specification is most appropriate.

A firm's market share will depend heavily on the definition of the market employed. Yet as we can see from the first-order conditions (16.52-16.54), these share values are crucial to understanding market dynamics. Indeed, they are crucial to understanding whether or not the merger raises antitrust concerns in the first place.

The difficulties posed by the econometrics in merger analysis were dramatically illustrated by the proposed 1996 merger of two office superstore chains, Staples and Office Depot. ${ }^{26}$ Along with Office Max, the merging firms dominated the office superstore retail market. Of course, these three firms are not the only retailers of office supplies. While Staples and Office Depot had between 70 and 75 percent of the market defined by office superstores alone, their combined share of the retail sales of office supplies by all stores, including large discounters such as Wal-Mart, drug store chains, and stationery stores, was probably under ten percent. Accordingly, the question of whether the merger even crossed the threshold of concern established by the Merger Guidelines had to be addressed.

Moreover, even within the category of office superstores, market definition remains problematic. In the Staples case, it was widely agreed that there was not one national market but many local ones. In principle, this means that estimation of a demand structure like that in equation (16.55) would have to take the specific nature of each localized market into account. That is, account would need to be taken of variation across locations in the extent of competition. How should this variation be modeled?

The government argued that the local market boundaries were those of the Metropolitan Statistical Area (MSA) used by the Census Bureau. For any Staples store, competitors included all other Office Depot and Office Max stores in the same MSA. In contrast, the merging firms argued that there was a difference within an MSA depending on the actual distance between rivals. That is, an Office Depot store exerted greater price pressure on a Staples store if it were only 5 miles away than if it were 10 , or 20 miles away. Again, these seemingly small changes in specifying the competitive interaction can (and did) have a large effect on the results. Just this one alteration led to more than a three-percentage point difference between the firms' prediction that the merger would raise prices by about 0.8 percent and the government's estimate of a rise of 4.1 percent. Together these and other modest econometric modifications meant that the range in predicted price increases varied from less than 1 percent to almost 10 percent.

[^18]
## 422 Contractual Relations Between Firms

In short, simulating merger effects inevitably requires different estimation techniques and structural demand assumptions that vary according to the conceptualization of the market environment. Assumptions to ease the estimation burden do not alleviate other measurement and econometric issues. We can expect self interest to lead each side in a merger case to choose the framework and associated econometric technique that yield parameter values and other evidence most favorable to its own objective. Unfortunately, it is typically the case that each approach has some objective justification. It becomes very difficult even for economists to separate the truth from the self-interest in interpreting the results. It is even more difficult for the courts to resolve such debates. One of the striking features of the Staples case is that the final court decision never mentions the econometric evidence despite the fact that this case probably involved more econometric presentation than virtually any other merger litigation.

## Summary

Horizontal mergers are combinations of firms that are rivals within the same industry. Because they result in the joining of firms that were previously competitors, horizontal mergers raise obvious antitrust concerns. Such mergers may, in fact, be a means to create a legal cartel. One major puzzle in economic analysis is the merger paradox. This paradox reflects the fact that many commonly used economic models suggest that merger is not profitable for the merging firms and that the true beneficiaries of a merger are the nonmerging firms.

The clue to resolving the merger paradox is to find some means of credibly committing the newly merged firm to a profit-enhancing strategy. One way to do this in a quantity-setting models is to permit the merged firm to take on the role of Stackelberg leader whose increased production is credible. Another way is to consider merger decisions sequentially. Either of these approaches is capable of generating profitable mergers that also have adverse consequences for consumers. The sequential merger approach can also help explain the "domino effect," often observed, by which a merger of two firms in an industry is quickly followed by similar marriages among other firms in the same industry.

The merger paradox does not arise in markets where firms offer differentiated products and compete in price for customers. In these markets the merging firms can easily make a convincing commitment to specific locations or product designs-namely, those used by the firms before they merged. The ability to make such a commitment is sufficient to make merger profitable.

The ambiguous effects of mergers found in economic theory are also found in empirical
analysis. To date, there is little clear evidence that mergers have resulted in legalized cartels with significant monopoly power. Instead, what is clear is that the combination of theoretical and empirical ambiguity has led the legal authorities to take a much less aggressive and much less rigid stand against proposed mergers, a point to which we return in the policy discussion of the next chapter.

Policy also increasingly includes formal attempts to model the post-merger market and to evaluate mergers on a case-by-case basis. The theory behind this approach builds on the first-order conditions for profit maximization and using these to identify the pre-merger own- and crossprice elasticities to analyze the optimal pricing decisions of the merged firm and its rivals in the post-merger market. In practice, this is hard work and typically requires a number of simplifying assumptions to identify the needed parameters. However, there appears to be little alternative

In sum, there is no general rule regarding the impact of mergers. The merger paradox suggests that only mergers that are associated with large cost efficiencies will be profitable. Since firms do not pursue unprofitable opportunities, this suggests that any proposed merger must have very large cost efficiencies and, perhaps, should be approved. On the other hand we know if merged firms can acquire the ability to commit to a production level before others, then the merger can be profitable without cost savings and thus, would be anticompetitive. Antitrust authorities cannot rely solely on economic theory to determine whether or not a specific merger should be challenged. This is an area where empirical work based on advanced econometrics can be predicted to play an important role.

## Problems

For problems 1, 2, 3 and 4 consider a market containing four identical firms each of which makes an identical product. The inverse demand for this product is $P=100-Q$, where $P$ is price and $Q$ is aggregate output. The production costs for firms 1,2 , and 3 are identical and given by $C\left(q_{i}\right)$ $=20 q_{i} ;(i=1,2,3)$, where $q_{i}$ is the output of firm $i$. This means that for each of these firms, variable costs are constant at $\$ 20$ per unit. The production costs for firm 4 are $C\left(q_{4}\right)=(20+\gamma) q_{4}$, where $\gamma$ is some constant. Note that if $\gamma>0$, then firm 4 is a high-cost firm, while if $\gamma<0$, firm 4 is a low-cost firm $(|\gamma|<20)$. Note also that $Q=\sum_{i=1}^{4} q_{i}$

1. Assume that the firms each choose their outputs to maximize profits given that they each act as Cournot competitors.
a. Identify the Cournot equilibrium output for each firm, the product price, and the profits of the four firms. For this to be a "true" equilibrium, all of the firms must at least be covering their variable costs. Identify the constraint that $\gamma$ must satisfy for this to be the case.
b. Assume that firms 1 and 2 merge and that all firms continue to act as Cournot competitors after the merger. Confirm that this merger is unprofitable.
c. Now assume that firms 1 and 4 merge. Can this merger be profitable if $\gamma$ is positive so that firm 4 is a high-cost firm? What has happened to the profits of firm 2 as a result of this merger?
2. Now assume that each firm incurs fixed costs of $F$ in addition to the variable costs noted above. When two firms merge the merged firm has fixed costs of $b F$ where $1 \leq b \leq 2$.
a. Suppose that firms 1 and 2 merge and that $\gamma \geq 0$. Derive a condition on $b, F$ and $\gamma$ for this merger to be profitable. Give an intuitive interpretation of this condition.
b. Suppose by contrast that firms 1 and 4 merge. Repeat your analysis in a.
c. Compare the conditions derived in a. and b . What does this tell you about mergers that create cost savings?
3. Assume that if two firms merge, the merged firm will be able to act as an industry leader,
making its output decision before the nonmerged firms make theirs. Further assume that $\gamma=0$ so that the firms are of equal efficiency.
a. Confirm that a merger between firms 1 and 2 will now be profitable. What has happened to the profits of the nonmerged firms and to the product price as a result of this merger?
b. Confirm that the two remaining firms will also want to merge and join the leader group given that the leaders act as Cournot competitors with respect to each other (hint: this merger will create a leader group containing two firms and a follower group containing none). What does this second merger do to the market price?
4. Continue with the conditions of question 3 but now suppose that for a merger to be undertaken, the merging firms each have to incur a fixed cost, $f$ (this might include costs of identifying a merger partner, negotiating the terms of the merger, legal fees, and so on).
a. How high must $f$ be for the merger between firms 1 and 2 to be unprofitable?
b. How high must $f$ be for the subsequent merger between firms 3 and 4 to be unprofitable?
5. In the chapter it was shown that for a twofirm merger to be profitable, the following condition must be satisfied:

$$
\begin{aligned}
\pi_{l}^{L}(N-1, L+1) & =\frac{(A-c)^{2}}{B(L+2)^{2}(N-L-1)} \\
& >2 \cdot \pi_{f}^{F}(N, L) \\
& =2 \frac{(A-c)^{2}}{B(L+1)^{2}(N-L+1)^{2}}
\end{aligned}
$$

Assume as in questions 1 and 2 that $A=100$, $B=1, c=20$. Further assume that $\gamma=0$.
a. Assume that the number of firms in the market is ten, that is, $N=10$, and that, as in question 4, a two-firm merger requires that each of the merging firms incurs a fixed cost of $f$ prior to the merger. Derive a relationship, $f(L)$,
between $f$ and the size of the leader group, $L$, such that if $f>f(L)$, the twofirm merger will be unprofitable. Calculate $f(L)$ for $L=1,2,3,4$, and 5 to confirm that $f(L)$ is decreasing in $L$. Interpret this result.
b. Now assume that there are eight firms in the market, that is, $N=8$. Repeat your calculations in part (a) to show that the function $f(L)$ rises as $N$ falls. Interpret this result.
6. Normansville consists of a single High Street that is 1 mile long and has 100 residents uniformly located along it. There are three independent video rental stores located in the town at distances $1 / 6,1 / 2$ and $5 / 6$ of a mile from the left-hand edge of Normansville. Each resident rents one video per day provided that the price charged is no more than $\$ 5$. If a consumer is located $s$ miles from a store the transport costs is getting a video from that store is $\$ 0.50 \mathrm{~s}$.

Suppose first that the two stores do not price discriminate.
a. What rental charge will the three stores set given that they act as price competitors?
b. What profits do they earn?
7. Suppose that two neighboring stores in Normansville merge.

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a. What does this do to prices and profits?
b. Recalculate your answers to and 7 a assuming that the stores can perfectly price discriminate.
8. Recall that the first-order condition for maximizing profit may be written as: $p=\frac{\varepsilon-1}{\varepsilon} c$; where $\varepsilon$ is the absolute value of the firm's elasticity. Show that this result implies that, as an approximation, the proportional change in a firm's price as a result of a merger can be written as: $\frac{\Delta p}{p}=\frac{\Delta h}{h}+\frac{\Delta c}{c}$; where $h=\frac{\varepsilon-1}{\varepsilon}$. Suppose that as a result of a merger and decline in competitive pressure, a firm's demand elasticity falls by the proportion $\delta$, i.e., $\varepsilon^{\prime}=(1-\delta) \varepsilon$. Show that we may write $\frac{\Delta h}{h}=\frac{\delta}{(1-\delta) \varepsilon-1}$.
9. Use your results in question 9, to determine the necessary degree of cost efficiencies (i.e., the value of $\frac{\Delta c}{c}$ ), for the firm's price not to rise if its initial elasticity is $\varepsilon=2$, and if as a result of a merger, its demand elasticity falls by 10 percent, i.e., $\delta=0.1$. That is, by what proportion will costs have to decline in this case to keep $p$ constant?

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## Appendix A

## Bertrand Competition in a Simple Linear Demand System

Start with the inverse demand system of equation equations (16.46):

$$
\begin{align*}
& p_{1}=A-B q_{1}-s\left(q_{2}+q_{3}\right) \\
& p_{2}=A-B q_{2}-s\left(q_{1}+q_{3}\right)  \tag{16.A1}\\
& p_{3}=A-B q_{3}-s\left(q_{1}+q_{2}\right)
\end{align*} \quad(s \in[0, B))
$$

In order to identify the Bertrand-Nash equilibrium prices we first need to invert this demand system to get the direct demands. Some simple manipulation gives these demands as:

$$
\begin{aligned}
& q_{1}=\frac{A(B-s)-(B+s) p_{1}+s\left(p_{2}+p_{3}\right)}{(B-s)(B+2 s)} \\
& q_{2}=\frac{A(B-s)-(B+s) p_{2}+s\left(p_{1}+p_{3}\right)}{(B-s)(B+2 s)} \quad(s \in[0, B)) \\
& q_{3}=\frac{A(B-s)-(B+s) p_{3}+s\left(p_{1}+p_{2}\right)}{(B-s)(B+2 s)}
\end{aligned}
$$

Note that these make intuitive sense: demand for each firm is decreasing in the firm's own price and increasing in its rivals' prices.

## THE PRE-MERGER CASE

We begin by identifying the equilibrium when each firm acts independently. Profit to firm 1 is

$$
\begin{equation*}
\pi_{1}=\left(p_{1}-c\right) q_{1}=\left(p_{1}-c\right)\left[\frac{A(B-s)-(B+s) p_{1}+s\left(p_{2}+p_{3}\right)}{(B-s)(B+2 s)}\right] \tag{16.A3}
\end{equation*}
$$

Differentiating with respect to $p_{1}$ and simplifying gives the first-order condition for firm 1:

$$
\begin{equation*}
\frac{\partial \pi_{1}}{\partial p_{1}}=\frac{A(B-s)-2(B+s) p_{1}+s\left(p_{2}+p_{3}\right)+c(B+s)}{(B-s)(B+2 s)}=0 \tag{16.A4}
\end{equation*}
$$

There are similar best response functions for firms 2 and 3. Rather than use these to identify the equilibrium, we can take advantage of the knowledge that this equilibrium will be symmetric, i.e., in equilibrium $p_{1}^{*}=p_{2}^{*}=p_{3}^{*}=p_{n m}^{*}$. Substituting this into the best response function (16.11A4) gives: $\frac{A(B-s)-2 B p_{n m}^{*}+c(B+s)}{(B-s)(B+2 s)}=0$.

Solving for the equilibrium price gives

$$
\begin{equation*}
p_{n m}^{*}=\frac{A(B-s)+c(B+s)}{2 B} \tag{16.A5}
\end{equation*}
$$

Substituting these prices into the direct demand functions (16.11A2) gives the equilibrium output for each firm of $q_{n m}^{*}=\frac{(A-c)(B+s)}{2 B(B+2 s)}$ and substituting into the profit functions (16.11A3) gives the no-merger profit for each firm of

$$
\begin{equation*}
\pi_{n m}^{*}=\frac{(A-c)^{2}(B-s)(B+s)}{4 B^{2}(B+2 s)} \tag{16.A6}
\end{equation*}
$$

## MERGER OF FIRMS 1 AND 2

Now assume that firms 1 and 2 merge. Post-merger, the merged firm chooses its prices $p_{1}$ and $p_{2}$ to maximize its aggregate profit $\pi_{1}+\pi_{2}$ while the non-merged firm chooses $p_{3}$ to maximize its profit $\pi_{3}$. This gives the first-order conditions:

$$
\begin{align*}
& \frac{\partial\left(\pi_{1}+\pi_{2}\right)}{\partial p_{1}}=\frac{A(B-s)-2(B+s) p_{1}+2 s p_{2}+s p_{3}+c B}{(B-s)(B+2 s)}=0 \\
& \frac{\partial\left(\pi_{1}+\pi_{2}\right)}{\partial p_{2}}=\frac{A(B-s)-2(B+s) p_{2}+2 s p_{3}+s p_{3}+c B}{(B-s)(B+2 s)}=0  \tag{16.A7}\\
& \frac{\partial \pi_{3}}{\partial p_{3}}=\frac{A(B-s)-2(B+s) p_{3}+s\left(p_{1}+p_{2}\right)+c(B+s)}{(B-s)(B+2 s)}=0
\end{align*}
$$

Solving these for the equilibrium prices gives $p_{1}^{m}=p_{2}^{m}=\frac{A(2 B+3 s)(B-s)+c(2 B+s)(B+s)}{2\left(2 B^{2}+2 B s-s^{2}\right)}$ for the merged firm and $p_{3}^{n m}=\frac{A(B+s)(B-s)+c B(B+2 s)}{\left(2 B^{2}+2 B s-s^{2}\right)}$ for the non-merged firm. Substituting these prices into the profit equations (16.11A3) gives the profits of equation (16.48):

$$
\begin{equation*}
\pi_{1}^{m}=\pi_{2}^{m}=\frac{B(A-c)^{2}(B-s)(2 B+3 s)^{2}}{4(B+2 s)\left(2 B^{2}+2 B s-s^{2}\right)^{2}} ; \pi_{3}^{m}=\frac{(A-c)^{2}(B-s)(B+s)^{3}}{(B+2 s)\left(2 B^{2}+2 B s-s^{2}\right)^{2}} \tag{16.A8}
\end{equation*}
$$

## COMPARISON OF THE PRE-MERGER AND POST-MERGER CASES

Comparison of the pre- and post-merger profits looks on first sight to be difficult. However, if we define $\sigma=s / B$, where $\sigma$ lies in the interval $[0,1)$ since we have that $0 \leq s<B$, then we can write the non-merged profit as:

$$
\begin{equation*}
\pi_{n m}^{*}=\frac{(A-c)^{2}(B-s)(B+s)}{4 B^{2}(B+2 s)}=\frac{(A-c)^{2} B^{2}(1-\sigma)(1+\sigma)}{4 B^{3}(1+2 \sigma)}=\frac{(A-c)^{2}\left(1-\sigma^{2}\right)}{4 B(1+2 \sigma)} \tag{16.A9}
\end{equation*}
$$

and we can write profit of each division of the merged firm as:

$$
\begin{align*}
\pi_{1}^{m}=\pi_{2}^{m} & =\frac{B(A-c)^{2}(B-s)(2 B+3 s)^{2}}{4(B+2 s)\left(2 B^{2}+2 B s-s^{2}\right)^{2}}=\frac{B^{4}(A-c)^{2}(1-\sigma)(2+3 \sigma)^{2}}{4 B^{5}(1+2 \sigma)\left(2+2 \sigma-\sigma^{2}\right)^{2}} \\
& =\frac{(A-c)^{2}(1-\sigma)(2+3 \sigma)^{2}}{4 B(1+2 \sigma)\left(2+2 \sigma-\sigma^{2}\right)^{2}} \tag{16.A10}
\end{align*}
$$

Note that both profit equations (and post-merger the profit of firm 3) have the term $(A-c)^{2} / B$ in common. As a result, in comparing pre- and post-merger profits no generality is lost if we normalize this term to unity. The result is that profits are function solely of $\sigma$ and can be compared by plotting (16.11A8) and (16.11A9) in the interval $\sigma \in[0,1$ ). Doing so confirms that the merger increases the profits of the merged firm - and of the non-merged firm.

## Appendix B

## Equilibrium Prices in the Spatial Model without a Merger

We can take any one of the five firms as typical of the others. So consider firm 3. Demand for this firm from consumers to its left is $N r_{23}$, where $r_{23}$ is the marginal consumer given by

$$
\begin{equation*}
m_{3}+t r_{23}=m_{2}+t\left(\frac{L}{5}=r_{23}\right) \Rightarrow r_{23}=\frac{m_{2}-m_{3}}{2 t}+\frac{L}{10} \tag{16.B1}
\end{equation*}
$$

Similarly, demand from consumers to the right of firm 3 is $N r_{34}$, where $r_{34}$ is

$$
\begin{equation*}
r_{34}=\frac{m_{4}-m_{3}}{2 t}+\frac{L}{10} \tag{16.B2}
\end{equation*}
$$

Firm 3's profit is, therefore,

$$
\begin{equation*}
\pi_{3}=N m_{3}\left(r_{23}+r_{34}\right)=N m_{3}\left(\frac{m_{2}-m_{3}}{2 t}+\frac{m_{4}-m_{3}}{2 t}+\frac{L}{5}\right) \tag{16.11B3}
\end{equation*}
$$

Differentiating this with respect to $m_{3}$ to give the first-order condition for firm 3:

$$
\begin{equation*}
\frac{\partial \pi_{3}}{\partial m_{3}}=N\left(\frac{m_{2}+m_{4}}{2 t}-\frac{2 m_{3}}{t}+\frac{L}{5}\right)=0 \tag{16.B4}
\end{equation*}
$$

Since the five firms are identical, in equilibrium we must have $m_{3}=m_{2}=m_{4}$. Substituting this into (16.11B4) then gives the Bertrand-Nash equilibrium price:

$$
\begin{equation*}
m^{*}=t L / 5 \tag{16.B5}
\end{equation*}
$$

## EQUILIBRIUM PRICES IN THE SPATIAL MODEL AFTER FIRMS

2 AND 3 MERGE
Profit for each firm is easily identified by changing the firms' "labels" in equation (16B3), so that we have

$$
\begin{align*}
& \pi_{1}=N m_{1}\left(\frac{m_{5}-m_{1}}{2 t}+\frac{m_{2}-m_{1}}{2 t}+\frac{L}{5}\right) \\
& \pi_{2}=N m_{2}\left(\frac{m_{1}-m_{2}}{2 t}+\frac{m_{3}-m_{2}}{2 t}+\frac{L}{5}\right) \\
& \pi_{3}=N m_{3}\left(\frac{m_{2}-m_{3}}{2 t}+\frac{m_{4}-m_{3}}{2 t}+\frac{L}{5}\right)  \tag{16.C1}\\
& \pi_{4}=N m_{4}\left(\frac{m_{3}-m_{4}}{2 t}+\frac{m_{5}-m_{4}}{2 t}+\frac{L}{5}\right) \\
& \pi_{5}=N m_{5}\left(\frac{m_{4}-m_{5}}{2 t}+\frac{m_{1}-m_{5}}{2 t}+\frac{L}{5}\right)
\end{align*}
$$

After the merger, the merged firm chooses $m_{2}$ and $m_{3}$ to maximize aggregate profit $\pi_{2}+\pi_{3}$, while the remaining firms choose their prices to maximize their individual profits. This means there are five first-order conditions to solve:

$$
\begin{align*}
& \frac{\partial \pi_{1}}{\partial m_{1}}=N\left(\frac{m_{5}+m_{2}}{2 t}-\frac{2 m_{1}}{t}+\frac{L}{5}\right)=0 \\
& \frac{\partial\left(\pi_{2}+\pi_{3}\right)}{\partial m_{2}}=N\left(\frac{m_{1}+m_{3}}{2 t}-\frac{2 m_{2}}{t}+\frac{L}{5}\right)+N \frac{m_{3}}{2 t}=0 \\
& \frac{\partial\left(\pi_{2}+\pi_{3}\right)}{\partial m_{3}}=N\left(\frac{m_{2}+m_{4}}{2 t}-\frac{2 m_{3}}{t}+\frac{L}{5}\right)+N \frac{m_{2}}{2 t}=0  \tag{16.C2}\\
& \frac{\partial \pi_{4}}{\partial m_{4}}=N\left(\frac{m_{3}+m_{5}}{2 t}-\frac{2 m_{4}}{t}+\frac{L}{5}\right)=0 \\
& \frac{\partial \pi_{5}}{\partial m_{5}}=N\left(\frac{m_{4}+m_{1}}{2 t}-\frac{2 m_{5}}{t}+\frac{L}{5}\right)=0
\end{align*}
$$

Solving these equations simultaneously gives the prices in the text. In determining this equilibrium, we assume that no firm $i$ ever finds it profitable to price so low that it actually competes with firms beyond $i-1$ and $i+1$.


[^0]:    ${ }^{1}$ Mergerstat Review, January 2006 and 2007.
    2 See section 8.3, Chapter 8 .

[^1]:    ${ }^{3}$ The Department of Justice/Federal Trade Commission Merger Guidelines can be read at http:// www.ftc.gov/bc/docs/horizmer.htm. Section 2 on the potential adverse effects of mergers is particularly relevant.

[^2]:    4 A merger to monopoly is when all the firms in an industry combine into a single monopoly producer.
    5 The paradox was first formalized in a slightly different form by Salant et al. (1983).

[^3]:    ${ }^{6}$ You can check this equation by direct substitution of $a(N)$ in equation (16.9).

[^4]:    7 This is a special case of a much more sophisticated analysis by Farrell and Shapiro (1990) who show in a general setting that for consumers to benefit from a profitable horizontal merger of Cournot firms the merger has to create substantial cost synergies.

[^5]:    8 Note that this equilibrium only exists only if there is a limit on the disadvantage $b$ for firm 3. Specifically, firm 3's pre-merger output will be positive only if $b<210 / 90=7 / 3$, otherwise it will not operate in this market in the first place.

[^6]:    ${ }^{9}$ Perry and Porter (1985) assume that each firm's cost schedule declines with the total amount of capital it owns. Hence, by merging and gaining more capital, a firm lowers its costs. The scarcity of capital makes it difficult for other firms to do this and, because of rising costs, to free-ride as much on the merger of rivals.
    10 This analysis draws on the model A. F. Daughety (1990) who suggested this role for the merged firms.

[^7]:    11 The spatial model was first formulated in Hotelling (1929), and subsequently extended in Schmalensee (1978) and Salop (1979). We saw in Chapters 4, 7, and 10 that this sort of spatial model has proven insightful in analyzing a variety of topics in industrial organization, including brand proliferation in the ready-to-eat breakfast cereal industry, Schmalensee (1978), and the effects of deregulation of transport services such as airlines or passenger buses, Greenhut et al. (1991). It is not surprising that the spatial model is also useful in analyzing mergers of firms selling differentiated products.

[^8]:    12 An excellent example of the full analysis can be found in Davidson and Deneckere (1986).

[^9]:    13 A more general, but much more complicated version of this analysis can be found in Brito (2003).
    14 It bears repeating that the spatial or geographic interpretation of this model is only the most obvious one. See the discussion in Chapter 10.

[^10]:    15 If the reader is interested in working out the outcome for the case of $c \neq 0$, then we note here that in each case that we examine, the equilibrium price $m^{*}$ that we derive should be replaced by $c+m^{*}$.

[^11]:    ${ }^{16}$ We assume no firm prices so low as to lure buyers from beyond its two immediate neighbors. See Appendix to this chapter.

[^12]:    ${ }^{17}$ If the merger leaves products 2 and 3 under the control of separate, competing product divisions, prices will not change. It is important, in other words, that the merged firms take advantage of the opportunity they now have to coordinate their prices.
    18 There is one complication that we have ignored in this discussion. Judd (1985) argues that a merger that creates a multiproduct firm, as for example a merger of firms 2 and 3, may not be sustainable. The intuition is as follows: Assume that an entrant comes in exactly at firm 3's location after the merger of firms 2 and 3. Price competition will drive the price for this product down to marginal cost, in which case the entrant and the incumbent earn zero profits at this location (ignoring fixed costs). But the merged firm also loses money at the neighboring location 2 since the price war with the new entrant forces it to reduce the price there as well. If the merged firm were to close down its location 3 product, the entrant will raise price above marginal cost, and so the merged firm can raise the price at location 2 . There is, in other words, a stronger incentive for the merged firm to exit location 3 than for the entrant to do so. Hence, this kind of multiproduct merger may not be sustainable because it is not credible. This argument turns, however, on two important assumptions: that entry costs are not recovered on exit and that the merged firm has no incentive to try to develop a reputation for toughness. If only part of the entry costs are sunk (unrecoverable costs), or if reputation is important, the merged firm can sustain the multiproduct configuration.

[^13]:    19 A formal proof for he case of three firms is provided by Posada and Straume (2004).
    ${ }_{20}$ Refer to section 4.3, Chapter 4 for a definition and explanation of economies of scope.

[^14]:    ${ }^{21}$ This is discussed in Norman and Thisse (1996). They show that with a given number of firms, discriminatory pricing always benefits consumers. They also show, however, that the much more competitive environment of discriminatory pricing may cause enough firms to want to leave the market that prices actually increase for some consumers. In our example, there is no incentive to exit the market. See also Reitzes and Levy (1995).

[^15]:    22 In this type of merger the potential problems discussed in footnote 15 cannot arise if firms charge discriminatory prices. Consider, for example, entry at product 3's location this yields the price equilibrium in Figure 6a whether or not the merged firm exits this location. A potential entrant can correctly anticipate that there is no benefit to the merged firm exiting and so entry will not occur.

[^16]:    ${ }^{23}$ Refer to section 3.1, Chapter 3 for a discussion of concentration ratios.
    ${ }^{24}$ For a discussion of HHI again refer to section 3.1, Chapter 3.

[^17]:    25 Note though that these findings also raise doubts about any cost savings that mergers are alleged to generate.

[^18]:    ${ }^{26}$ For a more complete discussion of the econometric evidence in this case, see Ashenfelter et al. (2004)

