

CHAPTER 1.1

Abstract Entities

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One of the most puzzling topics for newcomers to metaphysics is the debate about abstract entities, things like numbers (seven), sets (the set of even numbers), properties (triangularity), and so on. The major questions about abstract entities are whether there are any, if so which ones there are, and if any do exist, what they are like.

My aim here is to provide a brief and accessible overview of the debates about abstract entities. I will try to explain what abstract entities are and to say why they are important, not only in contemporary metaphysics but also in other areas of philosophy. Like many significant philosophical debates, those involving abstract entities are especially interesting, and difficult, because there are strong motivations for the views on each side.

In the first section, I discuss what abstract entities are and how they differ from concrete entities and in the second section, I consider the most compelling kinds of arguments for believing that abstract entities exist. In the third section, I consider two examples, focusing on numbers (which will be more familiar to newcomers than other types of abstract objects) and properties (to illustrate a less familiar sort of abstract entity). In the final section, I examine the costs and benefits of philosophical accounts that employ abstract entities.¹

1 What are Abstract Entities?

Prominent examples of abstract entities (also known as *abstract objects*) include numbers, sets, properties and relations, propositions, facts and states-of-affairs, possible worlds, and merely-possible individuals (we'll see what some of these are in a bit). Such entities are typically contrasted with concrete entities – things like trees, dogs, tables, the Earth, and Hoboken. I won't discuss all of these examples, but will consider a few of the more accessible ones as case studies to help orient the reader.

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Numbers and sets Thought and talk about numbers are extremely familiar. We learn about the natural numbers (like three, four, and four billion), about fractions (rational numbers, like $\frac{2}{3}$ and $\frac{7}{8}$), and about irrational numbers (like the square root of 2 and e). And we learned a bit about sets in school – for example, the empty set, the set containing just 3 and 4, and the set of even numbers; we even learned to write names of sets using notation like “{3,4}.”

But what *are* numbers and sets? We cannot see them or point to them; they do not seem to have any location, nor do they interact with us or any of our instruments for detection or measurement in any discernible way. This may lead us to wonder whether there really are any such things as numbers, and whether, when we say things like “there is exactly one prime number between four and six,” we are literally and truly asserting that such a number exists (after all, what could it be?). But, as we will see in section 3.1, there are also strong philosophical arguments that numbers do exist. Hence a philosophical problem: do they or don’t they?

Properties and relations The world is full of resemblances, recurrences, repetitions, similarities. Tom and Ann are the same height. Tom is the same height now as John was a year ago. All electrons have a charge of 1.6022×10^{-19} coulomb. The examples are endless. There are also recurrences in relations and patterns and structures. Bob and Carol are married, and so are Ted and Alice; the identity relation is symmetrical, and so is that of similarity. Resemblance and similarity are also central features of our experience and thought; indeed not just classifications, but all the higher cognitive processes involve general concepts. Philosophers call these attributes of qualities or features of things (like their color and shape and electrical charge) *properties*. Properties are the ways things can be; similarly, relations are the ways things can be related.

Assuming for the moment that there are properties and relations, it appears that many things have them. Physical objects: The table weighs six pounds, is brown, is a poor conductor of electricity, and is heavier than the chair. Events: World War I was bloody and was fought mainly in Europe. People: Wilbur is six feet tall, an accountant, irascible, and married to Jane. Numbers: three is odd, prime, and greater than two. All of these ways things can be and ways they can be related are repeatable; two tables can have the same weight, two wars can both be bloody. The two adjacent diamonds in figure 1 are the same size, orientation, and uniform shade of gray.

Champions of properties hold that things like grayness (or being gray) and triangularity (or being triangular) are properties, and that things like being adjacent and being a quarter of an inch apart are relations. Since the goal here is just to give one prominent example of a (putative) sort of abstract object, I will think of properties as universals (as many, but not all, philosophers do). On this construal, there is a single,



Figure 1 Resemblances and Ways Things Can Be

universal entity, the property of being gray that is possessed or exemplified by each of the two diamonds in our figure. It is wholly present both *a* and *b*, and will be as long as each remains gray.

Philosophers who concur that properties exist may disagree about which properties there are and what they are like, but at least many properties (according to numerous philosophers, all) are abstract entities. Perhaps a property like redness is located in those things that are red, but where is justice, or the property of being a prime number, or the relation of life a century before? Such properties and relations exist outside space and time and the causal order, so they are rather mysterious. But, as we will see, there are also good reasons for thinking that properties and relations can do serious philosophical work, helping explain otherwise puzzling philosophical phenomena. This is a reason to think that they do exist. Another problem.

Propositions Two people can use different words to say the same thing; indeed, they can even use different languages. When Tom says “Snow is white” and Hans says “Schnee ist weiss,” there is an obvious sense in which they say the same thing. So whatever this thing is, it seems to be independent of any particular language. Philosophers call these entities *propositions*. They are abstract objects that exist independently of language and even thought (though of course many of them are expressed in language). Propositions have been said to be the basic things that are true or false, the basic truth-bearers, with the sentences or statements that express them being derivatively true or false.

Tom also believes that snow is white and Hans, who speaks no English, believes that Schnee ist weiss. Again, there is an obvious sense in which they believe the same thing. Some philosophers urge that the best way to explain this is to conclude that *there is* some one thing that Tom and Hans both believe. On this view, propositions are said to be the contents or meanings of beliefs, desires, hopes, and the like. They are also said to be the objects of beliefs. Thus the object of Tom’s belief that red is a bright color is the proposition that red is a bright color.

On this view propositions are abstract objects that express the meanings of sentences, serve as the bearers of truth values (truth and falsehood), and are the objects of belief. But like numbers, propositions are somewhat mysterious. We can’t see them, hear them, point to them. They don’t seem to do anything at all. This gives us reason to doubt their existence. But, there are also reasons to think that they exist. Problems, problems, problems.

1.1 What abstract entities are (nearly enough)

Debates about abstract objects play a central role in contemporary metaphysics. There is wide agreement about the paradigm examples of abstract entities, though there is also disagreement about the exact way to characterize what counts as abstractness. Perhaps this shouldn’t come as a surprise; if any two things are so dissimilar that their difference is brute and primitive and hard to pin down, abstract entities and concrete entities (*abstracta* and *concreta*) are certainly plausible candidates.

Even so, the philosophically important features of the paradigm examples of abstracta (like those listed above) are pretty clear. They are atemporal, non-spatial,

and acausal – i.e., they do not exist in time or space (or space-time), they cannot make anything happen, nothing can affect them, and they are incapable of change. Neither they, their properties, nor events involving them can make anything happen here in the natural world. We don't see them, feel them, taste them, or see their traces in the world around us. Still, according to a familiar metaphor of some philosophers, they exist "out there," independent of human language and thought.

Being atemporal, non-spatial, and acausal are not all necessary for being abstract in the sense many philosophers have in mind. Thus, many things that seem to be abstract also seem to have a beginning (and ending) in time, among them natural languages like Urdu and dance styles like the charleston. It may seem tempting to say that such things exist in time but not in space, but where exactly? Moreover, this claim can't be literally true in a relativistic world (like ours certainly seems to be), where space and time are (framework-dependent) aspects of a single, more basic thing, namely space-time.

And not all are sufficient. For example, an elementary particle (e.g., an electron) that is not in an eigenstate for a definite spatial location is typically thought to lack any definite position in space. The technicalities don't matter here; the point is just that although such particles may seem odd, they do have causal powers, and so virtually no one would classify them as abstract. Again, according to many religious traditions, God exists outside of space and time, but he brought everything else into existence, and so many would be reluctant to classify him as an abstract object.

All this suggests that the division into concrete and abstract may be too restrictive, or that abstractness may come in degrees. I won't consider such possibilities here, however, because the puzzles about abstract entities that most worry philosophers concern those entities that are, if they exist, atemporal, non-spatial, *and* acausal. And we don't need a sharp bright line between abstracta and concreta to examine these.

A philosopher who believes in the existence of a given sort of abstract entity is called a *realist* about that sort of entity, and a philosopher who disbelieves is called an *anti-realist* about it. Abstract entities are not a package deal; it is quite consistent, and not uncommon, for a philosopher to be a realist about some kinds of abstract entities (e.g., properties) and an anti-realist about others (e.g., numbers).

Not-quite existence Finally, some champions of abstract entities claim that there are such things, but grant them a lower grade of being than the normal, straight-forward sort of existence enjoyed by George Bush and the Eiffel Tower. They often devise esoteric labels for this state; for example, numbers, properties, and the like have been said to *have being*, to *subsist*, to *exist but not be actual*, or partake of one or another of the bewildering varieties of not-quite-full existence contrived by philosophers. Such claims are rarely very clear, but frequently they at least mean that a given sort of entity is real in some sense, but doesn't exist in the spatiotemporal causal order. Which is pretty much just to say it is abstract.

We will not pursue such matters here, however, since many of the same problems arise whether the issue about the status of abstracta is framed in terms of the existence or merely the subsistence or being of such things. Whatever mode of being the number two or the property of being abstract possesses, we still cannot perceive it, pick it out in any way, and it seems to make no difference to anything here in the natural world.

Because many of the most debated issues arise for all the proposed modes of being of abstract objects, I will focus on existence.

Why questions about abstracta matter Explicit discussion of abstract entities is a relatively recent philosophical phenomenon. Plato's Forms (his version of universal properties) have many of the features of abstract objects. They exist outside of space and time, but they seem to have some causal efficacy. We can learn about them, perhaps even do something like perceive them, though perhaps only in an earlier life (this is Plato's doctrine of recollection).

Soon after Plato, properties and other candidate abstracta – e.g., merely possible individuals (individual things, e.g., persons, that could have existed but don't) – were reconstrued as ideas in the mind of God. This occurred through the influence of Augustine and others, partly under the influence of Plotinus and partly under that of Christianity. Human beings were thought to have access to these ideas because of divine illumination, wherein God somehow transferred his ideas into our minds. In later accounts like Descartes' we had access to such ideas because God placed them in our minds at birth (they are innate). Such views persisted though medieval philosophy and well into the modern period. In this period, philosophers like Locke began to view what we thought of above as properties (e.g., redness, justice) as ideas or concepts in individual human minds.

It was really only in the nineteenth century, with work on logic and linguistic meaning by figures like Bernard Bolzano and Gottlob Frege, that abstract entities began to come into their own. They emerged with a vengeance around the turn of the twentieth century, with work in logic, the theory of meaning, and the philosophy of mathematics, and, more generally, because of a strongly realist reorientation of much of philosophy at this time in the English- and German-speaking worlds. After a few decades, interest in abstract entities subsided, but by the end of the twentieth century, there was perhaps more discussion of a wider array of abstract objects than ever before.

Although explicit discussion of abstract entities has a fairly recent history, they are central to debates over venerable philosophical issues, including the nature of mathematical truth, the meanings of words and sentences, the features of causation, and the nature of cognitive states like belief and desire. These debates also lie at the center of many perennial disputes over realism and anti-realism, particularly standard flavors of nominalism. Discussions about the existence of abstract objects may also illuminate the nature of human beings and our place in the world. If there are no abstract objects, nothing that transcends the spatiotemporal causal order, then there may well be no transcendent values or standards (e.g., no eternal moral properties) to ground our practices and evaluations. And if there is also no God, it looks like truth and value must instead be somehow rooted here in the natural order. We are more on our own.

2 Why Believe there are Abstract Objects?

The central questions about abstract objects are: Are there any? If at least some kinds of abstract objects exist, can we discover what they are like? How can we decide such issues? (This question is a problem because it seems to be difficult to make contact

with abstract objects in order to learn about their nature.) In this section I will offer an answer to the first question that also suggests an answer to the second.

A good way to get a handle on the issues involving abstract entities is to begin by focusing on the *point* of introducing them in the first place. Philosophers who champion one or another type of abstract object almost always do so because they think those objects are needed to solve certain philosophical problems, and their views about the nature of these abstracta are strongly influenced by the problems they think they are needed to solve and the ways in which they (are hoped to) solve them. Hence, our discussion here will be organized around the tasks abstracta have been introduced to perform. These tasks are typically *explanatory*, to explain various features of philosophically interesting phenomena, so to understand such accounts we need to ask about the legitimacy, role, and nature of explanation in metaphysics.

2.1 Philosophical explanations and existence

Ontology is the branch of metaphysics that deals with the most general issues about existence. Of course we know a great deal about what sorts of things exist just from daily life: things like trees, cats, cars, other people, the moon. And science tells us more about what sorts of things there are: electrons, molecules of table salt, genes. But ontology attempts to get at the most general categories or sorts of things there are, e.g., physical objects, persons, numbers, properties, and the like. Some philosophers doubt that the very enterprise of ontology makes sense (see chapter 9), but we will begin by assuming that it does.

For many centuries ontology aspired to be a demonstrative enterprise. On this traditional conception, ontology employs valid arguments to establish conclusions about what the most general and fundamental things in the universe are. It proceeds from obviously secure premises, step by deductively valid step, to obviously secure conclusions. The traditional standards for security were very high, requiring unassailable, necessary, self-evident “first principles.” These were supposed to be claims that couldn’t possibly be false and that no reasonable person could doubt.

The chief problem with this picture is that when we judge classical arguments in ontology by such standards, most not only fail – many fail miserably. There is, among other things, no consensus about which candidates for first principles are even true, much less necessarily so, and, in many cases, demanding valid arguments seems to be asking for too much. By these standards, even the best that the greatest philosophers could devise comes up far short.

Nowadays, many philosophers would gladly settle for premises that are uncontroversially true – or even just fairly plausible. But they still devote a good deal of time distilling arguments for (or against) the existence of one or another sort of abstract object down to a few numbered premises and a conclusion to write on the board, check for validity, then (most often) dismiss them. This approach is often invaluable, but it has limitations. For one thing, few philosophical arguments survive long when judged by the pass–fail standards of deductive validity (how likely is it, after all these centuries of inconclusive results, that Jones has just devised an unassailable demonstration that properties exist?). Indeed, it is quite possible that there are no deductively sound arguments beginning from true premises which do not mention abstracta and

end with conclusions that abstracta exist (“no abstracta in, no abstracta out”). However that may be, we often miss things of value if we write arguments off simply because they are not deductively valid. But if traditional and contemporary versions of the demonstrative ideal set the bar too high, how should we think about arguments and disagreements in ontology?

When we turn to the ways philosophers *actually* evaluate views about abstract objects, we typically find things turning on the pluses and minuses of one view compared to those of its competitors. And a very common feature of the (putative) pluses is that they involve explanation. For example, we are told that the existence of numbers would explain mathematical truth or that the existence or properties (like triangularity) would explain why it is that various objects are triangular and that it would also help explain how we recognize newly encountered triangles as triangles.

Moreover, even when the word ‘explain’ is absent, we frequently hear that some phenomenon holds in virtue of, or because of, this or that property, that a property is the ground or foundation or most enlightening account of some phenomenon, or that a property is (in part) the truthmaker, the *fundamentum in re* (as the medievals would have said) for the phenomenon. For example, it has been urged that the exemplification of a single, common property grounds the fact that our two items in figure 1 (above) are triangular; it makes it true that each is a triangle. The same property also helps to explain how we recognize that they are triangular and why the world ‘triangle’ applies to them.

Similar claims have been made on behalf of other abstracta. The role of expressions like ‘explain’ is to give reasons, to answer why-questions, which is a central point of explanation. My suggestion is that we should (re)construe arguments for the existence of abstract entities as inferences to the best overall available ontological explanation (we’ll return to this in sections 3 and 4; see also Swyer 1982, 1983, 1999a).

I will develop this idea in the course of examining the example of numbers, but first let’s see what morals we can draw from the view that arguments for the existence of abstract objects are *ampliative* (i.e., deductively invalid but capable of offering good, though not conclusive, support for their conclusions).

First, we should acknowledge at the outset that there will rarely (probably never) be knock-down arguments for (or against) the existence of any type of abstract entity. On this approach, metaphysics (including ontology) is a fallibilistic, ever-revisable enterprise. By way of example, twentieth-century physics presents us with a very surprising picture of physical reality, and it may well call for innovations in ontology. To note just one case, quantum field theory, that branch of physics that deals with things at a very small scale (quarks, electrons, etc.), strongly suggests that there are (at the fundamental level) no individual, particular things; there may be no fact about how many “particles” of a given kind there are in a particular region of space-time. If so, the traditional view that individuals or substances are a fundamental category of reality may be overthrown.

Second, although each specific argument for the existence of a certain kind of abstract entity may not be fully compelling, if there are a number of independent arguments that a given sort of entity exists, the claim that they do could receive *cumulative confirmation* by helping to explain a variety of phenomena.

Third, if some type of abstract entity is postulated to play particular explanatory roles, this *affords a principled way to learn about its nature*. We ask what such an entity would have to be like in order to play the roles it is postulated to fill. What, to take a question considered below, would the existence or identity conditions of properties have to be for them to serve as the meanings of predicates like ‘round’ or ‘red’?

If we are fortunate, we might devise a series of ontological explanations that employ the same entity. This increases information, because different explanations may tell us different things about what that entity is like. It also increases confirmation, because the sequence of explanation may provide cumulative support for the claim that the entity they all invoke actually exists.

Explanatory targets and target ranges An explanation requires at least two things. First, something to be explained, an *explanation target*. Second, something to explain it. In ontology, it is a philosophical theory (though “theory” is often a bit grandiose) like Plato’s theory of forms that does the explaining. We will be concerned with those theories that employ abstract objects in their explanation.

Explanation targets for ontology can come from anywhere. From the everyday world around us (e.g., different objects can be the same color, and a single object can change color over time); from mathematics (e.g., it is necessarily the case that three is a prime number); from natural languages (e.g., the word ‘triangle’ is true of many different individual figures); from science (e.g., objects attract one another because of their gravitational mass but may repel one another if they are different charges). Explanation targets for ontology can come from almost any area of philosophy (e.g., many moral values seem to be objective, but it’s a bit mysterious how this can be so). I will call a more-or-less unified collection of explanation targets a *target domain*.

In the next section I briefly discuss several target domains that have led some philosophers to postulate abstract entities. Although I believe that arguments in ontology are usually best construed as ampliative, much of what follows can be adapted fairly straightforwardly to the view that philosophical arguments should aim to be deductively sound.

3 Examples of Work Abstracta Might Do

When we turn to actual debates about abstract objects, we find few (arguably no) knock-down, iron-clad, settled-once-and-for-all arguments for, or against, the existence of most of the abstract objects that interest philosophers. Instead, the evaluation of the arguments involves the art of making trade-offs, the weighing of philosophical costs and philosophical benefits. I will urge that although there are widely shared, quite sensible criteria for this, they fall short of providing rules or a recipe that forces a uniquely correct answer to the question of which, if any, abstract entities exist. Benefits rarely come without costs, and we will examine some of the costs of abstracta in section 4. In this section we will consider some of their benefits.

There are many candidate abstracta and there is space to discuss only one. I will focus on the natural (0, 1, 2, and so on up forever), because this example will be familiar to readers with little background in philosophy.

3.1 Numbers

Target range for philosophy of mathematics There is no unanimity about precisely which mathematical phenomena are legitimate targets for philosophical explanation, but in the case of number theory (basic arithmetic), there is widespread agreement about the following.

- 1 The sentence ' $7 + 5 = 12$ ' is true, and its truth is independent of our beliefs and opinions. This is also the case for many other sentences of arithmetic. Similarly, many other sentences of arithmetic, like ' $7 + 5 = 13$ ' are false, and are so independently of what we happen to think. The *truth value* (either truth or falsity) is independent of our beliefs and opinions.
- 2 Statements of arithmetic *necessarily* have the truth values they do; ' $7 + 5 = 12$ ' could not have been false under any circumstances and ' $7 + 5 = 13$ ' could not have been true.
- 3 Quite apart from questions about language and truth, it is the case that $7 + 5 = 12$ but it is not the case that $7 + 5 = 13$. And the first is necessarily the case and the second, necessarily, is not.
- 4 There are infinitely many natural numbers, and necessarily so (there could not have been fewer).
- 5 The grammatical structure of the sentence '3 is prime' parallels that of 'Sam is tall'. In the later case the subject term, 'Sam', is standardly thought to denote a real object, the person Sam, and the sentence is true because the thing 'Sam' denotes is tall. This suggests that in '3 is prime' the numeral '3' might denote something, and that the sentence is true because the thing it denotes is a prime number.
- 6 We can employ standard logic in reasoning about arithmetic; the normal, logically valid patterns of inference apply. For example, the step from 'Sam is tall' to 'There is something that is tall' is a valid inference, both intuitively and in standard systems of logic. So too is the step from '3 is prime' to 'there is something that is prime'.
- 7 The claim 'there is something that is prime' follows from a true sentence ('3 is prime') and seems, quite independently, to be true. But the claim that there is such a thing is just our ordinary, paradigm way of saying that something exists, that it is genuine or really there. Perhaps this is not always the case, but it typically is. So we at least seem to be committed to the view that there is something that is a prime number.
- 8 It is possible to have reliable justified beliefs and, indeed, knowledge in mathematics.
- 9 Much of our mathematical knowledge is a priori. This means that we do not need to learn, and almost never justify, our claims in arithmetic by appeal to experience. Once we know what '1' and '2' and '+' and '=' mean, we just see that $1 + 1 = 2$.

The list isn't complete, and some of the items (e.g., 1) may be more central than others (e.g., 2). Still, the more of these targets a philosophical account can explain,

the better. As we will see, however, the features that enable a theory to explain some of these phenomena sometimes make it difficult for it to explain others.

Sample explanations in mathematics using abstracta A wide array of philosophical accounts have been developed to explain these targets. I will discuss one of the simplest approaches that employs abstracta.

Here is the metaphysical story. The natural numbers are objects or entities, though ones of a very special kind. They are abstract, existing outside of space and time and the causal order. There are infinitely many of them (what logicians call a denumerable infinity of them). They do not change. They exist necessarily (they could not have failed to exist), and they necessarily have the properties and stand in the relations that they do (it is necessarily the case that 13 is a Fibonacci number and that $13 > 7$).

This metaphysical picture allows us to explain item (3) in a very straightforward way. It's the case that $7 + 5 = 12$ because that is just how things are with these mind-independent, objective entities, the numbers – in particular with 7, 5, and 12. And there are infinitely many natural numbers item (4), because that is just how many of these entities there are (nothing deep here).

The purely metaphysical picture may also seem to explain (1) and (2), but to account for matters involving truth, we have to say something about meaning or semantics. Here, as is often the case with accounts of abstract entities, we need to make one or more additional assumptions, *auxiliary hypotheses*, in order to use those entities to explain the targets we want them to explain.

Here we need some *semantic* auxiliary hypotheses like the following. First, numerals are singular terms, ones that can occupy subject positions in sentences, and they denote the appropriate numbers ('0' denotes 0, '1' denotes 1, and so on out forever). Moreover, numerical terms like these would denote the same things in any possible situation (so they are what philosophers call "rigid designators"). Predicates like 'prime number' stand for the property of being a prime number and relational predicates like '<' stand for the relation of being a smaller natural number (I won't worry here about what these really are). Finally, function expressions like '+' and '' stand for numerical functions like the addition function (which outputs 5 when you input 2 and 3) and the multiplication function (which outputs 6 when you input 2 and 3). This isn't the entire story, but it is enough for us to see the basic ideas about how the explanations here work.

We then say that a sentence of the form ' n is P ', where n is a name of a natural number (e.g., the numeral '3') and P is a predicate (e.g., 'even'), is true just in case n refers to a number that has the property that the predicate P stands for. Similar stories are told for relation and function terms. All of this is a bit loose, but since the work of Alfred Tarski in the 1930s, we know how to make it completely precise. The interested reader can find the details in any good introductory text on symbolic logic, but they aren't needed to appreciate the basic ideas here.

We can now explain why '3 is prime' is true and '4 is prime' is false: '3' stands for an abstract object, the number three; 'prime' stands for the property of being prime, and three has that property. By contrast, '4' stands for an abstract object, the number four, that lacks the property. Similar accounts explain why ' $5 < 7$ ' and

' $7 + 5 = 12$ ' are true and ' $7 < 5$ ' and ' $7 + 5 = 13$ ' are false. This explains item (1) on the list. Moreover, since numerical terms necessarily stand for the things that they do, and because the natural numbers necessarily exemplify the properties and stand in the relations that they do, these claims necessarily have the truth values they do item (2).

Simple sentences of arithmetic appear to have a simple subject-predicate structure (item 5; when relation or function terms are involved there is more than one subject term, with '5' and '7' being the two subject terms of ' $5 < 7$ '). We can now explain this because, given the machinery invoked in our explanations thus far, this is exactly the structure such sentences *do* have. And we can apply standard logic in a completely straightforward way to explain why normal logical inference rules are valid when we apply them to arithmetical sentences (item 6). For example, existential generalization works because, if we take a true sentence like '3 is prime', we know that it is true because '3' stands for something (the number three) that is prime. Hence it follows that there is something (three) that is prime. And on this account this sentence does indeed make a true existence claim, telling us that there really is something (once again three) that is prime (item 7).

Items (8) and (9) differ from the preceding seven insofar as they involve notions like justification and knowledge. These are *epistemic notions*, ones studied in the philosophical field known as the theory of knowledge or epistemology (from the Greek *episteme*, 'knowledge', and *logos*, 'theory'). This is the area of philosophy that deals with knowledge and related concepts like justification. Although this is a different field from ontology, claims about ontology meet up with questions in epistemology when we ask whether, and if so how, we can know about abstract entities.

Justification in arithmetic (and in mathematics generally) often proceeds by way of calculations and, at more advanced levels, proofs. These are based on, indeed are little more than chains of, logically valid patterns of inference. Our previous machinery justifies the application of logic in arithmetic, and so explains some features of mathematical justification. If we are already justified in believing that '3 is odd', we are then also justified in believing 'there is something that is odd'. This is so because existential generalization is a mini-valid argument pattern, so if the first sentence is true, the second must be true as well.

But our reasoning must begin somewhere. How do we justify those of our arithmetical beliefs that we don't prove? How do we justify our belief (assuming we take it as basic) that $1 + 0 = 1$? Alas, accounts like the one so far that seem well equipped to explain phenomena (1)-(7) founder when we come to (8) and (9). (The classic discussion of this difficulty is Benacerraf, 1973.)

The basic problem is that since numbers are abstract, they lie completely outside the spatiotemporal order. We seem unable to achieve *any* sort of contact with them. We can't see numbers, touch them, point to them, measure them. Nor do they cause things we can see or touch or point to or measure. So how do we ever learn anything about numbers? Since all of us know that five is an odd number, we do, somehow, know something about them. On the present account, this knowledge is about an abstract object, namely the number five, though of course a person may not think of it as being an abstract object, perhaps never having heard of such things. But how? The problem is serious enough that we will defer it in order to treat it in some detail below.

There are competing explanatory accounts of our nine phenomena that employ abstracta other than numbers (especially sets, but also properties, categories, and structures). They have many of the same costs and benefits as our simple account using numbers, however, and I will not discuss them here. Finally, we should note that strategies like the one sketched above can be applied in many other parts of mathematics by postulating additional abstract objects, e.g., irrational numbers, complex numbers, and other sorts of mathematical entities (like points, lines, groups, vector spaces).

Lessons the explanations teach us about these abstracta We know a good deal about numbers before we ever study philosophy, so the present philosophical explanations aren't likely to provide much novel information about their nature (other than telling us that they are abstract). But in the case of less familiar abstracta (like properties or propositions), the explanations might well shed light on the nature of the entity in question. I will say something about what this involves here in the case of numbers to illustrate the sort of thing that is involved in inquires about the nature of any sort of abstract entity.

There are at least four things philosophers often want to know about a given sort of entity: its existence conditions, its identity conditions, its modal status, and its epistemic status.

Existence conditions There may not seem to be much philosophical interest in the existence conditions of natural numbers, since we already know which numbers there are (0, 1, 2, 3, . . .; anything you can get by starting with 0 and adding 1 as many times as you like). But with less familiar notions, like that of complex numbers or vector spaces, we typically want to know their existence conditions. Under what conditions is something a complex number? Which (putative) items of that sort exist? The aim is to provide necessary and sufficient conditions for something being a complex number. To take another example, in set-theory very elaborate conditions are laid down for telling us which sets exist.

This model is sometimes carried over from mathematics to philosophy, where philosophers ask for the existence conditions for various sorts of non-mathematical abstracta like properties and propositions. It is a matter of debate whether asking for necessary and sufficient conditions of this sort unreasonably assimilates philosophy to mathematics, but obviously the more their proponents can say about which properties or propositions there are, the better. For example, can there be properties that are not exemplified? Again, assuming that being round and being square are properties, are there also properties like being round or square or being round and square?

Identity conditions If x and y are abstract objects, can we provide necessary and sufficient conditions for them being one and the same object (in the way that 2 and the positive square root of 4 are the same, but 2 and the negative square root of 4 are not)? In the case of numbers, we can typically answer specific questions of this sort by calculation or proof, but can we give general identity conditions that apply to *all* natural numbers in a fell swoop? If x and y have exactly the same numerical properties

and stand in exactly the same numerical relations, it then turns out that they must be identical, the self-same number. But we might like conditions that throw more light on what it is to be a natural number. By way of example, if x and y are sets, then x and y are identical just in case they contain exactly the same members. Here we get identity conditions that are specifically geared to sets (in terms of the notion of set membership), and so are more enlightening about their specific nature.

Identity conditions are important in mathematics, and as with existence conditions it is possible to worry that requiring identity conditions for a given sort of abstract object as a precondition to granting its existence (or even to discussing whether it exists or not) is an unreasonable demand. After all, philosophers have thus far not been very successful at spelling out precise identity conditions for physical objects or for persons – but we all know perfectly well that such things exist. Of course an account of a given sort of abstract object should tell us as much as possible about that object, so we would like to know as much as possible about when x and y are identical, even if this falls short of full necessary and sufficient conditions for identity.

Modal status Do the abstract entities invoked in our explanations exist necessarily (they simply couldn't have failed to have existed) or merely contingently (they might not have existed)? Second, we may ask which features of, and relations among, these entities (e.g., being an even number) belong to them necessarily (in any circumstances in which they could exist) and which features only belong to them contingently (they could have existed without having them).

Our hope is that if the answers to questions about the nature of a given sort of abstract entity aren't obvious before developing explanations employing that entity, the explanations themselves will help us answer these questions. In the present case, we hope to see what the modal status of a postulated abstract entity *must be* in order to explain some of the targets it is supposed to explain. In the explanation sketches of the nine phenomena (listed above on page ●●), the answer is that the numbers necessarily exist and that they necessarily have the properties and stand in the relations that they do. We must conclude this in order to explain items (2), (3), and (4) above.

Epistemic status The most basic epistemological question about an abstract entity we have reason to believe exists is how we can know about it. We can't reasonably expect a detailed scientific answer to such questions at this stage in history, but it would be very useful to be given a general idea. By way of analogy, there is much that we don't currently understand about visual perception. But we have enough of a general idea how visual perception works to see that it is a normal, natural, causal process involving the reflection of light off objects to the backs of our retinas, there stimulating nerves and setting off various electro-chemical reactions that, in turn, trigger processes in the visual cortex and other parts of the brain. Admittedly, we don't understand the conscious aspects of the visual experience itself, but at least they occur in time, surely in space (the brain – besides, you can't really separate time and space), and involve some sorts of natural, neural causal processes. It would be good to have at least a little detail of this general sort in order to shed light on the way we know about abstract objects.

In the process of answering these questions, we may get an answer to the further epistemic question of whether our knowledge about a given sort of entity (here, arithmetical knowledge about the natural numbers) is a priori or not. We began by assuming it was, as is traditional, though the account we examined didn't yield a very satisfying explanation of how this could be so (there are other accounts that do, but they have trouble explaining earlier items on the list). But in the case of at least some other abstract entities, for example properties, there is some debate as to whether our knowledge about them is a priori, i.e., attainable independently of experience (save for enough experience to acquire the concept of them) or a posteriori (based on experience). In those cases we might hope that our explanations of our knowledge about entities that did the jobs properties were invoked to do had to be a priori if properties were to do those jobs.

Evaluating explanations in ontology We can rarely explain much with the bald assertions that numbers exist or that properties exist. These claims are typically part of a longer story, a philosophical theory, that tells us something about what the relevant abstract entity is like. The theory also needs to explain how the entity is related to other things, including other abstract entities (if any – since theories often invoke more than just one sort of abstract entity, e.g., accounts in semantics often employ both properties and propositions). The account also needs to tell us how its abstracta are related to the phenomena around us that led us to postulate them in the first place.

To take a non-mathematical example, a full account of properties should tell us something about which sorts of things have properties (e.g., can properties themselves exemplify properties?), and should at least provide the resources for dealing with questions like whether properties include colors, shapes, and masses. How are properties related to those things that have them, i.e., what does exemplification amount to? Answers to such questions help us *apply* the theory of the abstract entity, bridging the gap between the abstract realm and the typically concrete phenomena we want to account for. And an especially important part of an account of abstracta is to tell us at least enough to see that their connection to our cognitive faculties is not hopelessly problematic.

Desiderata There are various desirable features of ontological explanations, features that, other things being equal, make an explanation more compelling.

Do more with less This injunction can take various forms. The fewer unexplained (primitive) entities, the better. If two primitive abstract entities will explain the targets in a domain, don't use six to do so. The motivation here is general and somewhat vague, but it is important and has a venerable history. The great Medieval philosopher, William of Ockham (c. 1287–1347), counseled philosophers “not to multiply entities beyond necessity.”

This precept has become known as Ockham's Razor, but, as everyone who writes on the matter soon observes, Ockham's exhortation was to avoid multiplying entities beyond necessity. So the relevant question is always whether a given sort of abstract entity is necessary, which typically means: is it *required* in order to *explain* any philosophical targets? The answers to such questions are often

controversial, so although we can agree that, if a short simple theory works as well as a long and complicated one, the former is better, in practice, the wielding of Ockham's Razor is often contentious.

Breadth and depth of coverage are important The more of the nine arithmetical phenomena (and, indeed, the more additional phenomena) a philosophical theory can explain, the better. Similarly, it counts in favor of a theory of meaning based on properties if it can explain the semantic behavior of different constructions of English (e.g., 'Sam is tall' and 'Sam is taller than Jill').

Explain why rival accounts work as well as they do It is useful if an explanation illuminates why competing accounts work (in those places where they do work) and fail (where they fail).

Explain which things need explaining It is also good if an account can illuminate what should, and what should not, be on the list of targets it is used to explain. And if it explains a traditional target away (showing that it doesn't really exist), it needs to provide arguments for doing so.

Don't solve one problem only to create another just as bad It is important to explain a target (e.g., in semantics) without creating new problems elsewhere (e.g., in epistemology).

This list isn't exhaustive, but it illustrates some of the commonly accepted and central desiderata for explanations in ontology. Unfortunately, these desiderata can be in tension. For example, we can sometimes get by with fewer primitives by sacrificing breadth of coverage. Hence, these goals do not add up to rules or recipes that always tell us which of several competing philosophical explanations is best, and this remains the case even if we add further plausible desiderata. But it is important that they often can tell us that certain explanations are not very good.

Constraints There are also constraints on ontologically satisfactory explanations. Some are nearly universal (e.g., consistency, though even that has been challenged lately). Others vary with time or schools of thought, and some reflect quite idiosyncratic philosophical scruples or ideals. Constraints and desiderata fade into one another, but the importance of the former should not be underestimated.

For example, in various periods there have been religious constraints on metaphysical explanations. In medieval disputations about properties, issues involving faith, reason, and the nature of God were never far from view. Indeed, these matters often provided explanatory targets for metaphysicians. Philosophical orientations also provide constraints. For example, many philosophers have argued that knowledge must be grounded in experience. We cannot simply reason out what the world is like from the armchair; we have to go and check. Today, naturalistic world-views are popular, and these are often thought to allow only physical entities, or at most only entities that exist in space-time.

In concrete historical settings, constraints can seem very real, sometimes inevitable, even if at a latter time they seem arbitrary, even quaint. This needn't make metaphysics "subjective" in any debilitating sense (so that whatever a particular culture happens to think about it is "true for them"). But it is a useful reminder that metaphysics, like

any other intellectual enterprise, is a human endeavor that takes place in, and is highly colored by, a time, culture, and tradition.

Difficulties with competitors The best available ontological explanation must meet some minimal threshold of goodness to justify belief in its conclusion. Moreover, the notion of the best available explanation is comparative; a theory doesn't get many points for explaining something if a rival theory explains it much better. Hence, arguments for the existence of a particular sort of abstract entity often need to be bolstered by criticisms of opposing theses. For example, the view that properties are needed to play the role of semantic values of predicates is stronger when accompanied by arguments that other sorts of entities, e.g., sets (of the things in the extension or the predicate), cannot play the role nearly as well. Finally, being the best explanation doesn't mean being perfect. Virtually all philosophical accounts have open problems, and trying to solve them is part of the day-to-day work of philosophers.

Quandaries and doubts I have spoken as though inference to the best ontological explanation were relatively unproblematic, but there are various places where objections to it, and especially to its use in philosophy, can be raised. I have discussed these matters elsewhere (e.g., Swoyer 1983; 1999a; 1999b), however, and will not pursue the matter here.

4 Pluses and Minuses

Metaphysics, like life, often presents us with diverse, not fully compatible, goals that require us to make trade-offs, weigh costs against benefits, make hard decisions. In this section we consider the chief benefits, and costs, of abstract entities.

4.1 Benefits

The primary philosophical attraction of abstract entities is that they seem to offer so much explanatory power. For example, when we encounter words or phrases that look like denoting singular terms (e.g., '3', 'courage') we can explain this very neatly by arguing that they *are* singular terms and that they denote an abstract entity (a number, a property). The realist can often avoid denying the existence of relatively obvious phenomena (like the existence of mathematical truth, as some anti-realists about numbers and sets do), needn't urge that we have been badly in error about entire realms of discourse (like mathematics), and can avoid resorting to tortured paraphrases to evade ontological commitment. Indeed, the more luxuriant lines of abstracta (e.g., Russell 1903; Zalta 1988; Bealer 1982) contain so much metaphysical machinery that it is almost a foregone conclusion that they can explain any phenomenon that comes their way. All this sounds a little too good to be true. Is it?

4.2 Costs

Ockamist impulses and ontological economy Few philosophers like ontological bloat. Other things being equal, a good explanation of a philosophical target

that doesn't rely on abstracta is preferable to a good explanation that does. But other things are rarely equal. Abstract objects often add enough explanatory power that theories invoking them can give broader and smoother explanations of a target than theories that do not. For example, it is very difficult (though a number of philosophers believe not impossible) to give an account of mathematical truth that does not employ abstracta of any sort. So while ontological economy is important, other things are rarely equal, so it is rarely decisive.

Anti-realism: there *are* alternatives There is always anti-realism, so perhaps we shouldn't feel driven to abstracta as the only game in town. There are many forms that opposition to realism takes nowadays, as new positions (e.g., fictionalism, projectivism, error theories) spill over from the philosophy of mathematics, the philosophy of science, and meta-ethics into philosophy generally. Furthermore, with the demise of behaviorism, philosophy's linguistic turn is beginning to show its age, and the rise of cognitive science, and various flavors of conceptualism, are once again on the menu (e.g., Swoyer 2005). Still, none of these alternatives provides a strong reason for avoiding the need for a given sort of abstract object to explain a legitimate philosophical target unless the anti-realist explanation (or dismissal) of it is spelled out in a reasonably detailed and compelling way. So again, we must consider each approach case by case.

Epistemic access Epistemology is the Achilles' heel of realism about abstracta. We are biological organisms thoroughly ensconced in the natural, spatiotemporal causal order. Abstract entities, by contrast, are atemporal, non-spatial, and causally inert, so they cannot affect our senses, our brains, or our instruments for measuring and detecting.

A few philosophers have postulated a cognitive faculty of intuition that provides some sort of non-causal access to numbers or other abstracta. The nature of this access has never been explained, however, and many of us find nothing like it in our own perception and thought. Scientists have no inkling where it is located in the brain, and it has yet to turn up in any empirical studies. Empirical investigation of thought that (might) seem to be about abstracta is becoming more common (e.g., Boroditsky and Ramscar 2002), and it may eventually illuminate the issues here. At present, however, it doesn't get at the most basic problems that have worried philosophers about our cognitive access to abstracta.

Perhaps knowledge about abstracta doesn't require contact with them. The only remotely plausible story about this would seem to be that such knowledge is innate. This may well be true of our rudimentary knowledge of arithmetic, but it doesn't scale up well to knowledge about tensor algebra or the semantic values of words for describing the nuances of medieval chivalry.

The epistemic problems here do not stem from any (almost certainly hopeless) causal theory of knowledge, but simply from the fact that our acquisition and justification of beliefs about things lying outside the spatiotemporal causal order is more than a little mysterious. Indeed, even if abstracta did exist, it is difficult to see how they could make any difference to our cognitive processes. Things would seem just the same whether they existed or not, or if they existed up until tomorrow, then suddenly vanished.

Reference and non-uniqueness Nowadays a major reason for postulating abstracta is to use them as semantic values in semantic accounts of natural languages. Unfortunately, the epistemic problems abstracta generate make it difficult to use them for this purpose. We can't make epistemic contact with abstracta, so it is difficult to see how we could get our words to latch onto them. We can't single numbers out, by pointing or in any other obvious way, and say 'that is 0', 'that is 37', and so on. We might try to pick 0 out by saying that '0 is the first of the natural numbers', but this doesn't really help unless we have pinned down the reference or extension of 'natural number' and (less obviously) that of 'first' (as it applies to the sequence of natural numbers). So we are back with the original problem. We can't make identifying reference in language because we can't make identifying reference in thought.

In some cases, particularly in mathematics, we can specify the structure of a given realm of abstract entities. For example, we can pin down the structure of the natural numbers with some sophisticated logic (with what is known as a second-order version of Peano's Postulates). But if there is one group of things with this structure, there are many, and there is little reason to suppose that any one of them gives the unique metaphysical truth about "What Numbers Really Are" (cf. Benacerraf 1965). Because we lack epistemic contact with numbers, we can only describe the structure of the realm of numbers, and such descriptions underdetermine the denotations of our numerical vocabulary. So, ironically, the apparent success of our earlier explanations for the semantic features of numbers seems undermined by the problems with epistemic phenomena.

5 Conclusion

So . . . are there abstract entities? And if so, which ones? The answers depend on the answers to three prior questions. Is inference to the best available overall ontological explanation ever legitimate? If so, when? And when it is, how do we adjudicate among competing explanations? My answers are more tentative than I would like, but this is a conclusion, so I will end by drawing some.

Is the game optional? If someone won't play the metaphysical game, there are no knock-down, non-question-begging arguments to show she is wrong. We can cite reasons for, and against, the possibility of inference to the best ontological explanation, but none of them comes close to being conclusive. Indeed, if I am right, differences of beliefs in ontology very often stem from differences of beliefs about the legitimacy and nature of inference to the best explanation in ontology.

Evaluating competing explanations The gist of the discussion thus far is that evaluation of rival explanations in ontology is a global affair that requires sound philosophical judgment rather than a reliance on hard and fast rules (the problem is that there are no such rules, though there are rough but generally accepted guidelines, so that not just anything goes). The process is global or holistic, in the sense that it depends on the weighing of many different considerations at the same time. And

although the decisions that must be made in evaluating competing programs are usually made in light of shared philosophical values, there doesn't seem to be any uniquely correct way to trade such values off against each other. For example, other things being equal, more explanatory power, breadth of coverage, and simplicity are better than less. But then, when are things ever equal? And when they are not, is it better to have a richly detailed explanation of a narrower range of phenomena or a less detailed explanation of a wider range?

Disagreements about simplicity Arguments over simplicity play a prominent role in debates in ontology, sometimes crowding out consideration of other important explanatory virtues. The verdict of simplicity is rarely unequivocal, however, and judgments about it differ from one philosopher to another. Still, some philosophical disputes actually come down in print to questions about whether two basic, undefined, primitive objects and one basic, undefined, primitive relation are simpler than one primitive object and two primitive relations. Such considerations are surely much too fragile to support conclusions about the "ultimate nature of reality," as if "What There Really Is" could come down to whether an account employs two primitive notions, rather than three.

The fundamental ontological trade-off There is an even more fundamental trade-off that we face at every turn in philosophy, from ethics to philosophy of science to philosophy of mathematics to metaphysics. I will call it the *fundamental ontological trade-off*. This is the trade-off between explanatory power, on the one hand, and epistemic credibility, on the other; between a rich, lavish ontology that promises a great deal of explanatory punch, and a more modest ontology that promises more epistemological security and believability. How a philosopher strikes a balance in this trade-off goes a long way to determining whether or not she will believe there are abstract entities.

The more machinery (especially abstract machinery) we postulate, the more we might hope to explain – but the harder it is to believe in the existence of all that machinery. Russell makes this sort of point in his famous theft-over-honest-toil passage: "The method of 'postulating' what we want has many advantages; they are the same as the advantages of theft over honest toil. Let us leave them to others and proceed with our honest toil" (Russell 1919: 71). But without at least a little postulation, it is very difficult to even get started.

The upshot Once ontological explanation is allowed and (rough-and-ready) ground rules are set, there can be winners and losers and perhaps a spectrum of views in between, but it is important that not everyone who plays the metaphysical game gets to win. For example, although Goodman and Quine's (1947) celebrated attempt to provide an account of mathematics that avoided all abstracta remains impressive, it simply cannot account for enough features of mathematics to be judged a success – even by Quine. But once we eliminate the more unpromising explanations, we may well be left with more than one contender.

In short, if ontological explanations are legitimate, it is unlikely that there will be uniquely correct explanations, and so unlikely that we will arrive at a single picture

about which abstracta (if any) there are. Perhaps we can make slow progress to this goal, with a series of explanations zeroing in more and more on the existence and nature of various abstracta. But such a series may instead lead to a fragmentation of entities, with a corresponding fragmentation of our views about them.

If epistemology isn't a problem, then abstracta win

If inferences to the best ontological explanation are legitimate, and if the epistemic problems about cognitive access to abstract entities can be overcome, then the case for at least some abstract entities is very strong. This is so because we can explain much more with them than without them. But the epistemological problems are severe.

A parting thought Still, would it really be so bad if the best we could do was to rule out some accounts in ontology and learn to live with more than one survivor? Perhaps developing a tolerance for more than one (which need not mean every) ontological framework is the best we can do. If we can do this without falling into some dreadful sort of relativism, maybe that's good enough.

Note

- 1 Many discussions of abstract objects are rather technical, but in the interests of accessibility I will steer clear of such complexities and avoid logical notation (the interested reader can find many of the more technical matters discussed in some of the works cited here). Because the existence of various sorts of abstract objects, and indeed abstract objects in general, is a matter of contention, prudence suggests constant qualifications like "putative" examples of abstract entities and talk that "seems" to be about them. But this becomes tiresome and I will mostly leave such hedges tacit. I am grateful to David Armstrong, Hugh Benson, Monte Cook, Brian Ellis, Ray Elugardo, Jim Hawthorne, Herbert Hochberg, Chris Menzel, Adam Morton, Sara Sawyer, Ted Sider, Shari Villani, and Ed Zalta for helpful discussions on the topics discussed here.

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