

## CHAPTER 1

# Basic Concepts of Logic

Logic is the study of correct reasoning. Logic pertains to all subjects, since people can reason about anything they can think about. Politics, the arts, literature, business, the sciences, and everyday problems are all subjects open to reasoning. Sometimes the reasoning is good; sometimes, not so good. People use logic to tell the difference.

Using logic, we can evaluate bits of reasoning as proper or improper, good or bad. Logic is not the study of how people *do* reason, but how they *should* reason. In this sense logic is like arithmetic. Arithmetic describes the rules for addition rather than the psychological process of addition. Just so, logic describes not the psychological process of reasoning but the rules for correct reasoning. Logic does not describe real reasoning, with its errors, omissions, and oversights; it prescribes methods for *justifying* reasoning; that is, for showing that a given bit of reasoning is proper. Logic thus describes an ideal that actual reasoning strives for but sometimes fails to reach.

Logic begins with the study of language. To develop a system of logic, it is necessary to understand how people actually reason. To eliminate the errors that creep into people's performance, we need to examine people's considered judgments about the correctness or incorrectness of inferences. No matter what mental processes people go through to achieve the right result, they try to follow rules for putting sentences together to form proper bits of reasoning. Logicians examine people's evaluations of bits of reasoning to say what the rules of correct reasoning are. Logic describes not the process of reasoning but the rules for correct reasoning.

### 1.1 ARGUMENTS

Arguments represent reasoning in language. Frequently, we think of arguments as heated debates, disagreements or disputes. Sometimes, however,

we speak of a politician arguing for the passage of a bill, a lawyer arguing a case, or a moviegoer arguing that *North By Northwest* is better than *The 39 Steps*. An argument in this sense starts with some assertions called premises and tries to justify a conclusion.

Many arguments in natural language are complicated. A lawyer arguing for the innocence of a client, for instance, offers many more specific arguments in presenting the case. The lawyer may argue that a piece of evidence is inadmissible, that results from a laboratory test are ambiguous, that the client could not have reached the scene of the crime by the time it was committed, and so on. All these smaller arguments form part of the larger argument for the client's innocence.

We can divide arguments, then, into two groups: extended arguments, which contain other arguments, and simple arguments, which do not. Extended arguments may have several conclusions. Such arguments may consist of several simple arguments in sequence. They may contain other extended arguments. And they may consist of a list of premises, followed by several conclusions stated at once.

Mathematical proofs are extended arguments. A mathematician may begin a proof by stating some assumptions. The mathematician then draws out consequences of the assumptions, perhaps making other assumptions along the way. Finally, the proof ends with a conclusion – the theorem it proves. A mathematical proof is thus a series of simple arguments.

A simple argument, like an extended argument, starts with premises justifying a conclusion. We will be so often concerned with simple arguments that we will drop the adjective simple and speak of arguments. (Later, when we examine proofs, we will just call them proofs.)

An *argument* consists of a finite sequence of sentences, called *premises*, together with another sentence, the *conclusion*, which the premises are taken to support.

An argument in ordinary language or in mathematics is a string or sequence of sentences. The sentences making up the argument are in a particular order, whether the argument is spoken, written, or encoded in a computer language. For our purposes in this text, the order of the premises makes no difference. So, we will not worry about order of presentation. But we will require that the string of premises be finite. No one has the patience to listen to an argument that runs on forever. If the premises never come to an end, the conclusion is never established.

Arguments consist of sentences. In this text, we will be interested only in sentences that can be true or false. Many ordinary sentences, including almost all in this book, fall into this category. They say something about the way the world is, and might be correct or incorrect in so describing it. But commands, for example, are different. *Shut the door* can be appropriate or inappropriate, irritating or conciliatory, friendly or hostile, but it cannot be true or false. Questions, such as *What is the capital of Zaire?*, and interjections, such as *Ouch!*, are likewise neither true nor false.

A sentence is true or false in a particular context: as used on a particular occasion by a particular speaker to a particular audience, in a given circumstance and as part of a discourse. Without contextual information, we cannot say whether a sentence such as *I love you* is true or false. Sentences have truth values – that is, are true or false – only relative to a context of use.

Nevertheless, very little in the following pages will involve context directly. So, we will generally speak of sentences as having truth values, trusting ourselves to remember that these values are relative to context.

A simple argument, according to our definition, contains one sentence that is its conclusion. This is an idealization: in natural language, a conclusion may be a clause in a sentence; it may be spread across several sentences; or it may be left unstated. The same is true of premises.

The definition does not specify how to pick out the conclusion of an argument. In English, certain words or phrases typically signal the conclusion of an argument, while others signal premises:

- conclusion indicators – *therefore, thus, hence, so, consequently, it follows that, in conclusion, as a result, then, must, accordingly, we may infer that*
- premise indicators – *because, for, since, as, given that*

All these words and phrases have other uses; they are not always premise or conclusion indicators. But these words and phrases can, and often do, serve as indicators because they can attest to relations of support among the sentences of an argument. *Since Fred forgot to go to the interview, he won't get the job* presents a simple argument within a single English sentence. The word *since* indicates that we should take *Fred forgot to go to the interview* as a premise, supporting the conclusion *he won't get the job*. Similarly, *Jane's business must be doing well; she drives a Mercedes* constitutes an argument. The auxiliary verb *must* marks *Jane's business is doing well* as the conclusion, supported by the evidence in *she drives a Mercedes*.

Premise indicators often signal not only that one or more sentences are premises, but also that a certain sentence is a conclusion. *Since*, for example, exhibits a relation of support between the sentences it links. Its occurrence in *Since Fred forgot to go to the interview, he won't get the job* points out, not only that the sentence immediately following it is a premise, but also that the sentence *he won't get the job* is a conclusion. Similarly, the occurrence of *for* in *Northern Indiana Public Service will not pay its usual dividend this quarter, for the court refused to allow expenditures on its now-cancelled nuclear project into the rate base* indicates both that *the court refused to allow expenditures on its now-cancelled nuclear project into the rate base* is a premise and that *Northern Indiana Public Service will not pay its usual dividend this quarter* is a conclusion.

Indicators provide important clues to the structure of arguments. Often, however, no explicit indicators appear. Sometimes the conclusion is not even stated. In such cases, we must consider the point of the argument. What is the author trying to establish? Knowing a language and its uses in context often allows us to recognize even unstated assumptions and conclusions.

Consider an example:

Suppose we argued that what was true was true for us, that two assertions met on no common ground, so that neither was “really true” or “really false.” This position went further than skepticism and declared the belief in error itself to be erroneous. Royce called this view that of the total relativity of truth, and he had an argument against it. If the statement “There is error” is true, there is error; if it is false, then there is, ipso facto, error. He could only conclude that error existed; to deny its existence was contradictory.

Bruce Kuklick, *The Rise of American Philosophy*

This is an extended argument. The conclusion of Royce's smaller argument is plainly *error exists*; the words *conclude that* make this obvious. Royce then uses this conclusion to argue that the view of the total relativity of truth is false. The sentence *error exists* thus functions as the conclusion of one argument and as a premise of another, all within the same extended argument.

When we write an argument “officially,” in standard form, we will list the premises in the order in which they are given, and then list the conclusion. So, in our official representations, conclusions will always come last. This is not always true in natural language; conclusions may appear at the beginning, in the middle, or at the end of arguments, if they are stated at all. In addition, we'll preface the conclusion with the symbol  $\therefore$ , which means “therefore.”

To see how these representations work, let's write Royce's smaller argument in standard form:

If the statement "There is error" is true, there is error.  
 If the statement "There is error" is false, there is error.  
 ∴ There is error.

Royce's larger argument, then, is:

Error exists.  
 The view of the total relativity of truth holds that the belief in error is erroneous.  
 ∴ The view of the total relativity of truth is false.

To take another example:

If it were permitted to reason consistently in religious matters, it is clear that we all ought to become Jews, because Jesus Christ was born a Jew, lived a Jew, and died a Jew, and because he said that he was accomplishing and fulfilling the Jewish religion.

Voltaire

Voltaire seems to be arguing for the conclusion we all ought to become Jews. Here the key word is *because*, which indicates that the rest of the argument is a list of premises. Voltaire, a satirist, is really aiming not at this conclusion but at another. Everything he says is supposed to follow from the hypothetical *if it were permitted to reason consistently in religious matters*. Like Royce, he is offering an argument within an extended argument. The conclusion of the extended argument is not stated. Nevertheless, it is easy to see that Voltaire is trying to establish that it is not permitted to reason consistently in religious matters. The conclusion of the smaller argument – we all ought to become Jews – is an observation that few Christians in Voltaire's intended audience would be willing to accept, even though, according to Voltaire, their own doctrine commits them to it.

We can express Voltaire's two arguments in standard form:

Jesus Christ was born a Jew, lived a Jew, and died a Jew.  
 Jesus Christ said he was accomplishing and fulfilling the Jewish religion.  
 ∴ If it were permitted to reason consistently in religious matters, it is clear that we all ought to become Jews.

If it were permitted to reason consistently in religious matters, it is clear that we all ought to become Jews.

(It is not clear (to religious Christians) that we all ought to become Jews.)

( $\therefore$  It is not permitted to reason consistently in religious matters.)

A final example is a mathematical proof – the traditional proof that the square root of 2 is irrational:

[Suppose] for the sake of argument that  $\sqrt{2}$  is rational, i.e. that there are two integers, say  $m$  and  $n$ , which are mutually prime and which are such that  $m/n = \sqrt{2}$  or  $m^2 = 2n^2$ . From this it follows that  $m^2$  must be even and with it  $m$ , since a square number cannot have any prime factor which is not also a factor of the number of which it is the square. But if  $m$  is even,  $n$  must be odd according to our initial supposition that they are mutually prime. Assuming that  $m = 2k$ , we can infer that  $2n^2 = 4k^2$ , or  $n^2 = 2k^2$ ; and from this it can be shown by a repetition of the reasoning used above that  $n$  must be even. Our hypothesis, therefore, entails incompatible consequences, and so it must be false.

W. Kneale and M. Kneale, *The Development of Logic*

Like almost any proof, this one is an extended argument; in fact, it is a series of simple arguments. The proof begins with the assumption that  $\sqrt{2}$  is rational. The first simple argument concludes that  $m^2$  must be even; very quickly follows another simple argument concluding that  $m$  must also be even. The third simple argument concludes that  $n$  is odd. The fourth concludes that  $2n^2 = 4k^2$ ; the fifth, that  $n^2 = 2k^2$ ; the sixth, that  $n$  is even. Finally, the proof ends with a seventh simple argument that the hypothesis that the square root of 2 is rational is false:

$\sqrt{2}$  is rational, i.e., there are two integers, say  $m$  and  $n$ , which are mutually prime and which are such that  $m/n = \sqrt{2}$  or  $m^2 = 2n^2$ .  
 $\therefore m^2$  is even.

$m^2$  is even.

A square number cannot have any prime factor that is not also a factor of the number of which it is the square.

$\therefore m$  is even.

$m$  is even.

$m$  and  $n$  are mutually prime.

$\therefore n$  is odd.

$m = 2k$

$\therefore 2n^2 = 4k^2$

$2n^2 = 4k^2$

$\therefore n^2 = 2k^2$

$$n^2 = 2k^2$$

(A repetition of the reasoning used above.)

$\therefore n$  is even.

The hypothesis that  $\sqrt{2}$  is rational entails incompatible consequences.

$\therefore$  The hypothesis that  $\sqrt{2}$  is rational is false.

### Problems

Which of the following passages contain arguments? Identify any conclusions you find.

1. Crime is common. Logic is rare. Therefore it is upon the logic rather than upon the crime that you should dwell. (Sir Arthur Conan Doyle)
2. Children make the most desirable opponents in Scrabble as they are both easy to beat and fun to cheat. (Fran Lebowitz)
3. Since we take an average of 45,000 car trips over the course of a lifetime, say statisticians, the chance of being in a serious accident is nearly one in two. (Jane Stein)
4. We owe a lot to Thomas Edison – if it wasn't for him, we'd be watching television by candlelight. (Milton Berle)
5. Cats are smarter than dogs. You can't get eight cats to pull a sled through snow. (Jeff Valdez)
6. One has to belong to the intelligensia to believe things like that; no ordinary man could be such a fool. (George Orwell)
7. Do not love your neighbor as yourself. If you are on good terms with yourself, it is an impertinence; if on bad, an injury. (George Bernard Shaw)
8. It is possible to own too much. A man with one watch knows what time it is; a man with two watches is never quite sure. (Lee Segall)
9. Every luxury must be paid for, and everything is a luxury . . . (Cesare Pavese)
10. Life does not agree with philosophy: there is no happiness that is not idleness, and only what is useless is pleasurable. (Anton Chekhov)
11. Ireland set out to crack down on alcohol-related traffic accidents. A spokesman for the Automobile Association in Dublin said it's time to stop blaming accidents on motorists: "In many cases the pedestrian is to blame. Often, he is lying prone in the roadway." (*Esquire*)
12. It is absurd to bring back a runaway slave. If a slave can survive without a master, is it not awful to admit that the master cannot live without the slave? (Diogenes of Sinope)

13. Man being a reasonable, and so a thinking creature, there is nothing more worthy of his being than the right direction and employment of his thoughts; since upon this depends both his usefulness to the public, and his own present and future benefits in all respects. (William Penn)
14. An Iron Curtain is being drawn down over their front. We do not know what lies behind it. It is vital, therefore, that we reach an understanding with Russia now before we have mortally reduced our armies and before we have withdrawn into our zones of occupation. (Winston Churchill)
15. . . . astrology was progressive. Astrology differed in asserting the continuous, regular force of a power at a distance. The influences of heavenly bodies on the events on earth it described as periodic, repetitious, *invisible* forces like those that would rule the scientific mind. (Daniel J. Boorstin)

Write each of the following arguments in standard form. If there are several arguments in a passage, write each separately.

16. The Bears did well this year, so they'll probably do well again next year.
17. John must have left already; his books are gone.
18. Few contemporary novels deal explicitly with political themes. The study of contemporary literature is therefore largely independent of the study of political culture.
19. Mary dislikes Pat. Consequently, it's unlikely that they'll work on the project together.
20. Most criminals believe that their chances of being caught and punished are small; thus, the perceived costs of a life of crime are low.
21. The building will generate large tax write-offs. As a result, it will be a good investment even if it yields little direct profit.
22. No one has ever constructed a convincing case that Bacon or someone else wrote the plays we generally attribute to Shakespeare. Shakespeare, then, almost certainly wrote the plays we attribute to him.
23. Nobody will ever find an easy way to get rich. People have been looking for centuries, and nobody's ever found one yet.
24. Swedish is an Indo-European language, but Finnish isn't. Hence Finnish is more difficult for English-speakers to learn than Swedish.
25. Many people are easily shocked by unusual or threatening events. No one who is thunderstruck can think clearly. It follows that the emotions can obstruct reason.
26. In Europe pupils devote time during each school day to calisthenics. American schools rarely offer a daily calisthenic program. Tests

- prove that our children are weaker, slower, and shorter-winded than European children. We must conclude that our children can be made fit only if they participate in school calisthenics on a daily basis. (LSAT test)
27. First, the personality and character – which are really synonymous – take their form during the first six or eight years of life. During this period of infancy and childhood, we select and develop the techniques which gain us satisfaction, defend us against threats, and become the tools in coping with the endless variety of problems situations that will be encountered later in life. It is during this time that we develop our methods of relating ourselves to other people and undergo the experiences which determine the strengths and weaknesses within our personalities. As adults we are not able to remember the details of these formative years. Therefore, we cannot understand our own behavior fully. (William Menninger)
28. One may well ask, “How can you advocate breaking some laws and obeying others?” The answer is found in the fact that there are two types of laws: There are just laws and there are unjust laws. I would be the first to advocate obeying just laws. One has not only a legal but a moral responsibility to obey just laws. Conversely, one has a moral responsibility to disobey unjust laws. I would agree with St. Augustine that “An unjust law is no law at all.” Now what is the difference between the two? How does one determine whether a law is just or unjust? A just law is a man-made code that squares with the moral law or the law of God. An unjust law is a code that is out of harmony with the moral law. To put it in the terms of St. Thomas Aquinas, an unjust law is a human law that is not rooted in eternal and natural law. Any law that uplifts human personality is just. Any law that degrades human personality is unjust. All segregation statutes are unjust because segregation distorts the soul and damages the personality . . . (Martin Luther King Jr.)
29. When you negotiate with people who take hostages you are obliged, in the negotiation, to give something. It may be just a little, it may be a lot, but you have to give something. Once you have given something, the kidnapper gains from his action. So what is his normal and spontaneous reaction? He does it again, thinking that it is a way of obtaining what he cannot obtain by other means. So you get caught in a process. Naturally you can get maybe two, three, or four hostages freed. But you immediately give the kidnapper an inducement to seize another three, four, five, or six. So it is an extraordinarily dangerous and irresponsible process. That is why I do not negotiate. (Jacques Chirac)

30. But we are convinced that the American national purpose must at some point be fixed. If it is redefined – or even subject to redefinition – with every change of administration in Washington, the United States risks becoming a factor of inconstancy in the world. The national tendency to oscillate between exaggerated belligerence and unrealistic expectation will be magnified. Other nations – friends or adversaries – unable to gear their policies to American steadiness will go their own way, dooming the United States to growing irrelevance. (Henry Kissinger and Cyrus Vance)
31. . . . Mars would be the next logical niche for human expansion in the universe. Why Mars? Clearly, Mars will have priority in any manned solar system exploration program because it offers the least severe environment for humans. Due to its atmosphere, its accessible surface, its probable availability of water and its relatively moderate temperatures . . . it is the most hospitable of all the planets other than Earth. Moreover, Mars resources include materials that could be adapted to support human life, including air, fuels, fertilizers, building materials, and an environment that could grow food . . . (James M. Beggs)
32. Computer makers must recognize that the old marketing rule is still golden: Listen to your customers. What corporate computer customers say they want is hardware and software that will allow them to tie their entire organization together in a true information network. Before the industry can give them that, individual manufacturers must agree on the uniform standards under which computers will “talk” to one another. This will be a complex effort. But as long as makers delay, customer frustration rises. (*Business Week*)
33. Being good liberals themselves, they had no ground in principle by which to justify indefinite Israeli rule over a rebellious Palestinian population. Nor could they answer the contention that continued Israeli occupation of the territories would ultimately erode the Jewishness of the state or transform it from a democracy into another South Africa. The only argument they could rely on was security: the argument that Israeli withdrawal in favor of a Palestinian state run by the PLO posed so great a danger to the “body” of Israel that, for the time being and for the foreseeable future, it had to take precedence over the danger to Israel’s “soul” admittedly posed by continued occupation. (Norman Podhoretz)
34. A struggle for existence inevitably follows from the high rate at which all organic beings tend to increase. Every being, which during its natural lifetime produces several eggs or seeds, must suffer destruction during some period of its life, and during some season

or occasional year; otherwise, on the principle of geometric increase, its numbers would quickly become so inordinately great that no country could support the product. Hence, as more individuals are produced than can possibly survive, there must in every case be a struggle for existence, either one individual with another of the same species, or with the individuals of distinct species, or with the physical conditions of life. It is the doctrine of Malthus applied with manifold force to the whole animal and vegetable kingdoms; for in this case there can be no artificial increase in food, and no prudential restraint from marriage. Although some species may now be increasing, more or less rapidly, in numbers, all cannot do so, for the whole world would not hold them. (Charles Darwin)

35. . . . Holmes [was] still carrying with him the stone which he had picked up in the wood. "This may interest you, Lestrade," he remarked, holding it out. "The murder was done with it."  
"I see no marks."  
"There are none."  
"How do you know, then?"  
"The grass was growing under it. It had lain there only a few days. There was no sign of a place whence it had been taken. It corresponds with the injuries. There was no sign of any other weapon."  
(Sir Arthur Conan Doyle)
36. "From the first, two facts were very obvious to me, the one that the lady had been quite willing to undergo the wedding ceremony, the other that she had repented of it within a few minutes of returning home. Obviously something had occurred during the morning, then, to cause her to change her mind. What could that something be? She could not have spoken to anyone when she was out, for she had been in the company of the bridegroom. Had she seen someone, then? If she had, it must be someone from America, because she had spent so short a time in this country that she could hardly have allowed anyone to acquire so deep an influence over her that the mere sight of him would induce her to change her plans so completely. You see we have already arrived, by a process of exclusion, at the idea that she might have seen an American. Then who could this American be, and why should he possess so much influence over her? It might be a lover; it might be a husband. Her young womanhood had, I knew, been spent in rough scenes, and under strange conditions. So far I had got even before ever I heard Lord St. Simon's narrative. When he told us of a man in a pew, of the change in the bride's manner, of so transparent a device for obtaining a note as the dropping of a bouquet, of her resort to her

confidential maid, and of her very significant allusion to claim-jumping, which in miners' parlance means taking possession of that which another person has a prior claim to, the whole situation became absolutely clear. She had gone off with a man, and the man was either a lover or was a previous husband, the chances being in favor of the latter." (Sir Arthur Conan Doyle)

Philosophers have advanced many arguments to prove that there is a God. Here are some famous arguments for God's existence. The first four are versions of the *ontological argument*. All the rest are versions of the *cosmological argument*. (Number 45 is a version of the *teleological argument*, also known as the *argument from design*.) Analyze the structure of these arguments, identifying premises and conclusions.

37. Although it is not necessary that I happen upon any thought of God, nevertheless as often as I think of a being first and supreme – and bring forth the idea of God as if from the storehouse of my mind – I must of necessity ascribe all perfections to it, even though I do not at that time enumerate them all, nor take note of them one by one. This necessity plainly suffices so that afterwards, when I consider that existence is a perfection, I rightly conclude that a first and supreme being exists. (René Descartes)
38. Even the Fool . . . is forced to agree that something, the greater than which cannot be thought, exists in the intellect, since he understands this when he hears it, and whatever is understood is in the intellect. And surely that, the greater than which cannot be thought, cannot exist in the intellect alone. For even if it exists solely in the intellect, it can be thought to exist in reality, which is greater. If, then, that, the greater than which cannot be thought, exists in the intellect alone, this same being, than which a greater cannot be thought, is that than which a greater can be thought. But surely this is impossible. Therefore, there can be absolutely no doubt that something, the greater than which cannot be thought, exists both in the intellect and in reality. (St. Anselm of Canterbury)
39. Certainly, this being so truly exists that it cannot even be thought not to exist. For something can be thought to exist that cannot be thought not to exist, and this is greater than whatever can be thought not to exist. Hence, if that, the greater than which cannot be thought, can be thought not to exist, then that, the greater than which cannot be thought, is not the same as that, the greater than which cannot be thought, which is absurd. Therefore, something, the greater than which cannot be thought, exists so truly that it cannot even be thought not to exist. (St. Anselm of Canterbury)

40. You exist so truly, Lord my God, that You cannot even be thought not to exist. And this is as it should be. For, if a mind could think of something better than You, the creature would rise above its creator and judge its creator, and that is completely absurd. In fact, everything else, except You alone, can be thought not to exist. You alone, then, of all things most truly exist, and therefore of all things possess existence to the highest degree; for anything else does not exist as truly, and possesses existence to a lesser degree. (St. Anselm of Canterbury)
41. The first and most obvious way is based on change. Certainly, our senses show us that some things in the world are changing. Now anything changing is changed by something else. For nothing changes except what can but does not yet have some actuality; something that causes change has that actuality already. For to cause change is to bring into being what was before only potential, and only something that already is can do this. Thus, fire, which is actually hot, causes wood, which can be hot, to become actually hot, and so causes change in the wood. Now it is impossible for the same thing to be simultaneously actually *F* and potentially *F*, though it can be actually *F* and potentially *G*: the actually hot cannot at the same time be potentially hot, though it can be potentially cold. It is therefore impossible that something undergoing a change cause itself to undergo that very change. It follows that anything changing must be changed by something else. If this other thing is also changing, it is being changed by another thing, and that by another. Now this does not go on to infinity, or else there would be no first cause of the change and, consequently, no other changes. The intermediate causes will not produce change unless they are affected by the first change, just as a stick does not move unless moved by a hand. Therefore, it is necessary to arrive at some first cause of change, itself changed by nothing, and this all understand to be God. (St. Thomas Aquinas)
42. The second way is based on the nature of causation. In the observable world causes are to be found ordered in series; we never observe, or even could observe, something causing itself, for this would mean it preceded itself, and this is impossible. Such a series of causes, however, must stop somewhere. For in all series of causes, an earlier member causes an intermediate, and the intermediate a last (whether the intermediate be one or many). If you eliminate a cause you also eliminate its effects. Therefore there can be neither a last nor an intermediate cause unless there is a first. But if the series of causes goes on to infinity, and there is no first cause, there would

be neither intermediate causes nor a final effect, which is patently false. It is therefore necessary to posit a first cause, which all call “God.” (St. Thomas Aquinas)

43. The third way depends on what is possible and necessary, and goes like this. We observe in things something that can be, and can not be, for we observe them springing up and dying away, and consequently being and not being. Now not everything can be like this, for whatever cannot be, once was not. If all things could not be, therefore, at one time there was nothing. But if that were true there would be nothing even now, because something that does not exist can be brought into being only by something that already exists. So, if there had been nothing, it would have been impossible for anything to come into being, and there would be nothing now, which is patently false. Not all things, therefore, are possible but not necessary; something is necessary. Now what is necessary may or may not have its necessity caused by something else. It is impossible to go on to infinity in a series of necessary things having a cause of their necessity, just as with any series of causes. It is therefore necessary to posit something which is itself necessary, having no other cause of its necessity, but causing necessity in everything else. (St. Thomas Aquinas)
44. The fourth way is based on the gradation observed in things. For some things are found to be more good, true, and noble, and other things less. But “more” and “less” describe varying degree of approximating the maximum; for example, things are hotter and hotter the more they approach the hottest. Something, therefore, is the best and truest and noblest of things, and consequently exists to the highest degree; for Aristotle says that the truest things exist to the highest degree. Now when many things have a common property, the one having it most fully causes the others to have it. Fire, the hottest of all things, causes the heat in all other things, to use Aristotle’s example. Therefore, something causes all other things to be, to be good, and to have any other perfections, and this we call “God.” (St. Thomas Aquinas)
45. The fifth way is based on the rule-governed character of nature. The ordering of actions toward an end is observed in all bodies obeying natural laws, even when they lack awareness. For their behavior hardly ever varies, and will practically always turn out well; this shows that they truly tend toward a goal, and do not merely hit it by accident. Nothing, however, that lacks awareness tends toward a goal, except under the direction of someone aware and intelligent. The arrow, for example, requires an archer. All things

in nature, therefore, are directed toward a goal by someone intelligent, and this we call "God." (St. Thomas Aquinas)

46. Whatever has being must either have a reason for its being, or have no reason for it. If it has a reason, then it is contingent. . . . If on the other hand it has no reason for its being in any way whatsoever, then it is necessary in its being. This rule having been confirmed, I shall now proceed to prove that there is in being a being which has no reason for its being. Such a being is either contingent or necessary. If it is necessary, then the point we sought to prove is established. If on the other hand it is contingent, that which is contingent cannot enter upon being except for some reason which sways the scales in favour of its being and against its not-being. If the reason is also contingent, then there is a chain of contingents linked one to the other, and there is no being at all; for this being which is the subject of our hypothesis cannot enter into being so long as it is not preceded by an infinite succession of beings, which is absurd. Therefore contingent beings end in a Necessary Being. (Avicenna)
47. Possible existents must of necessity have causes which precede them, and if these causes again are possible it follows that they have causes and that there is an infinite regress; and if there is an infinite regress there is no cause, and the possible will exist without a cause, and this is impossible. Therefore the series must end in a necessary cause, and in this case this necessary cause must be necessary through a cause or without a cause, and if through a cause, this cause must have a cause and so on infinitely, and if we have an infinite regress here, it follows that what was assumed to have a cause has no cause, and this is impossible. Therefore the series must end in a cause necessary without a cause; i.e., necessary by itself, and this necessarily is the necessary existent. (Averroes)
48. . . . *nothing happens without a sufficient reason*; that is to say, that nothing happens without its being possible for him who should sufficiently understand things, to give a reason sufficient to determine why it is so and not otherwise. . . . Now this sufficient reason for the existence of the universe cannot be found *in the series of contingent things*, that is, of bodies and of their representation in souls; for matter being indifferent in itself to motion and to rest, and to this or another motion, we cannot find the reason of motion in it, and still less of a certain motion. And although the present motion which is in matter, comes from the preceding motion, and that from still another preceding, yet in this way we make no progress, try as we may; for the same question always remains. Thus

it must be that the sufficient reason, which has no need of another reason, be outside this series of contingent things and be found in a substance which is the cause, or which is a necessary being, carrying the reason of its existence within itself; otherwise we still should not have a sufficient reason in which we could rest. And this final reason of things is called *God*. (Gottfried Leibniz)

## 1.2 VALIDITY

Some arguments are good; others are not. What distinguishes good from bad arguments? What makes a good argument good? A good argument links its premises to its conclusion in the right way. There is a special connection between the premises and the conclusion.

To see what this special sort of connection is, consider an argument that has true premises and a true conclusion, but is nevertheless bad:

Harrisburg is the capital of Pennsylvania.  
 Richmond is the capital of Virginia.  
 ∴ Austin is the capital of Texas.

What is wrong with this argument? The facts cited in the premises have nothing to do with the truth or falsehood of the conclusion. Texas could move its capital – to, say, Del Rio – while Harrisburg and Richmond remained the capitals of their respective states. That is, the conclusion of this argument could turn out to be false, even when the premises were true. The truth of the premises does nothing to guarantee the truth of the conclusion. This is the mark of a *deductively invalid* argument: its premises could all be true in a circumstance in which its conclusion is false.

In a *deductively valid* argument, the truth of the premises guarantees the truth of the conclusion. If the premises are all true, then the conclusion has to be true. Consider, for example, this argument:

All Canadians are North Americans.  
 Jeff is a Canadian.  
 ∴ Jeff is a North American.

In any circumstance in which the premises of this argument are true, the conclusion must be true as well. It is impossible to conceive of a state of affairs in which, while all Canadians are North Americans, Jeff is a Canadian but not a North American. In a deductively valid argument, the truth of the premises guarantees the truth of the conclusion. Or, to say the same thing, if the conclusion of a deductively valid argument is false, at least one premise must also be false.

An argument is *deductively valid* if and only if its conclusion is true whenever its premises are all true.

It is possible, then, for a deductively valid argument to have true premises and a true conclusion; (at least some) false premises and a false conclusion; and false premises and a true conclusion. But no deductively valid argument has true premises and a false conclusion:

#### Some deductively valid arguments

<i>True premises, true conclusion</i>	<i>False premises, false conclusion</i>	<i>False premises, true conclusion</i>
You can read.	You are a duck.	You are a duck.
All readers are mortal.	All ducks eat dirt.	All ducks can read.
∴ You are mortal.	∴ You eat dirt.	∴ You can read.

Each of these arguments is deductively valid: In each case, the conclusion is true in every possible circumstance in which the premises are all true. How could it be true that you are a duck, and true that all ducks eat dirt, but false that you eat dirt? Whether the premises and conclusion are actually true or false makes little difference to the validity of the argument. We evaluate deductive validity *as if* the premises were true. What matters is that *if* the premises are true the conclusion cannot be false.

Thus, not every argument with true premises and a true conclusion is deductively valid, as the argument concerning state capitals shows. Similarly, many arguments with false premises and a true conclusion are deductively invalid. The same is true for arguments with false premises and a false conclusion. So, although valid arguments can have any of these three combinations of truth and falsity, not every argument with those combinations is valid. An argument is deductively invalid if it is possible for the premises to be true while the conclusion is false. Similarly, an argument is deductively valid just in case its conclusion has to be true if its premises are all true.

Some deductively invalid arguments nevertheless have some legitimate force in reasoning. Although the truth of the premises of such an argument does not guarantee the truth of its conclusion, it does make the truth of the conclusion probable. Consider for example, this argument:

Every crow that has been observed is black.  
∴ All crows are black.

It is possible for the premise to be true while the conclusion is false. There may be white crows that nobody has ever seen. So, the argument is deductively invalid. Nevertheless, the premise lends some support to the conclusion. The argument is *inductively strong*; how strong depends

on how many crows have been observed, among other things. Inductively reliable arguments are extremely important in both scientific and everyday reasoning. They constitute the subject matter of chapter 13. Until then, we will focus on deductive validity.

When we imagine a circumstance in which some sentences would be true, and others would be false, we normally imagine a situation that settles the matters that those sentences involve, but that leave lots of other things unsettled. Above, for example, we imagined a case in which Texas moved its capital, but Pennsylvania and Virginia didn't. That was all we said, or, apparently, needed to say to convince ourselves that the argument was invalid. But that was not even close to a complete description of an entire world. We said nothing about what happened to Montana, or Alaska, or Afghanistan, or the pennant hopes of the Mets, or the price of pork bellies on the Chicago Board of Trade. The case we've described, therefore, is not very determinate. There are many different ways the world might be that all agree in fitting our description. So, it might be more correct to say that we imagined, not a single case, but a kind of case in which the premises are all true and the conclusion is false. Many circumstances might fit the description we gave.

The logic we study in this book assumes that some circumstances are so comprehensive that they determine whether each declarative sentence of a language is true or false. Every sentence that can be true or false at all must, in such a complete situation, be either true or false. The logic studied throughout most of this book is a *bivalent* logic because it says that, given any sentence capable of truth or falsehood, the question, "Is this sentence true, or false, or whatever?" always has only two possible answers: "True" and "False." In other words, classical logic allows only two truth values: truth and falsehood. The truth value of a sentence is truth, if the sentence is true, and falsehood or falsity, if it is false. In chapter 11 we will consider what happens if we allow for the possibility that some sentences are neither true nor false.

Deductively valid arguments always preserve truth; if they begin with true premises, they carry us to true conclusions. Usually we want to have an argument that not only preserves truth but has some truth to preserve. A *sound* argument meets both criteria for success. It has true premises; it is valid. Furthermore, since, in any valid argument, the truth of the premises guarantees the truth of the conclusion, it also has a true conclusion.

An argument is <i>sound</i> if and only if (1) it is valid and (2) all its premises are true.
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Sound arguments, then, are often paradigms of successful arguments. They derive the truth of their conclusions by arguing validly from true premises.

Nevertheless, most of this book will focus, not on soundness, but on validity. This focus is easy to understand. First, validity is obviously a crucial component of soundness. We can't evaluate whether an argument is sound without first determining whether it is valid. Second, evaluating soundness requires judging the actual truth or falsehood of premises. This, however, is the job, not of logical theory, but of those who know enough physics, history, business, or whatever facts are relevant to the argument at hand. Third, although we usually want to argue from true premises, many useful arguments start from false ones. Some arguments try to show that a certain sentence is false by using it as a premise to reach an outrageous or absurd conclusion. Others adopt a premise purely as a hypothesis, to see what would follow if it were true. Aristotle first realized how important such arguments are; he characterized them as having *dialectical* rather than *demonstrative* premises. These forms of argument are much more common and useful than most people would imagine. A simple example occurs at the beginning of this chapter in the proof of the irrationality of the square root of two. The proof starts with the assumption that  $\sqrt{2}$  is rational, and deduces from it a contradiction. The point of this argument is precisely to show that the premise that  $\sqrt{2}$  is rational is false. The argument's success, therefore, depends solely on validity, not on soundness. For all three of these reasons, our study of reasoning will focus on validity.

### Problems

Evaluate these arguments as valid or invalid. If the argument is invalid, describe a circumstance in which the premises would be true but the conclusion would be false. Do any of the invalid arguments nevertheless make their conclusions probable, in your opinion?

1. John and Mary came to the party. Hence, Mary came to the party.
2. If Frank takes the job in Cleveland, he'll make a lot of money on the sale of his house. Frank won't take the job in Cleveland. It follows that Frank won't make a lot of money on the sale of his house.
3. If the rain continues, there will be a real danger of floods. The rain will continue. Therefore, flooding will be a real danger.
4. If Sally has pneumonia, she needs penicillin and lots of rest. Sally does need penicillin and lots of rest. So, Sally has pneumonia.

5. Congress will agree to the cut only if the President announces his support first. The President won't announce his support first, so, Congress won't agree to the cut.
6. I have already said that he must have gone to King's Pyland or to Mapleton. He is not at King's Pyland, therefore he is at Mapleton. (Sir Arthur Conan Doyle)
7. Nobody saw what happened. If nobody witnessed it, nobody can testify. If nobody can testify, you can't be convicted. So, you can't be convicted.
8. We will let you out of the lease only if you pay us two months rent. You can't pay us two months rent. So, we won't let you out of the lease.
9. If Socrates died, he died either while he was living or while he was dead. But he did not die while living; moreover, he surely did not die while he was already dead. Hence, Socrates did not die. (Sextus Empiricus)
10. A man cannot serve both God and Mammon. But if a man does not serve Mammon, he starves; if he starves, he can't serve God. Therefore a man cannot serve God.
11. Either we ought to philosophize or we ought not. If we ought, then we ought. If we ought not, then also we ought (to justify this view). Hence in any case we ought to philosophize. (Aristotle)
12. If Lynn testifies against the mobsters, she'll endanger her life. So, she won't testify against them, since she won't put her own life in danger.
13. Pamela played Shelley for the tournament trophy. Consequently, Pamela played either Shelley or Tracy for the trophy.
14. The patient will surely die unless we operate. We will operate. Therefore the patient will not die.
15. Jerry will take the job unless we match the salary offer. Since we won't match the offer, Jerry will take the job.
16. The launch will be delayed if the weather doesn't clear. So, if the weather clears, the launch won't be delayed.
17. The meeting will take place only if both parties agree on the agenda. So, if the parties don't agree on the agenda, the meeting will not take place.
18. Marilyn will finish the brief on time only if she gets an extension on the Morley case. Therefore, if Marilyn gets an extension on the Morley case, she will finish the brief on time.
19. Nancy will not marry Alex unless he signs a prenuptial agreement. So, if Alex signs a prenuptial agreement, Nancy will marry him.
20. This album will sell only if it contains at least one hit song. Hence, unless it contains a hit song, this album will not sell.

21. Some illegal acts go unpunished. All blatantly wrong acts are punished. Therefore, some illegal acts are not blatantly wrong.
22. All who do not remember the past are condemned to repeat it. No one condemned to repeat the past looks forward to the future with eagerness. So, everyone who eagerly looks forward to the future remembers the past.
23. Lori is unhappy with some people who didn't write thank-you notes. Lori will send presents next year to everyone with whom she's happy. Therefore, some people who didn't write thank-you notes won't get presents from Lori next year.
24. Anyone who is not an idiot can see that Jake is lying. Some people in this room can't tell that Jake is lying. Hence, some people in this room are idiots.
25. Henry doesn't know anyone. So Henry doesn't know Kim.
26. Rocky has beaten everyone he's faced. Thus, Rocky has beaten Mad Moe, if he's faced him.
27. Some politicians are demagogues, but no demagogues are good leaders. Hence, some politicians are not good leaders.
28. All scientists have a deep interest in the workings of nature. All who devote their lives to the study of the physical world have a deep interest in the workings of nature. Consequently, all scientists devote their lives to the study of the physical world.
29. Few students fully appreciate the value of an education while they are in school. Only those who fully appreciate the value of their education while they are in school devote themselves to their studies as much as they ought to. Therefore, most students don't devote themselves to their studies as much as they ought to.
30. Corporate taxes result in higher prices for consumer goods, increases in interest rates, reduced employment at lower wages, and reduced levels of savings and investment, depending on whether corporations pass along the cost of taxation to the consumer, borrow to replace these funds, take steps to reduce labor costs, or reduce the return they offer to shareholders. Consequently, corporate taxes should be repealed.
31. Most Americans who travel in Europe know no language other than English. All Americans who travel in Europe are affluent. Thus, most affluent Americans know no language other than English.
32. For while every man is able to judge a demonstration (it would not deserve this name if all those who consider it attentively were not convinced and persuaded by it), nevertheless not every man is able to discover demonstrations on his own initiative, nor to present them distinctly once they are discovered, if he lacks leisure or method.

(Gottfried Leibniz) [Therefore, not all those who can judge demonstrations can discover them.]

33. 1. You had chalk between your left finger and thumb when you returned from the club last night. 2. You put chalk there when you play billiards to steady the cue. 3. You never play billiards except with Thurston. 4. You told me four weeks ago that Thurston had an option on some South African property which would expire in a month, and which he desired you to share with him. 5. Your cheque-book is locked in my drawer, and you have not asked for the key. 6. You do not propose to invest your money in this manner. (Sir Arthur Conan Doyle)
34. “I see that you are professionally rather busy just now,” said he, glancing very keenly across at me.  
 “Yes, I’ve had a busy day,” I answered. “It may seem very foolish in your eyes,” I added, “but really I don’t know how you deduced it.”  
 Holmes chuckled to himself.  
 “I have the advantage of knowing your habits, my dear Watson,” said he. “When your round is a short one you walk, and when it is a long one you use a hansom. As I perceive that your boots, although used, are by no means dirty, I cannot doubt that you are at present busy enough to justify the hansom.”  
 “Excellent!” I cried.  
 “Elementary,” said he. (Sir Arthur Conan Doyle)
35. “And in practice again, I observe. You did not tell me that you intended to go into harness.”  
 “Then how do you know?”  
 “I see it, I deduce it. How do I know that you have been getting yourself very wet lately, and that you have a most careless and clumsy servant girl?”  
 “My dear Holmes,” said I, “this is too much. You would certainly have been burned, had you lived a few centuries ago. It is true that I had a country walk on Thursday and came home in a dreadful mess; but, as I have changed my clothes, I can’t imagine how you deduce it. As to Mary Jane, she is incorrigible and my wife has given her notice; but there again I fail to see how you work it out.”  
 He chuckled to himself and rubbed his long nervous hands together.  
 “It is simplicity itself,” said he; “my eyes tell me that on the inside of your left shoe, just where the firelight strikes it, the leather is scored by six almost parallel cuts. Obviously they have been caused by someone who has very carelessly scraped round the edges of the sole in order to remove crusted mud from it. Hence, you

see, my double deduction that you had been out in vile weather, and that you had a particularly malignant boot-slitting specimen of the London slavey. As to your practice, if a gentleman walks into my rooms smelling of iodoform, with a black mark of nitrate of silver upon his right forefinger, and a bulge on the side of his top-hat to show where he had secreted his stethoscope, I must be dull indeed, if I do not pronounce him to be an active member of the medical profession.” (Sir Arthur Conan Doyle)

### 1.3 IMPLICATION AND EQUIVALENCE

A concept closely related to validity is implication. We might express the idea that an argument is valid by saying that its conclusion *follows from* its premises. Equivalently, we might say that its premises *imply* or *entail* its conclusion. At least part of what we mean, in either case, is that the truth of the premises guarantees the conclusion’s truth. If the premises are true, the conclusion has to be true too.

A set of sentences *implies* a given sentence just in case the truth of that sentence is guaranteed by the truth of all the members of the set.

If an argument is valid, the set consisting of its premises implies its conclusion.

We can also speak of a single sentence implying another sentence.

A sentence A *implies* another, B, if and only if A’s truth guarantees B’s truth.

One sentence implies another just in case, in every circumstance in which the first is true, the second must be true as well.

Consider these two pairs of sentences.

- (1.1) a. Mary likes Chinese food, but Bill hates it.  
b. Mary likes Chinese food.
- (1.2) a. Susan is going to spend her summer in Palo Alto or Pittsburgh.  
b. Susan is going to spend her summer in Pittsburgh.

Sentence (1.1)a implies (1.1)b. It is impossible to conceive of a situation in which it is true that Mary likes Chinese food, but Bill hates it, and false that Mary likes Chinese food. In such a circumstance, Mary would have to like and not like Chinese food; the sentence *Mary likes Chinese food* would have to be both true and false at the same time. There are no such circumstances. No sentence can be both true and false at the same time. So the truth of (1.1)a guarantees the truth of (1.1)b.

Does the truth of (1.2)a similarly guarantee the truth of (1.2)b? Obviously, the answer is no. Imagine a world in which Susan is going to spend her summer in Palo Alto, never setting foot outside California. In this situation, (1.2)a is true, but (1.2)b is false. So (1.2)a does not imply (1.2)b.

A sentence A implies a sentence B just in case B is true in all those possible circumstances in which A is true. B implies A, of course, just in case A is true in all the cases in which B is true. If A implies B and B implies A, then A and B must be true in exactly the same circumstances. In such a case, we say that A and B are equivalent.

A sentence A is *equivalent* to a sentence B if and only if A and B always agree in truth value.

If A and B are equivalent, then they must be true in the same circumstances, and false in the same circumstances. There could be no situation in which one would be true while the other would be false. Thus, equivalence amounts to implication in both directions. A is equivalent to B just in case A implies B and B implies A.

To make this more concrete, consider four more sentences:

- (1.3) a. No apples are oranges.  
       b. No oranges are apples.
- (1.4) a. All apples are fruits.  
       b. All fruits are apples.

The sentences in (1.3) are equivalent. Any circumstance in which no apples are oranges is one in which no oranges are apples, and vice versa. Both sentences say that nothing is both an orange and an apple. In (1.4), however, the sentences are obviously not equivalent. All apples are fruits, so (1.4)a is true. But not all fruits are apples, so (1.4)b is false. The real world is thus a case in which these sentences disagree in truth value.

## Problems

Consider the sentences in each pair: Are they equivalent? Does either sentence imply the other?

1. (a) Both Alan and Bob took their vacations in California. (b) Alan took his vacation in California.
2. (a) Vivian and Beth both majored in English in college. (b) Beth majored in English in college, and so did Vivian.
3. (a) Pittsburgh will face Dallas or New York in the championship game. (b) Either Pittsburgh will face Dallas in the championship game, or Pittsburgh will face New York.
4. (a) Neon and xenon are inert. (b) Xenon and neon are inert.
5. (a) Pluto or Uranus is now directly aligned with Neptune. (b) Pluto and Uranus are now directly aligned with Neptune.
6. (a) Not both whales and dolphins are fish. (b) Whales are not fish, and dolphins aren't either.
7. (a) Either Sam or Peter failed to give the play an appropriate sense of place. (b) Peter and Sam did not both give the play an appropriate sense of place.
8. (a) Aunt Alice will not come to the wedding, and neither will Uncle Harry. (b) Not both Uncle Harry and Aunt Alice will come to the wedding.
9. (a) Either the Babylonians or the Assyrians employed the *lex talionis*. (b) If the Assyrians employed the *lex talionis*, the Babylonians didn't.
10. (a) Either the physical world really exists, independently of our minds, or our senses systematically deceive us. (b) If our senses systematically deceive us, then the physical world doesn't really exist independently of our minds.
11. (a) If pay-per-view television catches on, cable companies will make huge profits. (b) If pay-per-view TV doesn't catch on, cable companies will not make huge profits.
12. (a) If Elizabeth did not sign this letter, then her assistant did. (b) If Elizabeth had not signed this letter, her assistant would have.
13. (a) No high-paying job is easy. (b) No easy job is high-paying.
14. (a) Most foods that are high in carbohydrates are high in calories. (b) Most foods that are high in calories are high in carbohydrates.
15. (a) Several cities with populations over 700,000 have no baseball franchises. (b) Several cities without baseball franchises have populations over 700,000.
16. (a) Anybody who can speak effectively can find a job in sales. (b) Anybody who can find a job in sales can speak effectively.

Consider the statement: *If a fetus is a person, it has a right to life.* Which of the following sentences follow from this? Which imply it?

17. A fetus is a person.
18. If a fetus has a right to life, then it's a person.
19. A fetus has a right to life only if it's a person.
20. A fetus is a person only if it has a right to life.
21. If a fetus isn't a person, it doesn't have a right to life.
22. If a fetus doesn't have a right to life, it isn't a person.
23. A fetus has a right to life.
24. A fetus isn't a person only if it doesn't have a right to life.
25. A fetus doesn't have a right to life only if it isn't a person.
26. A fetus doesn't have a right to life unless it's a person.
27. A fetus isn't a person unless it has a right to life.
28. A fetus is a person unless it doesn't have a right to life.
29. A fetus has a right to life unless it isn't a person.

Consider the statement: The patient will die unless we operate immediately. What follows from this, together with the information listed?

30. The patient will die.
31. The patient will not die.
32. We will operate immediately.
33. We won't operate immediately.

Consider this statement from IRS publication 17, *Your Federal Income Tax*: *If you are single, you must file a return if you had gross income of \$3,560 or more for the year.* What follows from this, together with the information listed?

34. You are single with an income of \$2,500.
35. You are married with an income of \$2,500.
36. You are single with an income of \$25,000.
37. You are married with an income of \$25,000.
38. You are single, but do not have to file a return.
39. You are married, but do not have to file a return.
40. You have an income of \$4,500, but do not have to file a return.
41. An old joke: Mutt says, "See you later;" Jeff answers, "Not if I see you first." Suppose that both statements are true. What follows?
42. Lao-Tzu said, "Real words are not vain, Vain words not real." Are these two statements equivalent? If not, in what circumstances could one be true while the other is false?
43. From an episode of *The Simpsons*:

Bart: If Lisa stays home, I stay home.

Lisa: If Bart stays home, I'm going to school.

- (a) Suppose that both statements are true. What follows?  
 (b) In the episode, Bart goes to school while Lisa stays home. That implies that one of the above statements is false. Which?
44. Donora, Pennsylvania, used to greet visitors with a road sign saying, “Donora. The nicest town on Earth, next to yours.” Could this be true for every visitor? If so, what does that imply about Donora?

## 1.4 LOGICAL PROPERTIES OF SENTENCES

Logic deals primarily with the logical connections between sentences. Nevertheless, it also classifies individual sentences. The overwhelming majority of sentences we use could, depending on what the facts are, be either true or false. There are possible circumstances in which they would be true and other cases in which they would be false. For instance, each of the following sentences would be true in some circumstances and false in others:

The snow is falling all over Ireland.

The King recognized that the some of the nobles would oppose him.

The earth is the third planet from the sun.

Francis Bacon wrote *The Merchant of Venice*.

Such sentences are contingent:

A sentence is *contingent* if and only if it is possible for it to be true and possible for it to be false.

Contingent sentences could be true, given the right set of circumstances. They could also be false, depending on the facts of the situation. They are useful precisely because they assert, in effect, that the real circumstance is among those in which they are true.

Some sentences, in contrast, cannot help being true. It’s simply impossible for them to be false. They are true in every possible circumstance. Such sentences are valid, or logically true:

A sentence is *valid* (or *tautologous*, or *logically true*) if and only if it is true in every possible circumstance.

If you doubt that there are any sentences that cannot be false, no matter what the facts may be, then try to imagine circumstances in which these sentences are false:

Either Lima is in Ecuador or it's not.  
 A rose is a rose.  
 Wherever you go, there you are.  
 It ain't over 'til it's over.  
 When you're hot, you're hot.

These sentences are true in every possible world. They also seem to say very little. But not all valid sentences are so straightforward and unsurprising. This, for example, is logically true:

If everyone loves a lover, and Sam doesn't love Jeanne, then Jeanne doesn't love Greg.

But it doesn't seem trivial. Notice, furthermore, that even those sentences can be useful. Sometimes they set up the structure of an argument, as when a mathematician begins a proof by saying, "the number  $n$  is either prime or not prime. If it is prime. . . ." At other times, they serve a function in discourse by forcing the listener to interpret certain terms as ambiguous. We normally assume that a speaker is making a good faith effort to communicate information. So, when Yogi Berra said "It ain't over 'til it's over," he presumably meant something like "it ain't over 'til it's *really* over"; that is, "the outcome isn't fully determined until the game ends." So interpreted, the sentence isn't valid at all, but contingent.

Some sentences, furthermore, could never be true. They are false, regardless of the facts. These sentences are *contradictory* (or *contradictions*).

A sentence is *contradictory* if and only if it is impossible for it to be true.

Here are some examples of contradictions:

Fred is both bald and not bald.  
 Sheila is irritated, and she's not.  
 This set belongs to itself if and only if it doesn't belong to itself.  
 Nobody's seen the trouble I've seen.

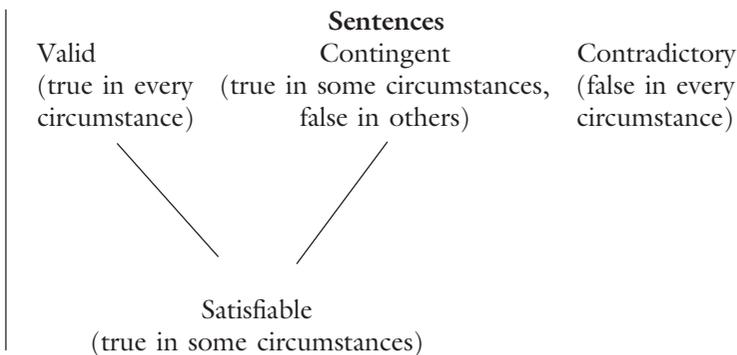
In no conceivable circumstance could any of these sentences be literally true. Try, for example, to imagine a situation in which Fred is both bald and not bald at the same time. Whatever the state of Fred's scalp, he's either bald, or not bald, but not both. No matter what Sheila's state of mind may be, she is either irritated or not. Similarly, the set in question must belong to itself or not. And, since *I've* seen the trouble I've seen, somebody (namely, me) has indeed seen the trouble I've seen.

Like logical truths, contradictions tend to signal that we should interpret some terms generously, since we assume that our colleagues in communication are trying to say something that could be true. Hearing *Nobody's seen the trouble I've seen*, then, we tend to read the *Nobody* as *Nobody else*, reading the sentence as a whole as if it were *Nobody else has seen the trouble I've seen*. Contradictions too may fulfill important functions in arguments. *This set belongs to itself if and only if it doesn't belong to itself*, for example, might be a crucial step in showing that the set under consideration cannot exist.

Nevertheless, contradictions are disruptive enough that it is worth having a term for sentences that, whether they are valid or contingent, at least are not contradictory. Such noncontradictory sentences are *satisfiable*.

A sentence is *satisfiable* if and only if it is not contradictory.

Obviously, a sentence is satisfiable just in case it's either contingent or valid. That is, it must be possible for the sentence to be true. Since every sentence is either valid, contingent or contradictory, the terms introduced in this section divide sentences into three groups:



## Problems

Classify these sentences as logically true, contradictory, or contingent.

1. Everything is what it is, and not another thing. (Bishop Butler)
2. The business of America is business. (Calvin Coolidge)
3. . . . what we are, we are . . . (Alfred, Lord Tennyson)
4. A lie is a lie, no matter how ancient; a truth is a truth, though it was born yesterday. (American proverb)
5. Nay, Sir, argument is argument. (Samuel Johnson)
6. . . . if it does not work, it does not work. (Prince Charles)
7. Bigness is bigness in spite of a hundred mistakes. (Jawaharlal Nehru)
8. All babies are young. (Benjamin Spock)
9. I am who I am.
10. All dogs are dogs.
11. Some dogs are not dogs.
12. Some cars are red.
13. All red cars are cars.
14. I know what I know.
15. Some people are friendly, and other's aren't.
16. Some people aren't friendly, but everybody is friendly.
17. There are many trees in Yosemite National Park.
18. Nobody loves everybody.
19. Everyone who loves everyone loves every loser.
20. Today is the first day of the rest of your life.
21. No batter ever made a hit with the bat on his shoulder. (John McGraw)
22. You are what you eat. (Ludwig Feuerbach)
23. When people are out of work, unemployment results. (Calvin Coolidge)
24. Our past has gone into history. (William McKinley)
25. The nobles are to be considered in two different manners; that is, they are either to be ruled so as to make them entirely dependent on your fortunes, or else not. (Niccolo Machiavelli)
26. Sudden death, though fortunately it is rare, is frequent. (*British Medical Journal*)
27. Nobody goes there anymore; it's too crowded. (Yogi Berra)
28. "Everybody that hears me sing it – either it brings tears to their eyes, or else –." "Or else what?" said Alice, for the Knight had made a sudden pause. "Or else it doesn't, you know." (Lewis Carroll)
29. There are two kinds of people in the world: those who divide the world into two kinds of people, and those who don't. (H. L. Mencken)

30. I am never less alone than when I am alone, nor less at leisure than when I am at leisure. (Scipio Africanus)
31. There comes a time to put principle aside and do what's right. (Michigan legislator)
32. I don't know what the previous speaker said, but I agree with him. (Texas legislator)
33. Say that a sentence A implies another sentence B. What can we conclude about B, if A is (a) logically true, (b) contingent, (c) satisfiable, or (d) contradictory?
34. Say that a sentence A implies another sentence B. What can we conclude about A, if B is (a) logically true, (b) contingent, (c) satisfiable, or (d) contradictory?
35. Say that a sentence A is equivalent to another sentence B. What can we conclude about A, if B is (a) logically true, (b) contingent, (c) satisfiable, or (d) contradictory?

## 1.5 SATISFIABILITY

A sentence is satisfiable just in case it is not contradictory; that is, just in case it can be true. Any true sentence, obviously, is satisfiable. But false sentences can also be satisfiable, so long as they are true in some other possible circumstance.

We can speak of sets of sentences, too, as satisfiable or contradictory. It is easy to think of sets of sentences that, in some sense, contain contradictions, even though each sentence in the set is itself satisfiable:

- (1.5)
  - a. Beer and sauerkraut are very good together.
  - b. Beer and sauerkraut aren't very good together.
- (1.6)
  - a. Many of my friends belong to the Flat Earth Society.
  - b. Nobody in the Flat Earth Society believes in modern science.
  - c. All my friends believe in modern science.

The sentences in (1.5), like those in (1.6), are not themselves contradictions. Taken individually, each could be true. Taken together, however, they describe an impossible situation. Although each could be true, the sentences in (1.5) or (1.6) could not be true together. In such cases, the *set* of sentences is contradictory, whether or not any individual sentence in the set is itself contradictory.

A set of sentences is *contradictory* if and only if it is impossible for all its members to be true. A set is *satisfiable* otherwise.

If a set is contradictory, we can also say that its members are *mutually inconsistent*, and that any member *contradicts*, or *is inconsistent with*, the set containing all the rest. If the set is satisfiable, then its members are *mutually consistent*, and each member is *consistent* or *compatible* with the set containing all the rest. Two sentences *contradict each other* just in case the set containing just the two of them is contradictory. If a set is satisfiable, then all its subsets are satisfiable: each member is consistent or compatible with each other member of the set.

From a logical point of view, contradictory sets of sentences can be described in two ways. First, the sentences in the set cannot all be true at the same time. Second, the set implies a contradiction. Although a contradictory set of sentences might not contain a contradiction, it must imply one. The sentences in (1.6), for example, together imply *Although many of my friends don't believe in modern science, all my friends do believe in modern science*. This is an outright contradiction.

To see that these two characterizations come to the same thing, recall that a set  $S$  of sentences implies a sentence  $A$  just in case  $A$  is true whenever every member of  $S$  is true. Contradictions, of course, are always false. When  $A$  is a contradiction, then, this amounts to the following:  $S$  implies  $A$  if and only if it is impossible for every sentence in  $S$  to be true. Therefore, a set of sentences implies a contradiction just in case it is itself contradictory. Or, to put it another way, satisfiability is freedom from contradiction.

Satisfiability is important: sets of sentences that are not satisfiable do not have a fighting chance at truth. They must contain at least one false sentence, no matter what the facts might be. A satisfiable set may also contain false sentences, but at least there is a possibility that all the sentences it contains are true.

This explains the significance of satisfiability in legal contexts. A lawyer may try to trap an opposing witness in a contradiction. The lawyer, in most cases, cannot alone provide any direct testimony relevant to the case. He or she may introduce witnesses of his or her own to dispute what the opposing witness says. If the opposing witness falls into a contradiction, however, then the witness must be saying something false, regardless of the facts of the case.

Even more fundamentally, people often use arguments to disprove someone else's contention. To refute an assertion, we have to recognize

when we have shown something that contradicts that assertion. So the notion of refutation depends on the notion of contradiction.

Satisfiability and validity are connected. If an argument is valid, the truth of the premises guarantees the truth of the conclusion. So, the premises cannot all be true while the conclusion is false. Thus, the set containing the premises and the *negation* of the conclusion – the conclusion prefaced with *It is not the case that* – is contradictory. As we have seen, this argument is valid:

All Canadians are North Americans.

Jeff is a Canadian.

∴ Jeff is a North American.

So, the set {All Canadians are North Americans, Jeff is a Canadian, It is not the case that Jeff is an American} is contradictory.

Finally, the concept of satisfiability has been very important in modern mathematics. Around the turn of the twentieth century, several mathematicians and logicians deduced contradictions from mathematical theories in use at the time. Ever since, mathematicians have been extremely cautious about the satisfiability of their theories, and have sought, whenever possible, proofs that theories are satisfiable. This concern led to some of the most important developments in twentieth-century logic, mathematics, and computer science.

### Problems

Evaluate these sets of sentences as satisfiable or contradictory.

1. The yard isn't white unless it's snowing. It's not snowing. But the yard is white.
2. If a student's GPA is very high, he or she will get into a good graduate school. Frank's GPA is not very high. Nevertheless, he'll get into a good graduate school.
3. John is a good guitarist. John is also an accountant, but not a good one.
4. Everyone who can cook a good chicken kung pao knows the value of hot peppers. Some who know the value of hot peppers don't themselves like hot food. Anybody who can cook a good chicken kung pao likes hot food.
5. If Marsha takes a job with a state commission, she'll gain much experience in new areas, although she won't get to travel. If she takes a job with a private company, she'll get to travel, and she'll be paid well, although she won't gain much experience outside her area. Marsha won't be paid well, but she will get to travel.

6. I like this painting, even though I don't think it's very good. I like everything that Elmer likes, and Elmer likes every painting that's good.
7. No drugs are approved for use without careful screening. Careful screening takes years. A few drugs in great demand, however, are approved for use in less time.
8. Few communist parties in Europe seek to identify themselves with the Russian party. Parties seeking to identify themselves with the Russians have a difficult time becoming part of coalition governments. Almost all European communist parties find it difficult, however, to become part of coalition governments.
9. Stocks of companies with high debt–equity ratios are fairly risky. If a stock is fairly risky, it must reward investors with better-than-average returns, or they will eschew the risk. But many stocks that fail to reward investors with better-than-average returns are those of companies with high debt–equity ratios.
10. People have a right to life. Fetuses are not people. If something has a right to life, it wrong to kill it. Abortion is the killing of a fetus. Abortion is wrong.
11. Few contemporary composers write anything that could reasonably be called twelve-tone compositions. If so, then atonal music is defunct. But atonal principles of composition still exert some influence on contemporary composers. And nothing that still exerts influence is defunct.
12. Many football stars never graduate from the colleges where they first become famous. Most of these colleges insist that almost all their football players receive degrees. These schools are telling the truth.
13. Most actresses begin their careers as successful models. Every woman who begins her career as a successful model is very glamorous. Nevertheless, few actresses are very glamorous.
14. Many well-known American novels deal with the character of a specific region of the country. Every well-known American novel, of course, portrays a certain conception of America itself. Nonetheless, many novels that portray a conception of America do not deal with any specific region of the country.
15. My barber, who lives and works in town, shaves every man in town who doesn't shave himself. Furthermore, my barber doesn't shave anyone in town who does shave himself.

True or false? Explain.

16. If a set of sentences is satisfiable, no member of that set implies a contradictory sentence.

17. If no member of a set implies a contradictory sentence, that set is satisfiable.
18. Some satisfiable sets of sentences imply contradictions.
19. Some satisfiable sets of sentences imply no contingent sentences.
20. Every contradictory set of sentences implies every contradiction.
21. No satisfiable sets of formulas imply every sentence.
22. Some contradictory sets of sentences imply every sentence.
23. If A implies B, then the set consisting of A and B together is satisfiable.
24. If the set consisting of just A together with B is contradictory, then A implies that B is false.
25. Any argument with a contradictory set of premises is valid.
26. Arguments with satisfiable sets of premises have satisfiable conclusions.
27. Every satisfiable set of sentences contains at least one true sentence.
28. Every contradictory set of sentences contains at least one false sentence.
29. Any set consisting of all valid sentences is satisfiable.
30. Any set consisting of all contingent sentences is satisfiable.

The Englishman William of Ockham (1285–1349), perhaps the most influential philosopher and logician of the fourteenth century, recorded 11 rules of logic in a chapter of his *Summa Totius Logicae*. Ten of these use concepts we have already developed. Say whether each is true, given the definitions of this chapter, and explain why.

31. The false never follows from the true.
32. The true may follow from the false.
33. Whatever follows from the conclusion of a valid argument follows from its premises.
34. The conclusion of a valid argument follows from anything that implies the argument's premises.
35. Whatever is consistent with the premises of a valid argument is also consistent with the argument's conclusion.
36. Whatever is inconsistent with the conclusion of a valid argument is also inconsistent with the argument's premises.
37. The contingent does not follow from the valid.
38. The contradictory does not follow from the satisfiable.
39. Anything whatsoever follows from the contradictory.
40. The valid follows from anything whatsoever.