Part I

HISTORICAL DEVELOPMENT OF LOGIC

1

Ancient Greek Philosophical Logic

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Ancient Greek logic was inseparable from ancient Greek philosophy. The formal theories developed by major logicians such as Aristotle, Diodorus Cronus, and Chrysippus were in large part influenced by metaphysical and epistemological concerns. In this brief essay, I will try to give some picture of this interrelationship. For reasons of space, I make no attempt to cover, or even to mention, every aspect of ancient Greek logic. I have preferred instead to concentrate on illustrating its philosophical aspects.

1 The Origins: Parmenides and Zeno

Greek philosophical logic originates with Parmenides (c. 510–c. 440 BCE). Though Parmenides cannot be said to have had a logic, or even an interest in studying the validity of arguments, his views did much to set the agenda out of which many things in Greek philosophy, including logic, later arose. His philosophical position is both simple and mystifying: being is, whereas not being is not and cannot either be thought or said. Consequently, any type of expression that implies that being is not or that not being is must be dismissed as nonsense. For Parmenides, this includes any reference to change (since it must involve the coming to be of what is not and the not being of what is) or multiplicity (since to say that there are two things is to say that something is not something else). The conclusion is that what is is one, unchanging, and uniform, without distinctions. Much of subsequent Greek philosophy is an effort to avoid these consequences and defend the coherence of talk of motion and multiplicity.

A second, and more explicitly logical, impact of Parmenides' thought on Greek philosophy is through its defense by Parmenides' follower Zeno of Elea (c. 490–c. 430 BCE). According to Plato's *Parmenides*, Zeno's goal was to defend Parmenides' views from the objection that they were absurd or in contradiction to our ordinary beliefs. In response, Zeno argued that the beliefs that there is motion and that there is a multiplicity of entities have consequences that are even more absurd because self-contradictory. This was the point of his celebrated arguments against motion and multiplicity.

To consider one example, Zeno gives the following argument (paraphrased) that motion is impossible:

> In order to move from point A to point B, you must first reach the point halfway between them. But before you can reach that point, you must reach the point halfway to it. Continuing in this way, we see that before you can reach any point, you must already have reached an infinity of points, which is impossible. Therefore, motion is impossible.

This argument rests only on the assumptions that motion is possible, that in order to move from one point to another one must first pass through the point halfway between, and that there is a point halfway between any two points.

Zeno's arguments take a particular form: beginning with premises accepted by his opponent, they derive conclusions that the opponent must recognize as impossible. Aristotle says that in introducing this form of argument, Zeno was the originator of 'dialectic'. The meaning of this word is contested by scholars, but we may note three features of Zeno's argument: (1) it is directed at someone else; (2) it takes its start from premises accepted by that other party; (3) its goal is the refutation of a view of that other party. These three characteristics can serve as a rough definition of a dialectical argument.

2 Dialectic and the Beginnings of Logical Theory

In the later fifth century BCE, professional teachers of oratory appeared in Athens. These were most often the same people called (by us, by their contemporaries, and often by themselves) 'Sophists'. We know that a number of the Sophists had interesting (and quite divergent) views on philosophical matters. Teaching oratory was a profitable occupation, and several Sophists seem to have amassed fortunes from it. The content of their instruction, to judge by later treatises on rhetoric, would have included such things as style and diction, but it would also have included some training in argumentation. That could have ranged from teaching set pieces of argument useful for specific situations, all the way to teaching some kind of method for devising arguments according to principles. One theme that emerges in several sophistic thinkers is a kind of relativism about truth. This is most forcefully put by Protagoras (c. 485–415 BCE), who began his treatise entitled *Truth* with the line, "Man is the measure of all things; of things that are, that they are, and of things that are not, that they are not." Plato tells us in his *Theaetetus* that this meant "whatever seems to be true to anyone is true to that person": he denied that there is any truth apart from the opinions of individuals. For Protagoras, this appears to have been connected with a thesis about the functioning of argument in a political situation. Whoever has the most skill at argument can make it seem (and thus be) to others however he wishes: in Protagoras' world, persuasive speech creates not merely belief but also truth.

Even apart from this perhaps extreme view, we find the themes of the variability of human opinion and the power of argument widespread in fifth-century Athens. Herodotus' history of the Persian Wars present a picture of opinions about right and wrong as merely matters of custom by displaying the variability in customs from one people to another. The treatise known as the *Twofold Arguments* (*Dissoi Logoi*) gives a series of arguments for and against each of a group of propositions; the implication is that argument can equally well support any view and its contradictory.

Contemporary with the Sophists was Socrates (469–399 BCE), whose fellow Athenians probably regarded him as another Sophist. Socrates did not teach oratory (nor indeed does he appear to have taught anything for a fee). Instead, he engaged people he encountered in a distinctive type of argument: beginning by asking them questions about matters they claimed to have knowledge of, he would lead them, on the basis of their own answers to further questions, to conclusions they found absurd or to contradictions of their earlier admissions. This process, which Plato and Aristotle both saw as a form of dialectical argument, usually goes by the name of 'Socratic refutation.' In overall form, it exactly resembles Zeno's arguments in support of Parmenides. Socrates insisted that he knew nothing himself and that his refutations were merely a tool for detecting ignorance in others.

Plato (428/7–348/7 BCE) did not develop a logical theory in any significant sense. However, he did try to respond to some of the issues raised by Parmenides, Protagoras, and others. In his *Theaetetus*, he argues that Protagoras' relativistic conception of truth is self-refuting in the sense that if Protagoras intends it to apply universally, then it must apply to opinions about Protagoras' theory of truth itself; moreover, it implies that the same opinions are both true and false simultaneously. He also partially rejects Parmenides' thesis that only what is can be thought or said by distinguishing a realm of 'becoming' that is not simply non-being but also cannot be said simply to be without qualification.

Plato's most celebrated philosophical doctrine, his theory of Forms or Ideas, can be seen as a theory of predication, that is, a theory of what it is for a thing to have a property or attribute. In very crude outline, Plato's response is that what it is for x (e.g. Socrates) to be F (e.g. tall) is for x to stand in a certain relation (usually called 'participation') to an entity, 'the tall itself,' which just is tall. In his *Sophist*, Plato begins to develop a semantic theory for predications. He observes that truth and falsehood are not properties of names standing alone but only of sentences produced by combining words. 'Theaetetus' and 'is sitting' are, in isolation, meaningful in some way but neither true nor false. We find truth or falsehood only in their combination: 'Theaetetus is sitting.' For Plato, a major achievement of this analysis is that it allows him to understand falsehoods as meaningful. In the sentence 'Theaetetus is flying,' both 'Theaetetus' and 'is flying' are meaningful; their combination is false, but it is still meaningful.

Aristotle (384–322 BCE), Plato's student, developed the first logical theory of which we know. He follows Plato in analyzing simple sentences into noun and verb, or subject and predicate, but he develops it in far greater detail and extends it to sentences which have general or universal (*katholou*, 'of a whole': the term seems to originate with Aristotle) subjects and predicates.

Aristotle also gives an answer to Protagoras and to related positions. Specifically, in Book IV of his *Metaphysics*, he argues that there is a proposition which is in a way prior to every other truth: it is prior because it is a proposition which anyone who knows anything must accept and because it is impossible actually to disbelieve it. The proposition in question is what we usually call the principle of non-contradiction: "it is impossible for the same thing to be both affirmed and denied of the same thing at the same time and in the same way" (*Met.* IV.3, 1005b19–20). He argues that it follows from this principle itself that no one can disbelieve it. At the same time, since it is prior to every other truth, it cannot itself be proved. However, Aristotle holds that anyone who claims to deny it (or indeed claims anything at all) already presupposes it, and he undertakes to show this through what he calls a "refutative demonstration" (*Met.* IV.4).

3 Aristotle and the Theory of Demonstration

When Aristotle says that the principle of non-contradiction cannot be proved because there is nothing prior from which it could be proved, he appeals to a more general thesis concerning demonstration or proof: no system of demonstrations can prove its own first principles. His argument for this appears in his *Posterior Analytics*, a work best regarded as the oldest extant treatise on the nature of mathematical proof. The subject of the *Posterior Analytics* is *demonstrative sciences*: a demonstrative science is a body of knowledge organized into demonstrations (proofs), which in turn are deductive arguments from premises already established. If a truth is demonstrable, then for Aristotle to know it just is to possess its demonstration: proofs are neither a means of finding out new truths nor an expository or pedagogical device for presenting results, but rather are constitutive of knowledge. Though he does not limit demonstrative sciences to mathematics, it is clear that he regards arithmetic and geometry as the clearest examples of them. Both historical and terminological affinities with Greek mathematics confirm this close association.

A demonstration, for Aristotle, is a deduction that shows why something is necessarily so. This at once imposes two critical limits on demonstrations: nothing can be demonstrated except what is necessarily so, and nothing can be demonstrated except that which has a cause or explanation (the force of the latter restriction will be evident shortly).

Since demonstrations are valid arguments, whatever holds of valid arguments in general will hold of them. Therefore, a natural place to begin the discussion of demonstrations would be with a general account of validity. Aristotle announces exactly that intention at the beginning of his *Prior Analytics*, the principal subject of which is the 'syllogism', a term defined by Aristotle as "an argument in which, some things being supposed, something else follows of necessity because of the things supposed." This is obviously a general definition of 'valid argument.' However, Aristotle thought that all valid arguments could be 'reduced' to a relatively limited set of valid forms which he usually refers to as 'arguments in the figures' (modern terminology refers to these forms as 'syllogisms'; this can lead to confusion in discussing Aristotle's theory).

Aristotle maintained that a single proposition was always either the *affirmation* or the *denial* of a single predicate of a single subject: 'Socrates is sitting' affirms 'sitting' of Socrates, 'Plato is not flying' denies 'flying' of Plato. In addition to simple predications such as those illustrated here, with individuals as subjects, he also regarded sentences with general subjects as predications: 'All Greeks are humans,' 'Dogs are mammals,' 'Cats are not bipeds.' (Here he parts company from modern logic, which since Frege has seen such sentences as having a radically different structure from predications.) Aristotle's logical theory is in effect the theory of general predications. In addition to the distinction between affirmation and denial, general predications can also be divided according as the predicate is affirmed or denied of all (universal) or only part (particular) of its subject. There are then four types of general predications:

	Affirmed (affirmative)	*Denied (negative)*
Universal	'Every human is mortal'	'No human is mortal'
Particular	'Some human is mortal'	'Not every human is mortal'

Aristotle then explores which combinations of two premises that share a term will imply a third sentence having the two non-shared terms as its subject and predicate. He distinguishes three possibilities based on the role of the shared term (the 'middle,' in his terminology) in the premises: it can be predicate of one and subject of the other (he calls this the 'first figure'), predicate of both ('second figure'), or subject of both ('third figure'). He carries out his investigation by first taking four combinations in the first figure as basic. He then systematically examines all other combinations in all the figures, doing one of two things for each of them: (1) in some cases, he shows that a conclusion follows by deducing that conclusion from the premises, using as resources one of the four basic forms and a limited stock of rules of inference; (2) in other cases, he shows that no conclusion follows by giving a set of counterexamples to any possible form of conclusion. As a result, he not only has an enumeration of all the valid forms of 'argument in the figures,' he also has shown that all of them can be 'reduced' to the basic four forms. He even shows that two of the basic forms can be derived from the other two using somewhat longer deductions. Following this treatment, he argues that every valid argument whatsoever can be 'reduced' to the valid forms of argument 'in the figures.' His defense of this is necessarily more complex, since it includes analysis of a variety of forms of arguments, for each of which he proposes ways to extract a figured argument.

I will not pursue here the details of his theory (see Corcoran 1973; Łukasiewicz 1957; Smiley 1974; Smith 1989). My concern instead is with the character of the whole enterprise. Aristotle's overriding concern is with demonstrating that every valid argument whatsoever can be reduced to a very small number of valid forms. This is not the sort of result that an author of a handbook for testing arguments for validity would want. It is, however, precisely the kind of result that someone interested in studying the structures of proofs would find valuable. And that is precisely the use we find Aristotle making of it. The only work of his that makes substantive use of the results proved in the *Prior Analytics* is the *Posterior Analytics*. Aristotle uses those results as the basis for a crucial argument to establish his position on the structures of demonstrative sciences. On this basis, I am persuaded that the theory contained in the *Prior Analytics* was developed largely to serve the needs of Aristotle's theory of demonstration, especially this argument: here, as in much of the early history of modern symbolic logic, logical theory arose to meet the needs of the philosophy of mathematics.

4 The Regress Argument of *Posterior Analytics* I.3

The argument to which I am referring is Aristotle's response to a problem about the possibility of demonstration that he presents in *Posterior Analytics* I.3: if demonstrations must rest on premises already demonstrated, then how is demonstration possible at all? Here is Aristotle's presentation of the positions in the debate:

> Some think that, because of the need to know the first things scientifically, there is no scientific knowledge. Others think that there is and that there is a demonstration of them all. Neither of these views is either true or necessary. Now, as for those who suppose that there is no scientific knowledge at all, they claim that it can be led into infinity, so that we do not know the posterior things from prior things of which none are first (and they are right, for it is impossible to go through infinite things). And if they do come to a stop and there are starting points, these will not be known just because there is no demonstration of them (which alone they say is scientific knowledge). And if it is not possible to know the first things, then neither is it possible to know those which follow from them scientifically, in the absolute or correct sense, but only from the assumption 'if these are so.' The other group agrees about scientific knowledge (that is, that it comes only through demonstration) but think that nothing prevents there being demonstration of everything because demonstration can be in a circle, that is, reciprocal. (*Posterior Analytics* I.3, 72b5–18)

Though this regress argument is frequently used as an early example of the kind of skeptical problem central to modern epistemology, a careful study of Aristotle's response to it shows that he has rather different concerns. He is really setting the stage for a complex and sophisticated argument about the structures of systems of mathematical proofs.

Even before Aristotle arrived in Athens, Plato's Academy was becoming a focal point for new developments in mathematics. In addition to proving new results and searching for the solutions to outstanding puzzles, Greek mathematicians had begun to arrange their accumulated knowledge systematically as a single structure of proofs. The ultimate outcome of this process, a century after Aristotle, was Euclid's *Elements*. However, though we do not know its contents, Hippocrates of Chios (fl. 440 BCE) composed an *Elements* in the late fifth or early fourth century, and Theudius of Magnesia (fl. c. 350? BCE) put together a treatise during Aristotle's lifetime that incorporated work by a number of other prominent mathematicians, including Archytas (428–347 BCE), Eudoxus (400–347 BCE), Leodamas (fl. c. 380 BCE), Theaetetus (c. 415–c. 369 BCE), and Menaechmus (c. 350? BCE). Euclid's *Elements* (c. 295 BCE) presupposes a certain overall structure for a mathematical system. At its basis are propositions which are not proved in the system; some of these are definitions, some are 'common conceptions' (*koinai ennoiai*), and some are 'things asked for' (*aitemata*: the customary translation is 'postulates'). Further propositions are added to the system by logical deduction from these first propositions and any others already proved; these are called theorems. Now, it is precisely this picture of a demonstrative system that is at issue in the passage quoted above from *Posterior Analytics* I.3, and one of the main goals of the treatise is to argue for it. Specifically, Aristotle argues that any demonstrative system must contain first propositions which are not demonstrated, or even demonstrable, in that system.

Aristotle's response to the regress argument appears at first to be a mere assertion: there are first principles that can be known without being demonstrated. We should then expect him to tell us straightaway what this other means of knowledge of these first principles is. Instead, he expends a great deal of argument trying to prove that the regress of premises always 'comes to a stop,' and it is in this argument that he needs the results established in the *Prior Analytics*. In order to appreciate the significance of this, we need to take note of an important difference between Aristotle's logical system and modern predicate (and propositional) logic. In Aristotle's logic, it is possible for there to be true propositions which cannot be deduced from any other set of true propositions whatsoever that does not already contain them. Aristotle's logic contains only predications, and the only rules of inference it knows about are those of the arguments in the figures. Now, a true sentence 'A belongs to every B' can only be deduced from premises of exactly one type: two premises of the forms 'A belongs to every C' and 'C belongs to every B.' If there are no such true premises, then 'A belongs to every B,' though true, is absolutely undeducible, and thus indemonstrable in a purely logical or semantic sense. Similar results hold for the other forms of sentence, though they are more complicated because there are multiple ways of deducing each of them.

Aristotle calls such true but undeducible sentences 'unmiddled' (*amesos*: the standard translation 'immediate,' though etymologically correct, is highly misleading). Since an unmiddled proposition cannot be deduced from anything, it obviously cannot be the object of a demonstration. Moreover, any premise regress that encounters such a proposition will come to a stop at that point. If every premise regress comes to a stop in unmiddled premises, then it might seem that we have a serious problem for the notion of demonstration, just as the anti-demonstrators of Aristotle's regress argument claimed. However, notice that it is a matter of objective fact which propositions are unmiddled in this way: given the sum total of all the true propositions, we can apply a set of mechanical procedures to find out which ones are unmiddled (Aristotle in effect gives us such a set of procedures in *Prior Analytics* I.27). Moreover, if we did have knowledge of just exactly the unmiddled propositions, then since they are the propositions in which every regress comes to a stop, and since a regress can be reversed to become a deduction, we would have knowledge of premises from which every other proposition could be deduced. Since unmiddled propositions cannot be known except by non-demonstrative means, it follows that the possibility of non-demonstrative knowledge of the unmiddled propositions is both a necessary and a sufficient condition for the possibility of demonstrations. Since there is no middle term explaining why an unmiddled proposition is true, there is no explanation of its truth: it is, in effect, uncaused and unexplained. Aristotle's view is precisely this: demonstrations, which give the causes why their conclusions must be true, ultimately rest on first premises for the truth of which there is no further explanation or cause.

This brief account of Aristotle's theory raises a host of important questions, most critically the question of how it is possible to have knowledge of these first indemonstrable premises. I will not try to pursue that issue further here (see Smith 1986 for a little more detail). The point I wish to emphasize is that Aristotle's logical theory arose in response to a philosophical question about the possibility of proof. Aristotle's logic is, at its core, a philosophical logic.

5 Time and Modality: The Sea-Battle and the Master Argument

Necessity and possibility were subjects of major importance for ancient logicians. This might be seen as part of the Parmenidean legacy, since Parmenides asserted that what is must be and what is not cannot be: from there it is not a long distance to the view that what is the case is necessary and what is not the case is impossible. On such a view, possibility and necessity collapse into one another. Only that which is, is possible; thus, what is possible is simply what is necessary, and there are no possibilities that are not actual. In other words, Parmenides' position appears to lead to a universal determinism or fatalism. Since such a view seems to rule out such things as free choice and deliberation, it runs into conflict both with common sense and with many philosophical views. Not surprisingly, we find considerable discussion of necessity and possibility in Greek philosophy. A good deal of that discussion involves the attempt to deal with these concepts in a logical system. Once again, we find that Greek logical theory developed in response to philosophical questions.

In *Metaphysics* IX.3, Aristotle ascribes the view that the modalities all collapse into one another to "the Megarians" and is at some pains to argue against it. Though he does not tell us who these Megarians were, we can supply a little history from other sources. Euclid of Megara (c. 430–c. 360 BCE), an approximate contemporary of Plato, was a member of Socrates' circle. He is said to have been influenced by Parmenides' views and to have maintained that "the good is one." We are told that he attacked arguments "not from their premises but from their conclusions;" what this means is not clear, but one possible interpretation is that Euclid followed Zeno in attacking rival positions by showing that they led to unacceptable consequences. A small circle of followers assembled around him, and from the beginning they appear to have had a strong interest in argumentation, especially in its dialectical form, in refutations, and in logical puzzles and paradoxes. Kleinomachus of Thurii, perhaps one of the first generation of Megarians, is said to have been the first to write on 'predications and propositions.' Eubulides, coming a generation or two later, is credited with the discovery of a number of paradoxes, including two of the most durable and difficult: the Liar and the Sorites. Eubulides engaged in a somewhat vitriolic controversy with Aristotle.

Now, Aristotle thought that the solution to Eleatic and Megarian arguments against motion and change could be found in a robust notion of potentiality. Aristotelian potentialities might be described as properties that point outside the present time. A lump of bronze, for instance, has the potentiality of being a statue, even though it is not one now, because it could, while remaining the same bronze, acquire the appropriate shape. Socrates, who is now seated, has the potentiality of standing up because he could, at some other time, acquire the property of standing up without ceasing to be Socrates. An intact garment has the potentiality of being cut up; a stone at the top of a hill has the potentiality of being at the bottom of the hill; a log has the potentiality of burning; an illiterate person has the potentiality of learning to read.

Potentialities make change possible, for Aristotle, since they allow him to describe change not at the coming to be of what was not but merely as the actualization of what was already in potentiality. For the bronze to become a statue, it is not necessary (as the Megarians might have it) that the lump of bronze cease to be and a new bronze statue emerges *ex nihilo*; instead, the same bronze persists, but a shape already possessed by it in potentiality becomes its actual shape. Aristotle extends this to a general definition of motion as "the actuality of what is in potentiality insofar as it is in potentiality." On this basis, he thinks that he can respond to Zeno's paradoxes of motion by claiming that a body in motion, while it is in motion, is never actually at any location: it is actually only in motion, only potentially at any of the points along its path. Were it to stop, of course, it would actually be located at some point; but then, it would no longer be in motion.

I will not discuss here whether this is an effective response to Zeno: what is important is that it depends on a notion of potentialities as properties which things can have at a given time without exhibiting them at that time. The potentiality (capacity, ability) which Socrates has of standing up does not manifest itself while he is seated, but it is there nonetheless: when he stands, of course, it is no longer a potentiality but an actuality. Precisely this point is what the Megarians denied. They held that the only possible evidence for the claim that Socrates can stand up is for him actually to do so: however, his standing will provide no evidence that he could have stood up a moment ago while he was sitting, but only evidence that he can stand now while he is standing.

So far, this may seem to be primarily a matter of metaphysics. In *On Interpretation* 9, however, Aristotle presents us with an argument resting on logical principles. The background of the argument is the notion of a 'contradiction' or 'contradictory pair' (*antiphasis*): two propositions with the same subject, one of which denies of that subject exactly what the other affirms of it (for example, 'Socrates is seated,' 'Socrates is not seated'). In general, Aristotle says that for any contradictory pair at any time, one of the pair is true and the other false. He finds a problem, however, if we allow this to extend to propositions about the future. All we need is the additional thesis that whatever is true about the past is now necessarily true and the general semantical principle that if a proposition is true, then whatever it says is the case is indeed the case. Imagine now that yesterday, I said, 'There will be a sea-battle tomorrow.' By the general principle governing contradictory pairs, either this sentence or its contradictory 'There will not be a sea-battle tomorrow' must have been true when I made my statement. If the sentence was true, then it is now a truth about the past that it was true, and therefore it is now necessary that it was true; therefore, it is now necessarily true that there is a sea-battle today. If, on the other hand, my statement was false, then by similar reasoning it is now necessarily false that there is a sea-battle today. Since my statement was either true or false, then it is now either necessary or impossible that there is a sea-battle today. But this can be generalized to any event at any time, since (as Aristotle says) surely it does not matter whether anyone actually uttered the sentence: thus, everything which happens happens of necessity, and there are no possibilities which do not become actual. It is far from clear just how Aristotle responds to this puzzle, except that he is certain that its conclusion must be rejected. One interpretation is that in order to avoid the repugnant conclusion, he restricts the application of the law of excluded middle to future propositions (the literature on this argument is enormous: see the Suggested Further Reading below for a few places to start).

Aristotle does not tell us the source of the argument to which he is responding in *On Interpretation* 9, though it is a reasonable guess that its author was Megarian. One piece of evidence in favor of that is the 'Master' argument developed by Diodorus Cronus (c. 360–c. 290 BCE). Our sources identify Diodorus as a Megarian (though some scholars have disagreed); his dates are unclear, and it is just possible that Aristotle is actually responding to Diodorus, though I think it more likely that he is replying to an ancestor of the Master developed by other Megarians. In any event, the Master began with a proof that the following three propositions form an inconsistent triad, so that the affirmation of any two entails the denial of the third:

1. What is past is necessary.
2. The impossible does not follow from the possible.
3. There is something possible which neither is nor will be true.

The first of these recalls the argument of *On Interpretation* 9. What the second means is not totally clear, but one reading is 'a possible proposition cannot entail an impossible one.' We do not know how Diodorus argued for the incompatibility of this triad, but we do know the conclusion he drew from it: he affirmed the first two propositions and deduced the denial of the third, so that for him 'possible' was equivalent to 'either now true or true in the future.' His view here conflicts directly with Aristotle, who asserts that there are possibilities that never become actual. What Diodorus may have been doing, in addition to defending a Megarian view of universal necessitation, was finding a way to talk about possibilities in a Megarian view of the world. That is, his position would allow him to assert that there is indeed a meaning for the word 'possible,' even though nothing can happen except what does happen.

The later history of the Master is closely associated with the Stoic school, which began with Zeno of Citium (335–263 BCE). Zeno learned logic from Megarian teachers, and Zeno and his follower Cleanthes (331–232 BCE) responded to the Master. Subsequently, Chrysippus (c. 280–207 BCE), the most distinguished logician among the Stoics and probably the most gifted and prolific logician of the Hellenistic period, affirmed the first and third propositions of the Master and denied the second: he argued that 'an impossible can follow from a possible.' To understand his response, we need first a brief sketch of his theory of propositions. For Chrysippus, a proposition – that is, what is true or false – is really an incorporeal entity, roughly the meaning of a sentence that expresses it (the Stoics called this a *lekton*, 'sayable,' which might plausibly be translated 'meaning' or 'sense'). There are similarities between this notion and, say, a Fregean notion of the sense of a proposition, though there are important differences. One important difference is that the Stoics thought of at least some propositions as changing their truth values over time, for example the proposition expressed by 'It is day' is at one time true and at another false while remaining the same proposition. Another Stoic thesis, and one that is crucial to Chrysippus' solution, is that propositions about individuals specified by demonstratives 'perished' when the individuals ceased to exist. If I point to Dion and say 'He is alive,' then I utter a proposition the subject of which is fixed by a demonstrative (in modern terms, an indexical). However, if Dion dies, then I can no longer point to Dion at all, since he does not exist; therefore, the proposition that was formerly expressed by 'He is alive' also ceases to exist rather than becoming false. Now, Chrysippus offers for consideration the proposition 'If Dion has died, then this one has died' (pointing to a living Dion, obviously). Since 'this one' refers to Dion, this conditional sentence is obviously true: its consequent follows from its antecedent. However, when Dion has died, the antecedent of the conditional becomes true while its consequent perishes: in fact, it is in a sense impossible for 'This one has died' ever to be true, since the condition for its truth is also the condition for its perishing. Therefore, we have an example of something impossible following from something possible.

Both the Stoics and Aristotle, then, investigated logical modalities in order to reconcile logical theory with their views about determinism. Chrysippus, who was a determinist, could nevertheless argue that his views did not entail that only what is necessary is possible, since he can produce an example of a proposition that is possible but that neither is nor will be true: 'This one has died.' Aristotle, who rejects universal necessitarianism and develops a complex theory of potentialities to accommodate his views on motion and deliberation, at least recognizes that his position will require some radical modification of his logical theory. (For more on the Master argument, see the readings cited below, especially Fine 1984; Gaskin 1995; Prior 1967.)

6 Sentential Logic in Aristotle and Afterwards

Aristotle never developed an account of sentential logic (the inferences that rest on sentential operators such as 'and,' 'or,' 'if,' 'not'). In my opinion, this is closely connected with his use of his logical theory in the *Posterior Analytics*. His argument that 'every regress terminates' can only work if the logic of arguments 'in the figures' is the only logic there is; and for that to be so, every proposition must either affirm or deny a predicate of a subject. In fact, Aristotle thinks that this is so, and he undertakes to show it in the *Prior Analytics.* This requires him to reject sentential composition: he does not recognize conjunctions, disjunctions, or conditionals as individual propositions. Precisely how this is to work is not clear, though we can discern a few details. For instance, because he treats affirmations and denials as two basic types of sentence, he does not think of negations as compound sentences; he appears to regard conjunctions not as single compound sentences but only as, in effect, collections of sentences (i.e. their conjuncts); and he treats conditionals not as assertions but as agreements to the effect that one sentence (the antecedent of the conditional) entails another (the consequent). Subsequent logicians, including Aristotle's own close associate Theophrastus, did not follow him in this and instead offered analyses of the role of sentential composition in arguments. With Chrysippus, this develops into a full-fledged sentential logic, resting on five 'indemonstrable' forms of inference. The Stoics stated these using ordinal numbers as place-holders for propositions:

1. If the first, then the second; the first; therefore the second.
2. If the first then the second; not the first; therefore not the second.
3. Not both the first and the second; the first; therefore not the second.
4. Either the first or the second; the first; therefore not the second.
5. Either the first or the second; not the first; therefore the second.

The Stoics then demonstrated the validity of other valid arguments by means of these indemonstrables (unfortunately, our knowledge of their views is very fragmentary: see Kneale and Kneale 1978; Mates 1953; Mueller 1978 for reconstructions). There may be some connection between the Stoic acceptance of sententially compound propositions and their views on the nature of propositions.

Aristotle may have another reason for being concerned about sentential logic. He wanted to allow for possibilities that never become actual, and to do that he analyzed possibility in terms of a notion of potentiality. This works best with subject–predicate sentences, where possibility can be seen as a matter of the subject possessing a potentiality; it is very difficult to extend it to compound propositions. In fact, Aristotle appears to have had some reservations about treating propositions as entities at all, perhaps because this appeared to give support to the argument from the necessity of past truth in *On Interpretation* 9. The Stoics, with their theory of 'sayables' as the bearers of truth and falsehood and their acceptance of a kind of determinism, had a much easier time developing a logic of sentential composition. Here again, a difference in logical theory may have been closely entwined with a difference in philosophical standpoint.

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