Understanding Financial Management: A Practical Guide Problems and Answers

Chapter 4 Time Value of Money

Note: You can use a financial calculator to check the answers to each problem.

4.2 Future Value of a Present Amount

1. If an investor deposits \$100,000 today, how much will she have 10 years from today if she earns an annual interest rate of (A) 6%, (B) 8%, and (C) 10%?

4.3 Present Value of a Future Amount

- 2. What is the present value of a \$750,000 payment that is expected to be received 20 years from today assuming an annual discount rate of (a) 6%, (b) 8%, and (c) 10%?
- 3. To settle a legal dispute, Lemon Inc. has agreed to pay damages to a competitor of \$1.25 million one year from today, \$1.5 million two years from today, and \$1.75 million three years from today. At a discount rate of 6%, what is the present value of the settlement payments?

4.4 Future Value of an Annuity

- 4. Janzen Corp plans to deposit \$2 million at the end of each of the next 15 years into a sinking fund account in order to have sufficient funds to retire a large bond issue. If Janzen can earn 8% per year on these funds, how much will the firm have in the sinking fund account 15 years from today?
- 5. To fund a future large capital expenditure, Lexor Inc. deposited \$1 million today and plans to also deposit \$500,000 at the end of each year for the next eight years. If Lexor can earn 9% annual return on these deposits, how much will the firm accumulate eight years from today?

4.5 Present Value of an Annuity

- 6. Compute the present value of 20-year annuity with annual payments of \$20,000 using a discount rate of 8%.
 - A. Assume payments occur at the end of each year.
 - B. Assume payments occur at the beginning of each year.

7. Xylar Company promises to pay a retiring employee \$10,000 at the end of each year for the next four years followed by \$15,000 at the end of each year for six more years. What is the present value of these future cash flows assuming a discount rate of 9%?

4.6 Present Value of a Perpetuity

8. Infinity Computer Inc. has just issued a preferred stock with no maturity that promises to pay dividends of \$80 a year. If investors require a 10% rate of return, what is the present value of these future dividend payments?

4.7 Compounding Frequencies

- 9. A small firm has recently invested \$50,000. If the firm expects to receive a return of 10% per year, compounded semi-annually, on this investment, what will be the future value of the investment five years from today? What if interest is compounded quarterly?
- 10. An investor currently has \$25,000 in his investment account. He plans to deposit an additional \$1,000 at the end of each month for the next four years. If he can earn 9% per year, compounded monthly, how much will the account be worth four years from now?
- 11. First Mortgage Inc. loaned a company \$300,000 at an annual interest rate (compounded monthly) of 12%. What is the end-of-the-month payment over the 30-year life of the mortgage loan?
- 12. An investor has deposited \$20,000 into an account that pays 9.5% per year, compounded continuously. How much will the account be worth six years from today?

4.8 Nominal and Effective Interest Rates

13. AMV Bank is offering a mortgage rate of 6.25% a year. If the bank requires end-of-the month payments, what is the effective annual interest rate on the mortgage?

4.9 Solving for an Unknown Interest Rate

- 14. A firm just repaid a \$1 million loan by making five equal, end-of-year payments of \$300,000 in years 1 through 5. What is the implied interest rate on this loan?
- 15. An investment offers to pay investors \$125 at the end of each year for 10 years and to make an additional lump-sum payment of \$1,000 at the end of year 10. The investment is currently selling at a price of \$925. What is the implied interest rate on this investment?
- 16. An auto dealer is offering the following deal on a \$12,350 car: no down payment; \$245 per month payment for the first three years; \$295 per month payment for the following two years. What interest rate is implied by this deal?

4.10 Other Time Value Applications

- 17. Generous Motors is offering its customers two financing choices on its popular line of Ventura automobiles. Under Option A, customers receive a \$1,000 rebate. Under Option B, customers receive a special financing rate of 2.4% per year, compounded monthly. Assume a customer who chooses Option A can finance the full purchase price of the car over a four-year period at an interest rate of 9% per year, compounded monthly. Assume the customer will purchase the car and keep it for four years and can finance the entire purchase price.
 - A. Assume the purchase price of the new car is \$20,000 and the customer plans to keep the car for the entire four years. Which option should you choose?
 - B. What rebate amount would make the customer indifferent between Option A and Option B?

Answers

- 1A. The future value of a present amount: $FV = PV(1+r)^n$ $FV = PV(1+r)^n = \$100,000(1.06)^{10} = \$179,084.77$
- 1B. $FV = $100,000(1.08)^{10} = $215,892.50$
- 1C. $FV = $100,000(1.10)^{10} = $259,374.25$
- 2A. The present value of a future amount: $PV = FV\left[\frac{1}{(1+r)^n}\right]$

$$\mathsf{PV} = \mathsf{FV}\left[\frac{1}{(1+r)^n}\right] = \$750,000\left[\frac{1}{(1.06)^{20}}\right] = \$233,853.55$$

2B.
$$PV = $750,000 \left[\frac{1}{(1.08)^{20}} \right] = $160,911.16$$

2C. PV =
$$$750,000 \left[\frac{1}{(1.10)^{20}} \right] = $111,482.72$$

3. Calculate the present value of each future amount and then sum these amounts:

$$PV = \$1,250,000 \left[\frac{1}{(1.06)^{1}} \right] + \$1,500,000 \left[\frac{1}{(1.06)^{2}} \right] + \$1,750,000 \left[\frac{1}{(1.06)^{3}} \right] = \$3,983,573.69$$

- 4. The future value of the annuity: $FV = PMT\left[\frac{(1+r)^n 1}{r}\right]$ $FV = $2,000,000\left[\frac{(1.08)^{15} - 1}{0.08}\right] = $54,304,227.85$
- 5. Calculate the FV value, the FV of an ordinary annuity and then sum these amounts:

$$FV = \$1,000,000(1.09)^8 + \$500,000 \left[\frac{(1.09)^8 - 1}{0.09}\right] = \$1,992,562.64 + \$5,514,236.90$$
$$= \$7,506,799.54$$

6A. The present value of an ordinary annuity:

$$PV = PMT\left[\frac{1 - \frac{1}{(1+r)^{n}}}{r}\right] = \$20,000\left[\frac{1 - \frac{1}{(1.08)^{20}}}{0.08}\right] = \$196,362.95$$

6B. The present value of an annuity due:

$$PV = PMT \left[\frac{1 - \frac{1}{(1+r)^{n}}}{r} \right] (1+k) = \$20,000 \left[\frac{1 - \frac{1}{(1.08)^{20}}}{0.08} \right] (1.08) = \$212,071.98$$

7. Compute the PV of both annuities and then add them together:

$$PV = \$10,000 \left[\frac{1 - \frac{1}{(1.09)^4}}{0.09} \right] + \$15,000 \left[\frac{1 - \frac{1}{(1.09)^6}}{0.09} \right] \left[\frac{1}{(1.09)^4} \right] = \$32,397.20 + \$47,669.07$$
$$= \$80,066.27$$

8. The present value of the perpetuity:

$$\mathsf{PV} = \frac{\mathsf{PMT}}{\mathsf{r}} = \frac{\$80}{0.10} = \$800.00$$

9. The FV of the investment assuming semi-annual annual compounding:

$$FV = PV(1+r)^n =$$
\$50,000 $(1.05)^{10} =$ \$81,444.73

With quarterly compounding:

$$FV = $50,000(1.025)^{20} = $81,930.82$$

10. The future value of this investment account is:

$$FV = \$25,000(1.0075)^{48} + \$1,000\left[\frac{(1.0075)^{48} - 1}{0.0075}\right] = \$35,785.13 + \$57,520.71$$
$$= \$93,305.84$$

11. The loan amount is a PV. We use the present value of an annuity equation to solve for the payment:

$$PV = PMT \left[\frac{1 - \frac{1}{(1+r)^{n}}}{r} \right]$$

\$300,000 = PMT $\left[\frac{1 - \frac{1}{(1.01)^{360}}}{0.01} \right]$

Rearranging and solving for PMT, the monthly payment is \$3085.84.

12. The future value amount using continuous compounding:

$$FV = PVe^{m} = $20,000e^{(.095)(6)} = $20,000e^{(.57)} = $35,365.34$$

13. Solve the effective annual interest rate using m = 12 periods per year:

$$r_{eff} = \left[1 + \frac{r_{nom}}{m}\right]^m - 1 = \left[1 + \frac{0.0625}{12}\right]^{12} - 1 = 0.0643 \text{ or } 6.43\%$$

14. Solve the present value of an annuity formula for the unknown interest rate:

$$1,000,000 = 300,000 \left[\frac{1 - \frac{1}{(1 + r)^5}}{r} \right]$$

Using trial and error: guess 15.24%

$$1,000,000 = 300,000 \left[\frac{1 - \frac{1}{(1.1524)^5}}{0.1524} \right]$$

Therefore, the implied interest rate is 15.24%.

Using the BA II Plus[®] calculator, the correct interest rate is found as follows:

- 1. 2nd CLR TVM
- 2. 5 N
- 3. 1,000,000 PV
 4. 300,000 +/-
- 5. CPT I/Y

The implied interest rate is 15.24%.

15. Solve for the unknown interest rate:

$$\$925 = \$125 \left[\frac{1 - \frac{1}{(1+r)^{10}}}{r} \right] + \$1,000 \left[\frac{1}{(1+r)^{10}} \right]$$

Solve for r using trial and error. Guess 13.93%

$$925 = 125 \left[\frac{1 - \frac{1}{(1.1393)^{10}}}{0.1393} \right] = 1000 \left[\frac{1}{(1.1393)^{10}} \right]$$

Therefore, the implied interest rate is 13.93%.

Using the BA II Plus[®] calculator, the correct interest rate is found as follows:

- 1. 2nd CLR TVM
- 2. 10 N
- 3. 925 PV
- 4. 125 +/- PMT
- 5. 1,000 +/- FV
- 6. CPT I/Y

The implied interest rate is 13.93%.

16. Solve for the unknown implied interest rate:

$$12,350 = 245 \left[\frac{1 - \frac{1}{(1+r)^{36}}}{r} \right] + 295 \left[\frac{1 - \frac{1}{(1+r)^{24}}}{r} \right] \left[\frac{1}{(1+r)^{36}} \right]$$

Solving for the unknown interest rate algebraically involves an interative procedure. Using trial and error, the result is r=0.8291%

$$245\left[\frac{1-\frac{1}{(1.008291)^{36}}}{0.008291}\right]+295\left[\frac{1-\frac{1}{(1.008291)^{24}}}{0.008291}\right]\left[\frac{1}{(1.008291)^{36}}\right]=12,350$$

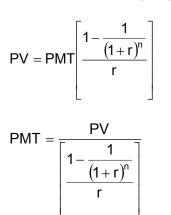
Thus, r = 0.8291% per month. r = 9.95% per year (0.8291 × 12)

Using the BA II Plus[®] calculator, the correct interest rate is found as follows:

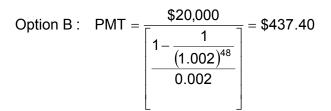
1. $[CF] \rightarrow [2^{nd}] \rightarrow [CLR WORK]$ 2. $12,350 \rightarrow [+/-] \rightarrow [ENTER]$ 3. $[\downarrow] \rightarrow 245 \rightarrow [ENTER]$ 4. $[\downarrow] \rightarrow 36 \rightarrow [ENTER]$ 5. $[\downarrow] \rightarrow 295 \rightarrow [ENTER]$ 6. $[\downarrow] \rightarrow 24 \rightarrow [ENTER]$ 7. $[IRR] \rightarrow [CPT]$

Thus, r = 0.8291% per month. r = 9.95% per year (0.8291 × 12).

17A. Compute the monthly payment for each option using PV of annuity formula:

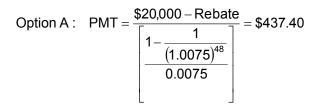


Option A : PMT =
$$\frac{\$19,000}{\left[\frac{1-\frac{1}{(1.0075)^{48}}}{0.0075}\right]} = \$472.82$$



The customer should choose Option B with the lower monthly payment of \$437.40 versus \$472.82 over the 48 month period.

17B. Solve for the PV amount under Option A that would give a monthly payment of \$437.40:



Discounting 437.40 for 48 periods at 0.75% a month gives a present value of 17,576.82. Thus, 20,000.00 - 17,476.82 = 2,423.18, which is the amount of the rebate.