Understanding Financial Management: A Practical Guide
Problems and Answers

Chapter 4
Time Value of Money

Note: You can use a financial calculator to check the answers to each problem.

4.2 Future Value of a Present Amount

1. If an investor deposits $100,000 today, how much will she have 10 years from today if she earns an annual interest rate of (A) 6%, (B) 8%, and (C) 10%?

4.3 Present Value of a Future Amount

2. What is the present value of a $750,000 payment that is expected to be received 20 years from today assuming an annual discount rate of (a) 6%, (b) 8%, and (c) 10%?

3. To settle a legal dispute, Lemon Inc. has agreed to pay damages to a competitor of $1.25 million one year from today, $1.5 million two years from today, and $1.75 million three years from today. At a discount rate of 6%, what is the present value of the settlement payments?

4.4 Future Value of an Annuity

4. Janzen Corp plans to deposit $2 million at the end of each of the next 15 years into a sinking fund account in order to have sufficient funds to retire a large bond issue. If Janzen can earn 8% per year on these funds, how much will the firm have in the sinking fund account 15 years from today?

5. To fund a future large capital expenditure, Lexor Inc. deposited $1 million today and plans to also deposit $500,000 at the end of each year for the next eight years. If Lexor can earn 9% annual return on these deposits, how much will the firm accumulate eight years from today?

4.5 Present Value of an Annuity

6. Compute the present value of 20-year annuity with annual payments of $20,000 using a discount rate of 8%.
   A. Assume payments occur at the end of each year.
   B. Assume payments occur at the beginning of each year.

7. Xylar Company promises to pay a retiring employee $10,000 at the end of each year for the next four years followed by $15,000 at the end of each year for six more years. What is the present value of these future cash flows assuming a discount rate of 9%?

4.6 Present Value of a Perpetuity

8. Infinity Computer Inc. has just issued a preferred stock with no maturity that promises to pay dividends of $80 a year. If investors require a 10% rate of return, what is the present value of these future dividend payments?

4.7 Compounding Frequencies

9. A small firm has recently invested $50,000. If the firm expects to receive a return of 10% per year, compounded semi-annually, on this investment, what will be the future value of the investment five years from today? What if interest is compounded quarterly?

10. An investor currently has $25,000 in his investment account. He plans to deposit an additional $1,000 at the end of each month for the next four years. If he can earn 9% per year, compounded monthly, how much will the account be worth four years from now?

11. First Mortgage Inc. loaned a company $300,000 at an annual interest rate (compounded monthly) of 12%. What is the end-of-the-month payment over the 30-year life of the mortgage loan?

12. An investor has deposited $20,000 into an account that pays 9.5% per year, compounded continuously. How much will the account be worth six years from today?

4.8 Nominal and Effective Interest Rates

13. AMV Bank is offering a mortgage rate of 6.25% a year. If the bank requires end-of-the month payments, what is the effective annual interest rate on the mortgage?

4.9 Solving for an Unknown Interest Rate

14. A firm just repaid a $1 million loan by making five equal, end-of-year payments of $300,000 in years 1 through 5. What is the implied interest rate on this loan?

15. An investment offers to pay investors $125 at the end of each year for 10 years and to make an additional lump-sum payment of $1,000 at the end of year 10. The investment is currently selling at a price of $925. What is the implied interest rate on this investment?

16. An auto dealer is offering the following deal on a $12,350 car: no down payment; $245 per month payment for the first three years; $295 per month payment for the following two years. What interest rate is implied by this deal?

4.10 Other Time Value Applications

17. Generous Motors is offering its customers two financing choices on its popular line of Ventura automobiles. Under Option A, customers receive a $1,000 rebate. Under Option B, customers receive a special financing rate of 2.4% per year, compounded monthly. Assume a customer who chooses Option A can finance the full purchase price of the car over a four-year period at an interest rate of 9% per year, compounded monthly. Assume the customer will purchase the car and keep it for four years and can finance the entire purchase price.
    A. Assume the purchase price of the new car is $20,000 and the customer plans to keep the car for the entire four years. Which option should you choose?
    B. What rebate amount would make the customer indifferent between Option A and Option B?

Answers

1A. The future value of a present amount: $FV = PV(1+r)^n$

$$FV = PV(1+r)^n = \$100{,}000(1.06)^{10} = \$179{,}084.77$$

1B. $FV = \$100{,}000(1.08)^{10} = \$215{,}892.50$

1C. $FV = \$100{,}000(1.10)^{10} = \$259{,}374.25$

2A. The present value of a future amount: $PV = FV\left[\frac{1}{(1+r)^n}\right]$

$$PV = FV\left[\frac{1}{(1+r)^n}\right] = \$750{,}000\left[\frac{1}{(1.06)^{20}}\right] = \$233{,}853.55$$

2B. $PV = \$750{,}000\left[\frac{1}{(1.08)^{20}}\right] = \$160{,}911.16$

2C. $PV = \$750{,}000\left[\frac{1}{(1.10)^{20}}\right] = \$111{,}482.72$

3. Calculate the present value of each future amount and then sum these amounts:

$$PV = \$1{,}250{,}000\left[\frac{1}{(1.06)^1}\right] + \$1{,}500{,}000\left[\frac{1}{(1.06)^2}\right] + \$1{,}750{,}000\left[\frac{1}{(1.06)^3}\right] = \$3{,}983{,}573.69$$

4. The future value of the annuity: $FV = PMT\left[\frac{(1+r)^n - 1}{r}\right]$

$$FV = \$2{,}000{,}000\left[\frac{(1.08)^{15} - 1}{0.08}\right] = \$54{,}304{,}227.85$$

5. Calculate the FV value, the FV of an ordinary annuity and then sum these amounts:

$$FV = \$1{,}000{,}000(1.09)^8 + \$500{,}000\left[\frac{(1.09)^8 - 1}{0.09}\right] = \$1{,}992{,}562.64 + \$5{,}514{,}236.90$$

$$= \$7{,}506{,}799.54$$

6A. The present value of an ordinary annuity:

$$PV = PMT\left[\frac{1-\frac{1}{(1+r)^n}}{r}\right] = \$20{,}000\left[\frac{1-\frac{1}{(1.08)^{20}}}{0.08}\right] = \$196{,}362.95$$

6B. The present value of an annuity due:

$$PV = PMT\left[\frac{1-\frac{1}{(1+r)^n}}{r}\right](1+k) = \$20{,}000\left[\frac{1-\frac{1}{(1.08)^{20}}}{0.08}\right](1.08) = \$212{,}071.98$$

7. Compute the PV of both annuities and then add them together:

$$PV = \$10{,}000\left[\frac{1-\frac{1}{(1.09)^4}}{0.09}\right] + \$15{,}000\left[\frac{1-\frac{1}{(1.09)^6}}{0.09}\right]\left[\frac{1}{(1.09)^4}\right] = \$32{,}397.20 + \$47{,}669.07$$
$$= \$80{,}066.27$$

8. The present value of the perpetuity:

$$PV = \frac{PMT}{r} = \frac{\$80}{0.10} = \$800.00$$

9. The FV of the investment assuming semi-annual annual compounding:

$$FV = PV(1+r)^n = \$50{,}000(1.05)^{10} = \$81{,}444.73$$

With quarterly compounding:

$$FV = \$50{,}000(1.025)^{20} = \$81{,}930.82$$

10. The future value of this investment account is:

$$FV = \$25{,}000(1.0075)^{48} + \$1{,}000\left[\frac{(1.0075)^{48}-1}{0.0075}\right] = \$35{,}785.13 + \$57{,}520.71$$
$$= \$93{,}305.84$$

11. The loan amount is a PV. We use the present value of an annuity equation to solve for the payment:

$$PV = PMT\left[\frac{1-\frac{1}{(1+r)^n}}{r}\right]$$

$$\$300{,}000 = PMT\left[\frac{1-\frac{1}{(1.01)^{360}}}{0.01}\right]$$

Rearranging and solving for PMT, the monthly payment is $3085.84.

12. The future value amount using continuous compounding:

$$FV = PVe^{rn} = \$20{,}000e^{(.095)(6)} = \$20{,}000e^{(.57)} = \$35{,}365.34$$

13. Solve the effective annual interest rate using m = 12 periods per year:

$$r_{eff} = \left[1+\frac{r_{nom}}{m}\right]^m - 1 = \left[1+\frac{0.0625}{12}\right]^{12} - 1 = 0.0643 \text{ or } 6.43\%$$

14. Solve the present value of an annuity formula for the unknown interest rate:

$$\$1{,}000{,}000 = \$300{,}000\left[\frac{1-\frac{1}{(1+r)^5}}{r}\right]$$

Using trial and error: guess 15.24%

$$\$1{,}000{,}000 = \$300{,}000\left[\frac{1-\frac{1}{(1.1524)^5}}{0.1524}\right]$$

Therefore, the implied interest rate is 15.24%.

Using the BA II Plus® calculator, the correct interest rate is found as follows:

1. 2nd CLR TVM
2. 5 N
3. 1,000,000 PV
4. 300,000 +/-
5. CPT I/Y

The implied interest rate is 15.24%.

15. Solve for the unknown interest rate:

$$\$925 = \$125\left[\frac{1-\frac{1}{(1+r)^{10}}}{r}\right] + \$1{,}000\left[\frac{1}{(1+r)^{10}}\right]$$

Solve for r using trial and error. Guess 13.93%

$$\$925 = \$125\left[\frac{1-\frac{1}{(1.1393)^{10}}}{0.1393}\right] = \$1{,}000\left[\frac{1}{(1.1393)^{10}}\right]$$

Therefore, the implied interest rate is 13.93%.

Using the BA II Plus® calculator, the correct interest rate is found as follows:

1. 2nd CLR TVM
2. 10 N
3. 925 PV
4. 125 +/- PMT
5. 1,000 +/- FV
6. CPT I/Y

The implied interest rate is 13.93%.

16. Solve for the unknown implied interest rate:

$$12{,}350 = 245\left[\frac{1-\frac{1}{(1+r)^{36}}}{r}\right] + 295\left[\frac{1-\frac{1}{(1+r)^{24}}}{r}\right]\left[\frac{1}{(1+r)^{36}}\right]$$

Solving for the unknown interest rate algebraically involves an interative procedure. Using trial and error, the result is r = 0.8291%

$$245\left[\frac{1-\frac{1}{(1.008291)^{36}}}{0.008291}\right] + 295\left[\frac{1-\frac{1}{(1.008291)^{24}}}{0.008291}\right]\left[\frac{1}{(1.008291)^{36}}\right] = 12{,}350$$

Thus, r = 0.8291% per month. r = 9.95% per year (0.8291×12)

Using the BA II Plus® calculator, the correct interest rate is found as follows:

1. [CF] →[2nd] → [CLR WORK]
2. 12,350 → [+/-] → [ENTER]
3. [↓] → 245 → [ENTER]
4. [↓] → 36 → [ENTER]
5. [↓] → 295 → [ENTER]
6. [↓] → 24 → [ENTER]
7. [IRR] → [CPT]

Thus, r = 0.8291% per month. r = 9.95% per year (0.8291×12).

17A. Compute the monthly payment for each option using PV of annuity formula:

$$PV = PMT\left[\frac{1-\frac{1}{(1+r)^n}}{r}\right]$$

$$PMT = \frac{PV}{\left[\frac{1-\frac{1}{(1+r)^n}}{r}\right]}$$

$$\text{Option A}: PMT = \frac{\$19{,}000}{\left[\frac{1-\frac{1}{(1.0075)^{48}}}{0.0075}\right]} = \$472.82$$

$$\text{Option B}: \quad PMT = \frac{\$20{,}000}{\left[\frac{1-\frac{1}{(1.002)^{48}}}{0.002}\right]} = \$437.40$$

The customer should choose Option B with the lower monthly payment of $437.40 versus $472.82 over the 48 month period.

17B. Solve for the PV amount under Option A that would give a monthly payment of $437.40:

$$\text{Option A: } \quad \text{PMT} = \frac{\$20{,}000 - \text{Rebate}}{\left[\dfrac{1 - \dfrac{1}{(1.0075)^{48}}}{0.0075}\right]} = \$437.40$$

Discounting $437.40 for 48 periods at 0.75% a month gives a present value of $17,576.82. Thus, $20,000.00 - $17,476.82 = $2,423.18, which is the amount of the rebate.