Understanding Financial Management: A Practical Guide
Guideline Answers to the Concept Check Questions

Chapter 4
The Time Value of Money

Concept Check 4.1

1. **What is the meaning of the terms *risk-return tradeoff* and *time value of money*?**

    Risk-averse investors in financial markets require higher rates of return for investing in risky securities to compensate them for their risk. Investors who want to avoid risk can invest in US Treasury bills in which the US government fully guarantees future payment. Since they are essentially risk free, US Treasury bills offer low rates of return. The securities issued by corporations such as stocks and bonds contain risk because investors do not know with certainty the future return they will earn. Investors require higher returns on these securities to compensate for the additional risk, establishing a *risk-return tradeoff* in the financial markets.

    A manager would not consider a $20 million net cash inflow that she expects to receive 20 years from now to have the same value as a $20 million cash inflow today. If the manager received the $20 million today, she could invest it at a positive rate of interest and have a much larger amount in 20 years. When making financial decisions that involve cash flows at various times in the future, financial managers must take into account *the time value of money*. Thus, *time value of money* refers to the process of quantifying the relation between cash flows at different points in time.

2. **Give several examples when the financial manager would use the time value of money.**

    When their firm issues bonds to investors, financial managers should compare the value of future cash payments owed to the bondholders in exchange for the cash received today. Similarly, when making capital investments, financial managers need to compare the value of expected cash flows in the future to the present cash outlay needed to undertake the investment.

Concept Check 4.2

1. **What is meant by the term *compounding of interest*?**

    When compounding interest, the interest earned and paid in future periods depends not only on the initial present value amount, but also on any previous interest earned (that

has not been withdrawn). Thus, with compounding, interest earns interest, and the future value amount increases geometrically over time.

2.   **How do the length of the compounding term and the interest rate affect future values?**

Future values increase as the level of the interest rate and length of the compounding term increase. For example, a firm will earn more on a $1 million invested over a 5-year period at 8 percent than at 6 percent. Similarly, holding the interest rate constant at 7 percent, the firm will receive a higher future value by compounding a $1 million over 20 years instead of 5 years.

Concept Check 4.3

1.   **What steps are needed to solve for the present value of a future amount?**

The first step involved in solving for the present value of a future amount is to identify the future value amount (FV), the interest rate per period (r), and the length of the discounting period (n). The second step is to substitute these amounts into the present value of a future amount formula to solve for the present value (PV):

$$PV = FV\left[\frac{1}{(1+r)^n}\right].$$

2.   **What is meant by the terms *discounting* and *discount rate*?**

Discounting refers to the process of computing the present value of a future amount. The *discount rate* is the interest rate used to compute the present value amount. Other terms for the discount rate include the opportunity cost, required rate of return, hurdle rate, and cost of capital.

3.   **How do the length of the compounding term and the interest rate affect present values?**

An inverse relationship exists between present values and interest rates. To illustrate, suppose a firm wants to set aside enough money today (a present value amount) in order to have $20 million to expand its production facilities in three years. By earning a higher interest rate on these invested funds, the firm could set aside a lower amount today to achieve its desired goal of $20 million in three years. An inverse relationship also exists between present values and the compounding term. In the previous example, if the firm had five years, instead of three years, to reach its investment goal, the firm could set aside a smaller present value amount today.

1.   **What steps are involved in solving for the future value of an annuity?**

     The first step in solving for the future value of an annuity is to identify the annuity amount (PMT), the interest rate per period (r), and the number of payments (n) in the annuity. The second step is to substitute these amounts into the future value of an annuity formula to solve for the future value (FV):

$$FV = PMT\left[\frac{(1+r)^n - 1}{r}\right].$$

2.   **When using the future value of an annuity formula (Equation 4.3), at what point in time is the future value computed?**

     When using the future value of an annuity formula, the future value is obtained at the time of the last annuity payment.

Concept Check 4.5

1.   **What are the steps involved in solving for the present value of an annuity?**

     The first step in solving for the present value of an annuity is to identify the following: the annuity amount (PMT), the interest rate per period (r), and the number of payments (n) in the annuity. The second step is to substitute these amounts into the present value of an annuity formula to solve for the present value (PV):

$$PV = PMT\left[\frac{1 - \dfrac{1}{(1+r)^n}}{r}\right].$$

2.   **What is the difference between an *ordinary annuity* and an *annuity due*?**

     With an *ordinary annuity*, cash flows occur at the end of each period. With an *annuity due*, cash flows occur at the beginning of each period. The present value formulas differ slightly for these two types of annuity.

Concept Check 4.6

1. **What is a perpetuity?**

   A *perpetuity* is an annuity with an infinite life. Preferred stock is an example of a perpetuity.

2. **What are the steps involved in solving for the present value of a perpetuity?**

   Solving for the present value of a perpetuity involves dividing the perpetuity payment (PMT) by the discount rate. There are no additional steps.

Concept Check 4.7

1. **What adjustments need to be made to the basic time value of money equations when interest is compounded semi-annually, quarterly, monthly, or daily?**

   Compounding more frequently than once a year requires adjusting the interest rate and number of periods to reflect the compounding basis. For example, suppose an investor earns interest at a rate of 10 percent per year, compounded semiannually over a 7-year period. The adjustments to the time value of money formulas involve halving the interest rate (10%/2 = 5% per period) and doubling the number of periods (2 x 7 = 14 periods).

2. **What is continuous compounding? What steps are involved in computing the future value of a present amount when interest is continuously compounded?**

   With continuous compounding, interest is compounded every infinitesimal part of a second. The first step is to identify the present value amount (PV), the interest rate per year (r), and the number of years (n). The second step is to substitute these amounts into the equation $FV = PVe^{rn}$. In this equation, e is the base of natural logarithm, approximately equal to 2.71828.

Concept Check 4.8

1. **How do you compute the effective annual interest rate, given a nominal rate?**

   The effective annual interest rate is computed by solving the equation:

   $$r_{eff} = \left[1 + \frac{r_{nom}}{m}\right]^m - 1.$$

In this equation, r_{eff} is the effective annual interest rate; r_{nom} is the nominal annual interest rate; and m is the number of compounding periods per year.

2. **Will the difference between the effective annual interest rate and the nominal rate increase or decrease as the compounding frequency (m) increases? Why?**

The difference between the effective annual interest rate and the nominal rate will increase as the compounding frequency increases. For any given interest rate, the greater the compounding frequency, the higher is the future value after one year for a given present value amount. The greater the future value after one year, the higher is the effective annual rate that would be needed to convert the present value amount into that future value.