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Social Insurance and the Public Budget

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Restraints on the public budget may limit the ability of the public sector to use financial markets for the diversification of shocks. This interferes with the role of the public budget as a buffer which may provide insurance by stabilizing income and thereby private consumption. We consider this insurance or stabilizing role of public budgets and show why pro-cyclical budgets and a progressive taxation system may be optimal even when tax distortions are taken into account. Balanced budget restrictions interfere with this insurance effect, and they do not necessarily imply that a lower level of public consumption is optimal.

INTRODUCTION

What is the role of public budget deficits? In a world with Ricardian equivalence, the answer is simple. Despite the voluminous theoretical and empirical literature addressing whether Ricardian equivalence holds (Seater 1993), there is a surprising scant literature dealing with the role of the public budget position when this equivalence result does not hold. This is particularly puzzling given that most observers would agree that Ricardian equivalence does not hold in practice.

Substantial interest has, of course, been devoted to analysis of the consequences of budget deficits in the absence of Ricardian equivalence, addressing the effects on interest rates, exchange rates and so on.[1] But this still leaves open the basic question of why there is a case for not balancing the budget. One interpretation is that a tendency towards systematic budget deficits arises when a bias in the political system causes a failure to finance all current expenditures, and budgetary policies boil down to a question of political conflicts over distributional issues. In an overview of the role of public deficits, Ball and Mankiw (1995) take this view by stating

> Thus, the winners from budget deficits are current taxpayers and future owners of capital, while the losers are future taxpayers and future workers. Because these gains and losses balance, a policy of running budget deficits cannot be judged by appealing to the Pareto criterion or other notions of economic efficiency. (Ball and Mankiw 1995, p. 108)

If budget deficits play a role only in relation to political conflicts over distribution, a straightforward solution would be to impose a balanced budget norm.[2] However, this may overlook the fact that budget deficits may improve efficiency in allocations precisely under the circumstances where Ricardian equivalence does not hold.

The primary budget position depends on the timing of taxation and expenditures. The insight of the 'tax-smoothing' principle (Barro 1979) is that minimization of the distortionary costs of income taxation (the dynamic Ramsey problem) calls for a constant tax rate. Accordingly, temporary increases in public expenditures or decreases in tax revenue would optimally be accommodated by running a public deficit. Barro (1979) developed this result for an income tax in a partial model with exogenous production, and it has later been cast in a general equilibrium setting by, e.g. Chamley (1985), and Lucas and Stokey (1983).

The timing of taxes should also take into account the possible ways in which taxes and deficits interfere with market failures (the dynamic Pigou problem). One potentially very important role here is the fact that the public budget may serve as a buffer to shocks impinging on the economy. Thereby, the public budget may stabilize, e.g. income and private consumption, providing an insurance or stabilization function to the economy. This idea can be traced back to Keynes and has played an implicit role in many macroeconomic analyses. Although modern macro models are cast in an explicit intertemporal setting with a modelling of economic decision-making and imperfections, there has surprisingly not been much work on the role of budget balances. The aim of this paper is to address the role of the public budget in a setting with capital market imperfections and where it accordingly may enhance economic efficiency by providing social insurance. By 'social insurance' we mean in broad terms the various ways in which public-sector activities mitigate the consequences of risk for individuals and society.[3]

For public budget positions, a key question is the ability of capital markets to diversify risk over time. This is so since idiosyncratic risks can be diversified even under a balanced budget, and since it is well known that the budget position is without real importance in the presence of complete capital markets. With increasing international integration of capital markets, the relevant question is the possibilities for risk diversification via international capital markets. By running deficits or surpluses, the government may use these markets to attain social insurance in the presence of aggregate shocks. A balanced budget restraint is effectively a constraint on the ability of the public sector to use capital markets. This may mean nothing if capital markets are complete or the public sector is unable to use capital markets better than the private sector. However, ample evidence indicates that capital markets are not complete[4] and that private agents are not able to exploit capital markets fully.[5] Under such circumstances, restraints on public budgets may have severe consequences.

We explore this issue in a small open economy with fluctuations driven by aggregate (productivity) shocks.[6] The focus is accordingly on the interplay between income shocks and income taxation. The optimal design of the income taxation system to finance a given level of public expenditures[7] is considered by taking account of both the insurance effects and the distortions caused by taxation.[8] This is compared with a situation in which there is a balanced budget regime. This makes it possible to evaluate both the welfare consequences of budget restraints and their implications for macroeconomic stability. We also analyse how the public budget rules affect the optimal level of public consumption.

The analysis makes use of a model for a small open economy with overlapping generations.[9] This is a convenient way to formulate a fully specified intertemporal general equilibrium model in which there is a capital market imperfection creating a role for social insurance. By the very nature of this setup, there is a market imperfection in the sense that private markets cannot fully diversify all risks—one fundamental reason being that this would require diversification across different generations, and there is no means by which current generations can extract resources from yet unborn generations and no mechanism by which the latter can ensure that resources are transferred to them (the problem of insurance at zero age). However, the government may be able to do so, and we analyse how this works in a small open economy with liberalized international capital markets.

Possibilities for diversification of aggregate risk in an open economy context have been analysed by Aizenman (1981). The idea is that the balance of payments is a shock absorber, and changes in the stock of international reserves can be used to diversify aggregate shocks and to smooth consumption so as to increase welfare. There is no capital market, and the scope for diversification is determined by the size of the stock of reserves. Gordon and Varian (1988) show how the government can implement a transfer (tax) scheme between different generations that are alive at a given period so as to improve risk allocation between generations and thereby improve welfare. In both cases, the capital market and the public budget play no role. Moreover, production is exogenous, and the issue of tax distortions is not addressed.

The paper is organized as follows. Section I sets up a small open overlapping-generations economy with liberalized capital movements. Section II develops the basic insurance implications of a balanced budget regime and a regime allowing for budget imbalances by considering the case of exogenous production, while Section III introduces tax distortions by endogenizing production. Finally, Section IV offers some concluding comments.

I. A Small Open Overlapping-Generations Economy

Consider a small open economy producing a commodity that is a perfect substitute for internationally traded goods being traded at a price P (in domestic currency) at the world market. The exchange rate is fixed, and there are no restrictions on access to international capital markets, implying that the rate of interest equals the world market interest rate.

Households

The population is constant, and individuals live for two periods. The generation born in period t consume as young (c_{1t}) in period t and as old (c_{2t+1}) in period $t+1$, and they work only as young (l_{1t}) in period t. Moreover, they obtain utility from access to a public good available in the amount g.[10] Lifetime utility for the representative household is given by a separable utility function, where the function U captures utility from work, v utility from leisure, and s the utility derived from public goods:[11]

$$U(c_{1t}, c_{2t+1}) - v(l_{1t}) + s(g);$$

$$u'_{c_j} > 0, \qquad u''_{c_j} < 0 \ (j = 1, 2), \qquad v' < 0, \qquad v'' > 0, \qquad s' > 0, \qquad s'' > 0$$

The consumer problem can conveniently be analysed in two steps: first, by considering the consumption decision given income, and second, by considering the labour supply decision to generate income. Households inherit ownership of firms and are entitled to profit income generated by firms.

For a given disposable income level I, the consumption problem is to maximize the utility of consumption subject to the budget constraint

$$c_{1t} + (1 + r_t)^{-1} c_{2t+1} = \frac{I_t}{P_t} = i_t,$$

where r_t denotes the real rate of interest and i_t, real income.

The consumption while young and old can now be stated:

$$c_{1t} = c_1(r_t, i_t),$$

$$c_{2t} = c_2(r_t, i_t).$$

The real rate of interest is exogenous owing to the small open economy assumption, and since the focus here is on income variability, we simplify and assume the real rate of interest to be constant. The utility of consumption following from the optimal consumption decision can now be summarized by the indirect utility function

$$U(i_t); \qquad U' > 0, \qquad U'' < 0,$$

where the real disposable income is given by[12]

$$i_t = (1 - \tau_t)(w_t l_t + \pi_t)$$

and w_t is the real wage rate, π_t real profits, and τ_t the tax rate applying to income.

Given the indirect utility function derived above, the labour supply decision is easily found as the solution to the following problem:

$$\max_{l_t} \; U[(1 - \tau_t)(w_t l_t + \pi_t)] - v(l_{1t}).$$

The labour supply decision is characterized by the following first-order condition:

$$(1) \qquad (1 - \tau_t) w_t U'(i_t) = v'(l_{1t}).$$

Firms

All firms are price and wage-takers and produce subject to a production function

$$y_t = a_t f(l_t); \qquad f' > 0, f'' < 0,$$

where l_t is labour input, and a_t is an indicator for productivity. The labour demand decision of the firms is characterized by the first-order condition

$$(2) \qquad a_t f'(l_t) = w_t.$$

Note that the production decision is taken under full certainty; i.e. the current shock is fully known. This also implies—under the assumed capital market structure—that it is inconsequential whether profits are distributed in period t or in $t+1$ as long as there is perfect information.

Shocks

Since the issue is social insurance, we want to rule out transfer/redistribution across time periods (generations) that is motivated by changes in the perception of the level of permanent income for the economy.[13] It is therefore convenient to specify a process for the productivity variable a_t such that it does not induce shifts in the perceived permanent income. This requires that the expected present value of the shock is time-invariant, i.e. that

$$E_t \sum_{j=0}^{\infty} (1+r)^{-j} a_{t+j} = \text{constant } \forall t$$

This condition is fulfilled by the following process:

$$a_t - \bar{a} = -(1+r)(a_{t-1} - \bar{a}) + \upsilon_t, \tag{3}$$

where $\bar{a}$ is the permanent level of a, and υ_t is i.i.d. having a symmetric density function $f(\upsilon)$ with support on $[\underline{\upsilon}, \bar{\upsilon}]$. This specification implies that there will be good and bad states, but it is *ex ante* uncertain which generation will be lucky and which ones will be unlucky. The assumption that the interest rate is constant effectively means that the shock considered is a country-specific shock. Allowing for correlated shocks across countries would not change anything qualitatively as long as the shocks are not perfectly correlated, since there would still be diversification possibilities across countries. The same applies if the interest rate is increasing in the amount borrowed. This would reduce but not eliminate the possibilities for diversification of shocks over time via international capital markets.

Note that, for a more general process for the shock variable, the constraint imposed on (3) can be used to define the transfers across generations that can be justified on pure insurance grounds.[14]

Government

The government supplies a public good g which is financed by an income tax. The real value of the primary public budget b_t in period t is

$$b_t = \tau_t y_t - g_t$$

The public sector has—like the private sector—access to the international capital market, and the real debt level d_t develops according to

$$d_t = -b_t + (1+r)d_{t-1}. \tag{4}$$

The initial debt level is assumed to be zero; i.e. $d_{t-1} = 0$.

We shall consider different budgetary regimes for the public sector. One regime has a continuously balanced budget; i.e.

$$b_t = 0 \qquad \forall t, \tag{5}$$

implying that the intertemporal solvency condition is automatically fulfilled. The other regime allows for budget imbalances within the constraint set by the intertemporal budget constraint which we operationalize by imposing that the expected budget balance is zero:[15]

$$E_{t-1}b_t = 0 \qquad \forall t,$$

which is sufficient to ensure that the expected level of debt is bounded, i.e.

$$E_t d_{t+j} < \overline{d} \qquad \forall t, j > 0$$

Note that the actual debt development is still determined by (4), but the regime is always expected to be feasible since public debt is bounded. This regime corresponds to the argument often made in policy debates that the budget should be balanced over the business cycle. We consider both how these financing schemes operate to finance a given level of public expenditures, and how they affect the optimal level of public consumption.

Equilibrium conditions

The labour market is competitive, and the equilibrium condition reads

$$l_t^d = l_t^s.$$

As the good produced is traded internationally, there is no product market equilibrium condition. The trade balance tb_t in period t reads

$$tb_t = y_t - c_t - g,$$

where c_t is total private consumption in period t, i.e. the sum of consumption by young and old given by

$$c_t = c_{1t} + c_{2t}.$$

II. Exogenous Production

To clarify the mechanisms through which the budget can provide social insurance, it is useful to start by considering the case with exogenous production. Labour is thus assumed to be supplied inelastically ($l = 1, v(l)$ = constant) and production is normalized such that ($y = af(1) = a$; $f(1) = 1$).

Consider first the problem of how a given level of public consumption should be financed. If the budget is required to be balanced period by period, it follows straightforwardly that the tax rate has to be

$$\tau(a_t) = \frac{g}{a_t}, \qquad \tau' = -\frac{g}{a_t^2} < 0; \tag{6}$$

that is, the tax rate moves countercyclically. In periods with high production, the given level of public consumption can be financed by a low tax rate and vice versa in periods with low production. The expected utility[16] to a member of any generation can be written

$$EU(a_t - g).$$

With a balanced budget, it follows that the public sector does not use the international capital market. Clearly, this may imply a welfare loss, as such markets offer a possibility of smoothing the tax burden and thereby allowing a diversification of shocks to aggregate production. One possibility for achieving this would be to choose a constant tax rate, avoiding the variation of taxes with the state of nature, i.e. setting the tax rate equal to

$$\tau = \frac{g}{\bar{a}}.$$

In this case expected utility of a period t generation becomes

$$EU\left(a_t - \frac{g}{\bar{a}}a_t\right).$$

Clearly, all generations are better off in terms of expected utility under a system with a constant tax rate compared with the balanced budget system, since the expected after-tax income is the same, i.e.

$$E\left(a_t - \frac{g}{\bar{a}}a_t\right) = \bar{a} - g,$$

but its variance is lower in the constant tax-rate regime, i.e.

$$\mathrm{Var}\left(a_t - \frac{g}{\bar{a}}a_t\right) < \mathrm{Var}(a_t - g).$$

It is easy to see why this policy can reduce risk. The budget is given by

$$b_t = \left(\frac{a_t}{\bar{a}} - 1\right)g.$$

In bad states there is a budget deficit, and good states a budget surplus. The public sector uses the international capital market to smooth the tax burden by letting tax payments be low when income is low and vice versa. Notice that this is not attainable by the private sector, because the shock is an aggregate and thus non-diversifiable shock within a given generation, and there are limited possibilities for private households to diversify such risk in the international capital market owing to their fixed lifetime.[17]

It is easily checked that the constant tax policy is feasible, since

$$E_t d_{t+1} = -E_t[(1+r)b_t + b_{t+1}] = -E_t\left(\frac{g}{\bar{a}}\upsilon_{t+1}\right) = 0.$$

Although holding a constant tax rate does give some insurance, it is not necessarily the optimal tax policy in the sense of being the best way of financing the given level of public expenditures so as to maximize expected utility across generations. To see this, there exists a tax policy that will remove all risk and thereby ensure a constant consumption level for all generations. This can be accomplished by the following tax function

$$(7) \qquad \tau(a_t) = \frac{g}{a_t} + \left(1 - \frac{\bar{a}}{a_t}\right); \qquad \tau' = \frac{\bar{a} - g}{a_t^2} > 0.$$

Comparing (7) with the balanced budget tax rate in (6), we find that the first term is the same, but that (7) includes an additional term capturing the insurance effect attainable by letting the tax rate depend on the difference between the actual and the mean value of the shock variable.

It is easily seen that the tax rate (7) implies that after-tax income becomes deterministic, i.e.

$$a_t(1 - \tau(a_t)) = \bar{a} - g,$$

and therefore the risk is completely absorbed by the public budget, leaving no variability in private consumption. It is easily verified that this policy is consistent with the budget constraint. Notice that the optimal policy (7) implies that the tax rate becomes procyclical—the tax rate is high when income is high and vice versa. This provides an argument for a progressive taxation system, since it automatically implies that tax rates move procyclically to aggregate shocks. Progressive taxation is a way of increasing the sensitivity of the public budget to the business cycle situation (moves procyclically) and thereby providing social insurance. It is worth stressing that it is an implication of the optimal tax policy given in (7) that, even if lump-sum taxation is feasible, it is not optimal to use this form of taxation since it is unconditional and therefore achieves no diversification.

Optimal public consumption

Having considered the optimal tax policy to finance a given level of public consumption (cf. (7)) for the case where budget imbalances are allowed, it is natural to question the extent to which the budget policy affects the optimal level of public consumption. Budget norms are often seen as instrumental to the objective of reducing the relative size of government. We consider public consumption to be of a type that cannot easily be changed (schools, infrastructure, etc.), and it is thus most plausible to consider the *ex ante* choice of public consumption before the state of nature is known. The optimal level of public consumption for a utilitarian government is found by maximizing the expected utility of households including the value of public goods. In the case of a balanced budget regime (indexed by B, for balanced budgets), the optimal level of public consumption is determined by the Samuelson condition

$$EU'(a_t - g_B) = s'(g_B),$$

while under the optimal tax rule (7) (indexed by N, for non-balanced budget) in the absence of a binding budget balance rule, it reads

$$EU'(a_t - g_N) = s'(g_N).$$

It follows that[18]

$$g_N \gtrless g_B \qquad \text{for } U''' \gtrless 0.$$

This shows that institutional budget rules, in general, influence the optimal level of public consumption even when the level of public consumption is decided before the veil of ignorance is lifted. However, the direction in which the rules affect the optimal level of public consumption is in general ambiguous. This implies that the often made conjecture that a balanced budget rule will reduce the level of public consumption is not generally supported.

III. Endogenous Production

Having clarified the basic insurance function that the public budget can play, the next step is to make production endogenous to allow for the double role played by taxes, namely, both to provide insurance, and to affect incentives.[19]

It is useful to start by considering in more detail how activity and utility depend on the state of nature for a given tax rate. Next, we consider the different budget regimes. Equilibrium employment can be written as a function of the variable $\hat{a} \equiv a(1 - \tau(a))$, which might be termed the after-tax value of the state of nature variable a, i.e. see Appendix (a).

$$(8) \qquad l = e(\hat{a}); \qquad \hat{a} \equiv a(1 - \tau(a)$$

and

$$\text{sign } e'(\hat{a}) = \text{sign}(1 - R_U); \qquad R_U \equiv -\frac{U''(i)i}{U'(i)} > 0.$$

To simplify the exposition, the time index is suppressed. Note that R_U is the measure of relative risk aversion for the indirect utility function U. Note also that an upward-sloping labour supply function is not sufficient to imply that the employment level is increasing in $\hat{a}$, because an increase in $\hat{a}$ also has an income effect via profit income.

Using (8), we can summarize the utility of consumption and the disutility of labour in equilibrium by an indirect utility function depending on $\hat{a}$ (see Appendix (b)), i.e.

$$V(\hat{a}) = \arg\max_{l} \; U(i) - v(l),$$

where

$$V' = U'f > 0.$$

One important finding is that, although the underlying direct utility function is characterized by risk aversion, this does not generally apply to the indirect utility, as

$$V'' \gtreqless 0 \qquad \textit{f}\text{or } R_U \gtreqless R_U^* \equiv \frac{\hat{a}f'e'}{f + \hat{a}f'e'}.$$

The reason is that the marginal utility of a change in $\hat{a}$ is given as the product of the marginal utility of consumption (U') and the production level (f). Hence, even if an increase in $\hat{a}$ increases consumption and thus lowers the marginal utility of consumption ($U'' < 0$), the effect on consumption may be counteracted by an increase in employment ($f'e'$).

Balanced budget

Consider first the case of a balanced budget regime where the tax rate is determined from the budget condition

$$\tau(a)af(e(\hat{a})) = g, \tag{9}$$

implying that

$$\tau' = -\frac{\tau f + \tau a f' e'(1-\tau)}{af - \tau a^2 f' e'}.$$

The tax rate thus may move pro- or countercyclically. A countercyclical tax rate implies that the effects of variations in a on $\hat{a}$ are amplified and vice versa if the tax moves procyclically. Note that the nominator gives the revenue effect of an increase in a for a given tax rate, and it is unambiguously positive if employment is increasing in a. The denominator gives the revenue effect of a change in the tax rate, and it is positive provided that we are on the 'right' side of the Laffer curve. Hence, a countercyclical tax rate is likely to be implied by a balanced budget norm.

Optimal taxation

It is easily shown that there exists a tax policy consistent with solvency (9) which dominates the balanced budget case even when production is endogenous.[20] The interesting question is how the interaction between the insurance motive and the distortions from taxation determines the optimal tax policy. We therefore start by considering the optimal taxation scheme to finance a given level of public expenditures.

The optimal tax policy solves the following problem:[21]

$$\max_{\{\tau(a)\}} EV(a - a\tau(a)) \tag{10}$$

subject to

$$EaR(a, \tau(a)) = g.$$

The revenue constraint is formulated in terms of the function

$$R(a, \tau(a)) \equiv \tau(a) f(e(\hat{a}));$$

that is, $aR(a, \tau(a))$ denotes the revenue attained in state a for a tax rate $\tau(a)$.

The first-order condition to the problem given in (10) can be written

$$\frac{V'(a - a\tau(a))}{R'_\tau(a, \tau(a))} = \lambda, \tag{11}$$

where λ is the Lagrange multiplier associated with the revenue constraint and

$$R'_\tau(a, \tau(a)) = f(e(\hat{a}))\left(1 - \varepsilon_{y\hat{a}}\frac{\tau}{1-\tau}\right),$$

where $\varepsilon_{y\hat{a}}$ is the elasticity of production with respect to $\hat{a}$. Note that

$$R'_\tau(a, \tau(a)) \neq f(e(\hat{a})) \qquad \text{for } e' \neq 0,$$

which reflects the presence of a tax distortion. For the latter reference, note that the marginal revenue effect of variations in the tax rate is state-dependent, since

$$R''_\tau(a, \tau(a)) = f'e'(1-\tau)\left(1 - \varepsilon_{y\hat{a}}\frac{\tau}{1-\tau}\right) - f\frac{\tau}{1-\tau}\frac{\partial \varepsilon_{y\hat{a}}}{\partial a} - f\varepsilon_{y\hat{a}}(1-\tau).$$

For two different states of nature, $a_1, a_2 (a_1 \neq a_2)$, we find from (11) that the optimal tax policy implies

$$\frac{V'(a_1 - a_1\tau(a_1))}{R'_\tau(a_1, \tau(a_1))} = \frac{V'(a_2 - a_2\tau(a_2))}{R'_\tau(a_2, \tau(a_2))}. \tag{12}$$

Condition (12) says that the optimal tax structure ensures that the marginal utility of private consumption relative to the 'marginal tax revenue' must be equal across states of nature. This has a number of important implications.

First, it is straightforward to show that full insurance, in the sense of fully eliminating the consequences of fluctuations in a on $\hat{a}$, is not optimal. This requires that

$$a_1 - a_1\tau(a_1) = a_2 - a_2\tau(a_2) \qquad \forall a_1, a_2 (a_1 \neq a_2).$$

For (12) to hold under this constraint we require that

$$R'_\tau(a_1, \tau(a_1)) = R'_\tau(a_2, \tau(a_2)),$$

a condition that is not generally fulfilled.[22] Notice that in the case where $f'e' = 0$, i.e. where there are no tax distortions, it follows that $R'_\tau(a, \tau(a)) = f(e(\hat{a}))$ and hence full insurance of variations in $\hat{a}$ is optimal.[23] Hence, with tax distortions, it is inoptimal via the public budget to provide full insurance, although it is a feasible option.

Second, it can be shown (see Appendix (c)) that the state dependency in the optimal tax rate is determined as

$$\text{sign } \tau' = -\text{sign}\left(\frac{V''}{V'}(1-\tau) - \frac{R''_{\tau a}}{R'_\tau}\right). \tag{13}$$

The first term reflects the insurance motive, which calls for the optimal tax rate to move procyclically if agents are risk-averse with respect to fluctuations in $\hat{a}$. The second term captures how the tax distortion varies with the state of nature, and this effect is in general ambiguous in sign. In general, the optimal sensitivity of taxes to the state of nature depends on both the insurance effect and the distortions. It is interesting that accounting for tax distortions does not necessarily weaken the case for procyclical tax rates. If the tax distortions are smaller in good states of nature, i.e. if $R''_{\tau a} \geqslant 0$, this reinforces the argument for letting tax rates be procyclical. This condition implies that the tax distortion is lower in a good state of nature (high a) than in a bad state of nature (low a). Intuitively, this case arises if the elasticity of labour supply is increasing in the tax rate (an assumption taken to hold).

According to the 'tax-smoothing' principle, the optimal policy is a constant tax rate (Barro 1979).[24] This result takes into account only tax distortions, not insurance effects. By also including the insurance effects of taxation, we find that a constant tax rate is not in general optimal, although relative to the balanced budget case it does achieve some insurance. Moreover, it follows directly from (13) that, even in the case where agents are risk-neutral, the optimal tax policy is not a constant tax rate unless tax distortions are independent of the state of nature. Under plausible assumptions, the optimal tax rate is procyclical (progressive) even when agents are risk-neutral.

Finally, it should be pointed out that, even by allowing for lump-sum taxation, it is not optimal to fully finance public expenditures by this non-distortionary taxation. This shows that the insurance effect at the margin is strong enough to outweigh the distortions of income taxation.

Macroeconomic stability

The preceding has taken a welfare approach to the analysis of social insurance via the public budget. However, the role of procyclical budgets and tax rates has been extensively studied in the macro literature, and it is therefore of interest to consider the implications for key macrovariables such as production and consumption.

The financing regime for public expenditures has implications for macro economic volatility. For output, we find an elasticity with regard to the state of nature variable given as

$$\varepsilon_{ya} = 1 + \eta_y \varepsilon_{\hat{a},a},$$

where $\varepsilon_{yx} \equiv (\partial y/\partial x)(x/y)$ and $\eta_y = f'e'(\hat{a}/f) > 0$. It follows that output is more sensitive to the state of nature under a balanced budget rule (indexed by B) than under a non-balanced budget regime with an optimal tax structure (indexed by N):[25]

$$\varepsilon_{ya}\,|_B > \varepsilon_{ya}\,|_N$$

if $e' > 0$ (output and employment is increasing in the state of nature variable a) and optimal taxes are procyclical ($\tau' > 0, \varepsilon_{\tau a}$). This is consistent with the empirical findings of Gali (1994). As should be expected, this also lowers the sensitivity of consumption; i.e.

$$\varepsilon_{ca}\,|_B > \varepsilon_{ca}\,|_N.$$

It is also easily verified that both private and public net savings are increasing in the state variable $\hat{a}$ in this case. This implies that the trade balance moves procyclically, which is also in accordance with stylized empirical facts (see e.g. Backus and Kehoe 1992).

Optimal public consumption

Finally, we consider the optimal level of public consumption under a balanced budget (g_B) rule and under a non-balanced budget with an optimal tax policy (g_N). As for the case with exogenous production, we find that the result is ambiguous (see Appendix (d)); i.e.

$$g_N \gtreqless g_B.$$

It may be surprising that public consumption is not generally larger in the non-balanced budget regime as the budget balance restriction is lifted. One reason for this is that providing insurance may increase the expected marginal value of private consumption, and thereby reduce the marginal costs of public consumption.

IV. Concluding Remarks

The need to impose balanced budget norms often surfaces in the policy debate. In the United States there is a continuous debate on the Gramm–Rudman–Hollings amendment, and in Europe on the budget norms associated with the Economic and Monetary Union. This paper shows that restrictions on public deficits imply limitations on the possibilities for the public sector to use international capital markets for intertemporal substitution, which in turn conflicts with the insurance or stabilizing effects of 'automatic stabilizers' built into public budgets.[26]

Solving for the optimal tax policy, we find that under plausible assumptions it implies that both the tax rate (progressive taxation) and the primary public budget move procyclically; moreover, this also produces macroeconomic stability.

The present analysis has relied on separability in the utility function, but clearly the basic point on the insurance mechanisms running via the public budget does not depend on this assumption (which serves the purpose of deriving the basic results in a more transparent way). Crucial to the insurance argument is the presence of a capital market imperfection implying that the public sector has diversification possibilities for aggregate shocks that are not fully available to the private sector. While this possibility easily arises in an overlapping-generations' economy with an inoperative bequest motive, we think this is an illustrative way of modelling an aspect that is more a general than an intergenerational diversification of shocks, and which points to effects arising once capital markets are not complete.

The present analysis has not dealt with the political decision process as something that may influence debt policy and lead to a deficit bias (see e.g. Alesina and Perotti 1995). The present argument that there are welfare gains from allowing the public budget to be in imbalance suggests that there is a traditional rule *vs.* discretion problem in deciding whether budget restraints should be imposed (see e.g. Corsetti and Roubini 1997).

APPENDIX

(a) Equilibrium employment

Using the conditions determining labour supply and demand (1), (2), we can determine equilibrium employment from the relation

$$(1-\tau)af'(l)U'[(1-\tau)af(l)] - v'(l) = 0.$$

This gives equilibrium employment as an implicit function of $\hat{a} \equiv a(1-\tau(a))$; i.e.

(A1) $l = e(\hat{a})$.

Differentiation of (A1) yields

$$e' = \frac{(1/\hat{a})(R_U - 1)}{f''/f' - (1/l)(R_U\gamma - R_v)},$$

where

$$\gamma \equiv \frac{wl}{wl+\pi}; \qquad R_U = -\frac{U''i}{U'}; \qquad R_v \equiv -\frac{v''l}{v'}.$$

From the second-order condition to the household optimization problem, we have

$$-\frac{1}{l}(R_U\gamma - R_v) < 0.$$

Hence, given that $f'' < 0$, it follows that

$$\text{sign } e' = \text{sign}(1 - R_U)$$

(b) The indirect utility function: V*(*â*)*

Since $i \equiv (1-\tau)(wl+\pi) = (1-\tau)af(l)$ and $l = e(\hat{a})$, we can write the sum of utility of consumption and disutility of labour in equilibrium as

$$V(\hat{a}) \equiv U(\hat{a}f(e(\hat{a})) - v(e(\hat{a})).$$

We find by use of the first-order condition that

$$V' = U'f > 0$$

and

$$V'' = U''f^2 + f'e'U'(1 - R_U).$$

We have that

$$V'' \gtreqless 0 \qquad \text{for } R_U \lesseqgtr \frac{\hat{a}f'e'}{f + \hat{a}f'e'} \equiv R_U^* < 1.$$

(c) Progression of the optimal tax system with endogenous production

The first-order condition characterizing the optimal tax system reads

$$\frac{V'(a - a\tau(a))}{R'_\tau(a, \tau(a))} = \lambda,$$

where λ is the Lagrange multiplier associated with the budget constraint.

We find by differentiation with regard to a that

$$\frac{V''}{V'}(1-\tau-a\tau')=\frac{1}{R'_\tau}(R''_{\tau a}+R''_{\tau\tau}\tau'),$$

implying that

$$\tau'=\frac{(V''/V')(1-\tau)-R''_{\tau a}/R'_\tau}{R''_{\tau\tau}/R'_\tau+(V''/V')a}. \tag{A2}$$

Using the second-order condition to the optimization problem (10), we see that the denominator is negative. Hence, the sign of the state contingency of the optimal tax rate is determined by minus the sign of the nominator of (A2).

(d) The optimal level of public consumption and endogenous production

When solving for the optimal public consumption (and the optimal tax) in the non-balanced budget regime (indexed by N), the problem reads

$$\max_{g,\tau}\ E\{V[a(1-\tau_N(a))]\}+s(g_N)$$

subject to

$$g_N=E\{aR[a,\tau_N(a)]\}.$$

The shadow price of one extra unit of the public good λ_N, measured in terms of utility of the household, can be expressed as

$$\lambda_N=\frac{E\{aV'[a(1-\tau_N(a))]\}}{E\{aR_\tau[a,\tau_N(a)]\}}.$$

For the balanced budget regime (indexed by B), the problem reads

$$\max_{g,\tau}\ E\{V[a(1-\tau_B(a))]\}+s(g_B)$$

subject to

$$g_B=E\{aR[a,\tau_B(a)]\},$$

and the shadow price of public consumption is in this case

$$\lambda_B=E\left(\frac{aV'\{a[1-\tau_B(a)]\}}{\{aR_\tau[a,\tau_B(a)]\}}\right),$$

and in general

$$\lambda_N \gtreqless \lambda_B.$$

We know that the optimal level of public consumption is chosen such that $s'(g_i)=\lambda_i$ for $i=N, B$, and it follows that $g_N \gtreqless g_B$.

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NOTES

1. For a discussion, see e.g. Chang (1990) and Ball and Mankiw (1995).
2. This bias is discussed in a growing political economy literature; see e.g. Alesina and Perotti (1995) and Corsetti and Roubini (1997).
3. Social insurance is thus essential to the welfare society, and its potential effects on welfare arise from failures in private financial or insurance markets; cf. Atkinson (1991). Social insurance is used here rather than stabilization policy, since the former is based on an explicit welfare approach, and the latter traditionally has been based on postulated benefits from the stabilization of various aggregate measures such as output and employment.
4. There is typically a positive interest rate spread between similar types of private and public-sector bonds.
5. An important example of this is the failure for private agents fully to diversify consumption risk via international capital markets; see Lewis (1996, 1999).
6. Considering a real supply shock gives the analysis a non-Keynesian bias, and still the results turn out to support basic Keynesian insights.
7. Barro (1979) considers how distortionary taxes should be smoothed to finance variations in public demand driven by, e.g. wars.
8. Sinn (1995) considers the same issue for idiosyncratic shocks in a static setting.
9. The two-period overlapping-generations model is sometimes criticized as being inappropriate for the study of business cycle issues, since if the period length is interpreted literally it is well beyond the length relevant for business cycles. However, extending the number of periods in the OLG model to obtain a more plausible period length would add to the complexity of the analysis but not change the qualitative properties with respect to diversification of shocks over time.
10. This good may yield utility as young, old or both. This does not matter, as long as the level is exogenous to the agent and there is no uncertainty concerning the supply of the public good.
11. A prime is used to denote the first derivative, and a double prime the second derivative of the function with regard to the variable over which it is defined. If the function includes more than one variable, a subscript is used to denote the variable with regard to which the derivative is taken.
12. Notice that this formulation presumes that the only form of taxation is income taxation. It would also be possible to tax, say, capital income, but this is disregarded to focus on the interplay between income shocks and income taxation.
13. It is well known that changes in permanent income may be a reason for redistribution across agents; see e.g. Fátas (1997).
14. Note that this could be formulated alternatively such that a transitory shock should affect the current generation by a factor $r(1+r)^{-1}$, since this is implied by a smoothing of shocks over an infinite horizon. Consequently, not even transitory shocks should be fully diversified. The alternative formulation adopted here is more convenient, and captures the same qualitative insight.
15. This is a more strict condition than needed to have a sustainable debt level for the public sector; see e.g. Chang (1990).
16. This is the *ex ante* expectations, in the sense that it is unconditional on the history of the economy. It is thus the expected utility of any generation.
17. A direct transfer scheme between generations would attain some diversification; see Gordon and Varian (1988). However, this cannot be decentralized as a market outcome.
18. It is well known from the literature on uncertainty that comparative statics often depend on the third derivative of the utility function; see e.g. Lippman and McCall (1981). Since economic theory does not imply any restrictions on this, an ambiguity remains.
19. Note that employment decisions are taken under full certainty, hence the potential effects of insurance on employment decisions are not addressed; see Sinn (1995).
20. For example, let the tax rate be $\tilde{\tau}(a) = \tau(a) + \chi(a/a - 1)$, where $\tau(a)$ is the tax rate under a balanced budget. It follows that $\partial EV/\partial\chi$, evaluated for $\chi = 0$, is positive for $V'' < 0$ and negative for $V'' > 0$. That is, if agents are risk-averse, there is a potential welfare gain from making the tax rate more procyclical than implied by the balanced budget tax policy.
21. Notice that *ex ante* the expected level of productivity is the same for all generations, as a consequence of the specification of a process for the shock, implying a constant expected permanent income at any point of time.
22. Notice that full insurance is feasible, i.e. a equal to a constant κ is feasible, for a κ fulfilling $(\bar{a} - \kappa)f(e(\kappa)) = g$. A solution exists provided g is not too large.
23. This is consistent with the finding in Section II where production was exogenously given and therefore by assumption income taxation did not have any distortionary effects.
24. In Andersen and Dogonowski (2001) we show that an explicit modelling of tax distortions in a setup with intertemporal substitution in the labour supply does not support a constant tax rate as minimizing tax distortions.

25. See Röell and Sussman (1997) for a case where taxes provide implicit insurance, but where the optimal tax structure is not stabilizing.
26. In a European perspective, the insurance or stabilizing aspects of the public budget may be very important, as there is no federal budget to compensate for the loss of fiscal flexibility in member states if budget norms are implemented strictly in the Economic and Monetary Union.

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