

Appendix SELF-TEST Mathematical background

For each of questions 1-4, which of the mathematical statements is correct?

1.

- A. $\frac{1}{3} + \frac{1}{4} = \frac{1}{12}$
- B. $\frac{1}{3} + \frac{1}{4} = \frac{1}{7}$
- C. $\frac{1}{3} - \frac{1}{4} = \frac{1}{12}$
- D. $\frac{1}{3} + \frac{1}{4} = \frac{5}{12}$

C

2.

- A. $\log(2) + \log(3) = \log(5)$
- B. $\log(8) = 3\log(2)$
- C. $\log(8) = 8\log(1)$
- D. $\log(2) + \log(3) = \log(8)$

B

3.

- A. $2^{10} + 4^{10} = 6^{10}$
- B. $2^{10} + 2^6 = 2^{16}$
- C. $3 \times 2^{10} = 2^{30}$
- D. $8 \times 2^{10} = 2^{13}$

D

4.

- A. $\sum_{n=4}^{n=8} n^2 = \sum_{n=1}^{n=8} n^2 - \sum_{n=3}^{n=8} n^2$

- B. $\sum_{n=4}^{n=8} n^2 = \sum_{n=1}^{n=8} n^2 - \sum_{n=1}^{n=3} n^2$
- C. $\sum_{n=1}^{n=8} n^2 = \frac{n(n+1)(2n+1)}{6}$
- D. $\sum_{n=1}^{n=8} n^2 = 64!$

B

5. In a class of 20 students, the teacher asks for two volunteers. How many different ways can the 2 volunteers be chosen?

- A. 2^{20}
- B. 20×19
- C. $20 \times 19 / 2$
- D. $20!$

C

6. The class of 20 students is having a games lesson. How many different ways can the class be divided into 2 teams of 10?

- A. $20!$
- B. $10! + 10!$
- C. $20! / (10! \times 10!)$
- D. $20! / (2 \times 10! \times 10!)$

D

7. The value of the definite integral $\int_4^5 2x dx$ is

- A. 9
- B. 5
- C. 2
- D. 1

A – this is $5^2 - 4^2$

8. Which of the functions $f(x)$ is a solution of the following differential equation?

$$\frac{df}{dx} = 2f - 1$$

- A. $f(x) = \frac{1}{2}(e^{2x} + 1)$
- B. $f(x) = \frac{1}{2}e^{2x} + 1$
- C. $f(x) = e^2 + 1/2$
- D. $f(x) = e^x + 1/2$

A

9. A solution contains many RNA oligomers of length 6 bases. All 4 bases are present with equal frequency and are present in random orders in the oligomers. The proportion of oligomers containing exactly 2 guanines should be approximately:

- A. 0.2966
- B. 0.0625
- C. 0.0198
- D. 0.000244

A - $\left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^4 \frac{6!}{4! \times 2!} = 0.2966$

10. A DNA sequence of length 100 nucleotides is generated randomly such that each of the 4 bases A, C, G and T is chosen with equal frequency. The base at each position is chosen independently of the others. Let $P(n)$ be the probability that there are n Gs in the sequence. Which of the following statements is correct?

- A. $P(n)$ can be approximated by a Normal distribution with mean 20 and variance 75.
- B. $P(n)$ can be approximated by a Normal distribution with mean 25 and variance 18.75.
- C. $P(n)$ can be approximated by a Poisson distribution with mean 25 and variance 25.

- D. $P(n)$ is a bell-shaped curve with a peak at $n = 50$.

B - The mean is $100 \times \frac{1}{4} = 25$. The variance is $100 \times \frac{1}{4} \times \frac{3}{4} = 18.75$

11. A DNA sequence of length 100 is obtained from a real sequence database. The numbers of the 4 different bases are:

A.- 20; C - 27; G - 34; T - 19.

The 5% significance values in the χ^2 Table are

number of degrees of freedom	5% significance value
1	3.84
2	5.99
3	7.82
4	9.49

- A. The χ^2 test shows that the content of G+C bases differs significantly from 50%
- B. The χ^2 test shows that the content of G+C bases does not differ significantly from 50%.
- C. More information is needed to carry out a χ^2 test.
- D. A χ^2 test is not valid in this case.

A is correct.

The number of G+C = 61 and the number of A+T = 39. The expected numbers are 50.

$$\chi^2 = \frac{(61-50)^2}{50} + \frac{(39-50)^2}{50} = 4.84$$

There is one degree of freedom. 4.84 is greater than 3.84. Therefore the frequency differs significantly from 50% G+C at the 5% level.

12. A student answers a multiple choice exam paper with 12 questions on it. He guesses randomly from the 4 possible answers on each question. Which of the following is true?

- A. The probability of guessing all 12 questions correctly is slightly greater than 3%.
- B. The probability of getting exactly 3 correct answers is 0.0156.

- C. The probability of getting exactly 3 correct answers is 1.17×10^{-3} .
- D. The probability of guessing all answers wrong is greater than the probability of guessing all answers right.

A – wrong – The probability of guessing them all right is $(1/4)^{12} = 5.96 \times 10^{-8}$

B – wrong – The probability of guessing exactly three correct answers is

$$\left(\frac{1}{4}\right)^3 \binom{3}{4}^9 \frac{12!}{9! \times 3!} = 0.258.$$

C – wrong

D – correct – If this were not true we would not set multiple choice questions.