CONTENTS

ANSWERS TO QUESTIONS AND PROBLEMS

FUTURES MARKETS: INTRODUCTION

1. In purchasing a house, contracting to buy the house occurs at one time. Typically, closing occurs weeks later. At the closing, the buyer pays the seller for the house and the buyer takes possession. Explain how this transaction is like a futures or forward transaction.

The purchase of a house has many features of a forward contract. Contracting occurs at one date, with performance on the contract occurring later at the closing. In buying a house, there is usually a good faith deposit or earnest money put up at the time of contracting. The contract is tailored to the individual circumstances, with the performance terms and the closing date being agreed on between the buyer and seller. The contract is not like a futures, because there is no organized exchange, the contract terms are not fixed, and settlement can occur at any time instead of at a fixed date.

2. In the futures market, a widget contract has a standard contract size of 5,000 widgets. What advantage does this have over the well-known forward market practice of negotiating the size of the transaction on a case-by-case basis? What disadvantages does the standardized contract size have?

With standardized contract size and other terms, a futures contract avoids uncertainty about what is being traded. If these terms were not specified, traders would have to specify all of the features of the underlying good anew each time there is a contract. The futures style of trading has the disadvantage of losing flexibility due to the standardization. For example, the amount of the contract is fixed, as is the quality of the underlying good, and the time the contract will be settled.

3. What factors need to be considered in purchasing a commodity futures exchange seat? What are all the possible advantages that could come from owning a seat?

A seat on a commodity exchange is essentially a capital asset. The purchaser would want to consider the risk, including systematic risk, associated with such a purchase. The value of the seat depends mainly on the expected trading volume on the exchange, so we expect seat prices to be sensitive to the business cycle and to competition from foreign exchanges. Owning a seat allows one to trade on the exchange. Also, the seat holder can lease the seat for someone else to trade. Therefore, the seat offers the potential for cash inflows as well as capital appreciation.

4. Explain the difference between initial and maintenance margin.

Initial margin is the amount a trader must deposit before trading is permitted. Maintenance margin is the minimum amount that must be held in the trader's account while a futures position is open. If the account value falls below the amount specified as the maintenance margin, the trader must deposit additional funds to bring the account value back to the level of the initial margin.

5. Explain the difference between maintenance and variation margin.

Maintenance margin is the amount a trader must keep in the account to avoid a margin call. Variation margin is the payment a trader must make in a margin call. The margin call occurs when the account value drops below the level set for the maintenance margin. Upon receiving a margin call the trader must make a cash payment of the variation margin. The maintenance margin is a stock variable, while the variation margin is a flow variable.

6. On February 1, a trader is long the JUN wheat contract. On February 10, she sells a SEP wheat futures, and sells a JUN wheat contract on February 20. On February 15, what is her position in wheat futures? On February 25, what is her position? How would you characterize her transaction on February 20?

On February 15, the trader holds an intracommodity spread, being long the JUN and short the SEP wheat. On February 25, the trader is short one SEP wheat contract. The transaction on February 20 was a transaction offsetting the original long position in SEP wheat.

7. Explain the difference between volume and open interest.

Open interest is the number of contracts currently obligated for delivery. The volume is the number of contracts traded during some period. For every purchase there is a sale, and the purchase and sale of one contract generates one contract of trading volume.

8. Define "tick" and "daily price limit."

A tick is the minimum amount a futures contract can change. For example, in the T-bond contract, the tick size is $\frac{1}{32}$ of one point of par. This gives a dollar tick value of $31.25 per T-bond futures contract. The daily price limit is the amount the contract can change in price in 1 day. It is usually expressed as some number of ticks, and it is measured from the previous day's settlement price. No trade can be executed at a price that differs from the previous day's settlement price by more than the daily price limit.

9. A trader is long one SEP crude oil contract. On May 15, he contracts with a business associate to receive 1,000 barrels of oil in the spot market. The business associate is short one SEP crude oil contract. How can the two traders close their futures positions without actually transacting in the futures market?

The traders close their futures positions through an exchange for physicals transaction (EFP). In an EFP, two traders with futures positions exchange the physical good for cash and report this transaction to the exchange, asking the exchange to offset their futures contracts against each other. This transaction is also called an ex-pit or against actuals transaction.

10. Explain how a trader closes a futures market position via cash settlement.

For a contract satisfied by cash settlement, there is no delivery. Instead, when the futures contract expires, the final settlement price on the futures is set equal to the cash price for that date. This practice ensures convergence of the futures price and the cash price. Traders then make or receive payments based on the difference between the previous day's settlement price and the final settlement price on the contract.

11. Explain "price discovery."

In futures markets, price discovery refers to the revealing of information about future prices that the market facilitates. It is one of the two major social functions of the futures market. (The other is risk transference.) As an example, the futures price for wheat for delivery in 9 months reveals information to the public about the expected future spot price of wheat at the time of delivery. While controversial, there is some reason to believe that the futures price (almost?) equals the spot price that is expected to hold at the futures expiration. This price discovery function helps economic agents plan their investment and consumption by providing information about future commodity prices.

12. Contrast anticipatory hedging with hedging in general.

In anticipatory hedging, a trader enters the futures market and transacts before (or in anticipation of) some cash market transaction. This differs from a hedge of an existing position. For example, a farmer might sell wheat futures in anticipation of the harvest. Alternatively, a merchant holding an inventory of wheat might hedge the inventory by selling wheat futures. The farmer is engaged in anticipatory hedging, because he or she is expecting to have the cash market position and hedges this anticipation. The wheat merchant already has the cash market position, in virtue of holding the wheat inventory, and therefore is not engaged in anticipatory hedging.

13. What is "front running"?

Front running is a market practice in which a broker holds a customer's order for execution and executes a similar order of his or her own before executing the customer's order. This practice can be particularly pernicious if the customer's order is large, because the order may itself move prices. By front running, the broker seeks to capitalize on the privileged information that the order is coming to market. This practice is unethical and against the rules of the futures exchange.

14. Explain the difference in the roles of the National Futures Association and the Commodity Futures Trading Commission.

The National Futures Association (NFA) is an industry self-regulatory body, while the Commodity Futures Trading Commission (CFTC) is an agency of the federal government. The same law that instituted the CFTC also provided for the futures industry to establish self-regulatory bodies. The NFA enforces ethical standards on most futures industry members and provides testing for licensing of brokers and other futures industry professionals. The NFA operates under the supervision of the CFTC.

15. What are the two types of financial safeguard models used by futures clearinghouses?

Two main types of clearinghouse financial safeguard systems models are observed across the market. The first type is called "good to the last drop." In this model, the clearinghouse commits its capital to satisfy any default obligations not covered by (1) the margin posted by clearing members on behalf of customers and the member's proprietary accounts; or (2) a separately capitalized guarantee fund. In the good to the last drop model, the clearinghouse commits to satisfying all obligations to the point where the clearinghouse itself is insolvent.

The second type of financial safeguard model is the "live another day" model. In this model, the clearing members are protected primarily with guarantee funds and without committing the core capital of the clearinghouse. In this model, a primary objective is to sustain the clearinghouse so that it can continue to perform its risk-mitigating role during times of crisis when it is needed the most. In this model, default obligations are ultimately borne by clearing members who must absorb unpaid invoices.

16. What does it mean for an exchange to "demutualize?"

When a nonprofit member-owned exchange converts to for-profit status it is said to have demutualized. In a demutualization, the members receive shares of stock in the new corporation. Ultimately, the shares in the demutualized corporation may be offered to the public in an initial public offering (IPO). The shares can then be traded on the open market.

17. How did the Commodity Futures Modernization Act of 2000 alter the regulation of the futures industry?

The Commodity Futures Modernization Act of 2000 made sweeping changes to the way futures markets had previously been regulated. Key features include:

(a) Permitting futures trading on individual stocks and narrow-based stock indexes.
(b) Clarifying the legal status of privately negotiated swap transactions.
(c) Promoting competition and innovation in futures markets.
(d) Providing a predictable and calibrated regulatory structure tailored to the product, the participant, and the trading platform.
(e) Allowing exchanges to bring new contracts to market without prior regulatory approval.
(f) Establishing a set of core principles, or standards, that permit futures exchanges and clearinghouses to use different methods to achieve federal requirements.
(g) Giving the CFTC clear authority to stop certain illegal, foreign exchange transactions aimed at defrauding small investors.
(h) Giving the CFTC separate oversight authority with respect to clearinghouse organizations.

FUTURES MARKETS: REFINEMENTS

1. Explain the difference between an intercommodity and an intracommodity spread.

An intracommodity spread is a combined position in 2 or more delivery months for contracts on the same underlying commodity, such as long the JUN T-bond and short the SEP T-bond. An intercommodity spread is a combined position in two or more distinct, but related, commodities, such as long the SEP wheat and short the SEP corn.

2. A speculator buys a nearby and sells a distant silver futures contract. What must happen for the trader to make a profit from this combined position?

For there to be an overall profit on the spread, the profit on one leg must exceed the loss on the other. Assume silver prices rise in general. In this case, the spread earns a profit if the price of the nearby contract rises more than the price of the falls of the distant contract. If prices are falling generally, the spread earns a profit if the price of the nearby contract falls less than the price of the distant contract.

3. A speculator buys a silver futures contract and sells a gold futures contract for the same expiration month. What kind of spread is this? What must happen for the speculator to profit?

Because gold and silver are distinct, but related commodities, this is an intercommodity spread. For the spread to be profitable with rising metal prices, the silver contract price must rise more than that of the gold contract. In falling markets, the price of the silver contract must fall less than that of the gold contract.

4. What is "intermarket cross-margining"? Explain how cross-margining employs the ideas of portfolio theory.

Intermarket cross-margining is a system of futures and securities margining that considers a trader's entire portfolio across all markets. The idea is to require a level of margin that reflects the actual risk of the entire portfolio, no matter where the positions are held. For example, if a trader holds an S&P 500 long futures position, plus a long S&P 100 put, plus a short S&P 100 call, the trader holds positions on the Chicago Mercantile Exchange (futures) and the Chicago Board Options Exchange (both index options). The total risk of this position is lower than the futures or the options considered alone. Intermarket cross-margining would impose a margin requirement that reflects that risk. Without cross-margining, the total margin requirement on this position would be much greater, because the futures and options would be margined in isolation, and these positions considered individually do involve considerable risk.

5. If margins are maintained at levels that keep risk constant for individual contracts, but intermarket cross-margining is introduced, what is likely to happen to the overall pool of margin funds across all markets? Explain.

Cross-margining essentially considers the portfolio effects inherent in the positions that traders might hold across markets. Considering these portfolio effects will almost always show that a trader has a lower risk level than it appears from considering each position in isolation. If rules keep risk constant for the individual contracts but reflect these portfolio effects, the actual margin required from an individual trader will probably drop. This will lead to a reduction in the overall pool of margin funds in the system.

6. Consider a rainfall futures contract that might be written for cash settlement depending on the amount of rainfall by a certain date at various government weather stations. How would such a contract meet the conditions for success outlined in this chapter?

Table 2.3 lists 10 characteristics that promote futures contract success. The rainfall contract meets some conditions admirably and flunks on others. First, there is no cash market, so arbitrage between the futures and the cash would not be possible. Second, there is substantial volatility in rainfall, so this factor bodes well for the contract. Third, there is no cash market as such, but there is good information on actual rainfall. Fourth, there are substitutes for the rainfall contract but they may not be very good. For example, the soybean futures contract can serve as a proxy for midwest weather, but it certainly is not a perfect substitute. Also, crop prices depend on many factors other than rainfall, thereby suggesting that crop futures do not provide a very close substitute for the rainfall contract. Fifth, related contracts for spread trading are conspicuously absent. Sixth, good contract design would be a problem.

Manipulation is a potential problem, suggesting that a large number of stations might need to be included. Also, the contract would need some way to deal with snowfall instead of rain. Seventh, strong support from floor traders is important, and it depends largely on their perception of the contract's potential. Usually, this is difficult to judge until the contract is designed. Eighth, there is no deliverable supply as such, but with cash settlement this may not be a problem. The requirement of a large deliverable supply is a factor that prevents manipulation. Ninth, there appears to be no regulatory barrier, except for convincing the CFTC that the contract would have an economic purpose. Tenth, the underlying good (water) is pretty homogeneous.

7. Explain the different roles of a floor broker and an account executive.

A floor broker is located on the floor of the exchange and executes orders for traders off the floor of the exchange. Typically, the floor broker will either be an independent trader who executes orders on a contractual basis for a futures commission merchant (FCM), or the floor broker may be an employee of an FCM. An account executive is almost always employed by an FCM and is located off the floor of the exchange. The account executive is the person one typically thinks of as a broker. The account executive could be located in the local office of any major brokerage firm and has customers for whom he or she executes orders by communicating them to the exchange via the communication facilities of the FCM.

8. At a party, a man tells you that he is an introducing broker. He goes on to explain that his job is introducing prospective traders such as yourself to futures brokers. He also relates that he holds margin funds as a service to investors. What do you make of this explanation?

The guy is a fraud. First, a defining characteristic of an introducing broker (IB) is that the IB does not hold customers' funds. Instead, the IB is associated with an FCM who holds the customers' funds. Second, the last person the IB wants his customer to meet is another broker. The IB's income depends on executing orders for his customers, so the IB wants to keep his flock of customers away from the wolves (other brokers) that are hungry for customers.

9. Assume that you are a floor broker and a friend of yours is a market maker who trades soybeans on the floor of the Chicago Board of Trade. Beans are trading at $6.53 per bushel. You receive an order to buy beans and you buy one contract from your friend at $6.54, 1 cent above the market. Who wins, who loses, and why? Explain the rationale for making such practice illegal.

As described, this transaction costs your customer $0.01/bushel and transfers those funds to the friend from whom you purchase the contract at $6.54. On a 5,000 bushel contract, this amounts to $50. Thus, as described, the customer loses and the friend wins. It is important to see the motivation for the floor broker in engaging in this transaction. As described, the floor broker cheats her customer and helps the friend. Presumably, the motivation for such an action is the expectation that the friend will return the favor on another transaction. The rationale for making this transaction illegal is clear; it amounts to a direct theft from the customer.

10. Why are some futures exchanges wary of block trading? Why do some exchanges embrace block trading?

Although some futures exchanges have embraced block trading, many other exchanges are extremely wary of the practice because of the fear that block trading will fragment the central marketplace (i.e. the trading pit or electronic trading system) and undermine price discovery.

Another concern with block trading is that it may provide a means to evade surveillance systems aimed at detecting prohibited trading practices. Proponents of block trading argue that the practice actually helps integrate markets by bringing over-the-counter (OTC) transactions into an exchange-traded environment where prices and volumes are publicly reported. In addition, proponents argue that block transactions spawn futures transactions in the central market as block trading participants hedge their positions. They also argue that block trading activity leads to higher overall contract volume over time because it is common for positions opened via block trades to be closed out sooner or later in the central marketplace.

11. Back at the party after several more hours. Your buddy from Question 8 buttonholes you again and starts to explain his great success as a dual trader, trading both beans and corn. What do you think?

This guy is not bright. A dual trader is a person who trades for his or her own account and who executes orders for others at the same time.

12. You are having trouble escaping from your friend in Question 11. He goes on to explain that liquidation-only trading involves trading soybean against soyoil to profit from the liquidation that occurs when beans are crushed. Explain how your understanding of "liquidation-only trading" differs from your friend's.

We hope that your understanding of liquidation-only trading runs as follows: under liquidation-only trading each trade must result in a reduction of a trader's open interest. Every trade must be an offsetting trade. Liquidation-only trading essentially amounts to the closing of a market, and this is done during serious market disturbances, such as a manipulation. Liquidation-only trading has nothing in particular to do with beans or any other commodities.

13. A trader holds a long position in the MAR T-bond futures contract. She offsets this position and buys a JUN T-bond futures contract. What is the name for this kind of transaction?

The trader has rolled her position forward. After the transactions, she has the same position, except it has been transferred to a contract that expires later than her original position.

14. In what ways do futures exchanges compete?

To understand how exchanges compete, it is helpful to understand the economic incentives to which futures exchanges are responding. First, we must recognize that a futures exchange is a business firm that creates markets. The creation of markets is an entrepreneurial activity that entails substantial costs, such as gathering information, searching for trading partners, bargaining, and enforcing contracts. Futures exchanges economize on these costs by specifying the rules of trading, the terms of exchange contracts, the conditions of exchange membership, and the technology employed for order entry and trade execution.

By viewing futures exchanges as firms that create markets, we can see more clearly some of the ways exchanges compete. Among the ways exchanges compete is through innovation in the design of the contracts they offer, the technology they employ, the fees they charge, the business models they adopt, and the quality of trading information they provide to investors. Exchanges also compete directly with each other for the exclusive right to trade particular contracts. Finally, exchanges compete for business with markets offering related products. For example, futures exchanges compete with the OTC market and the market for exchange traded funds.

15. How do futures exchanges make money?

A futures exchange is a business. By far the biggest source of revenue at most exchanges is the fees they generate from executing transactions and clearing trades. Another large source of revenue is the sale of real-time data, that is, the quotation data fees. At some exchanges, such as the Chicago Board of Trade and the Minneapolis Grain Exchange, rent from real estate holdings is a major contributor to revenue.

16. What are bunched orders? What issues are involved in the posttrade allocation of bunched orders?

A bunched order is a collective trade placed on behalf of several accounts. In placing a bunched order, the CTA may not be able to fill the entire order at a single price. In other words, the CTA may receive “split fills” on his order. This means that the CTA will have to allocate the filled trades among the various accounts after the trade has been executed. Inevitably, some customers will receive more favorable, and some less favorable, fills because of the fact that portions of the order were executed at different prices. Although there is nothing wrong with this practice *per se*, it presents the CTA with the opportunity to favor some accounts over others by allocating the more profitable trades to favored accounts. It is for this reason that federal commodities laws prohibit brokers, advisors, and other market professionals, except in specific instances, from allocating orders among accounts after trades have been executed. This prohibition is aimed at preventing such persons from abusing their discretion in allocating trades.

17. What is an “out trade”? How do futures exchanges facilitate the reconciliation of out trades?

An “out trade” occurs when a discrepancy exists between the trade data submitted by the broker representing the buyer and the trade data submitted by the broker representing the seller. There are two types of out trades: those caused by a discrepancy in the reported price and those caused by a discrepancy in the reported quantity. Exchange rules usually require the brokers to choose between the trade data submitted by the buyer and that submitted by the seller. Any compromises or adjustments are handled by side payments between the brokers. If the brokers cannot resolve the discrepancy, exchange rules may enforce a predetermined solution.

18. What are the four elements of proof required for a futures market manipulation claim?

Federal courts use a four-pronged test, that has evolved though the common law, to determine by a preponderance of the evidence whether a set of facts are consistent with an alleged manipulation. The four elements of proof in manipulation cases are that

(1) the accused had the ability to influence market prices;
(2) the accused specifically intended to do so;
(3) an artificial price occurred; and
(4) the accused caused an artificial price.

19. In what ways is futures market manipulation deterred by the actions of the CFTC and the futures exchanges?

In addition to after-the-fact sanctions, there are other methods regulators use to achieve deterrence of manipulation. For example, the CFTC employs a staff of economists and futures market specialists who conduct real-time market surveillance of the positions of traders. The CFTC also has emergency powers to force would-be manipulators to liquidate their positions. Other anti-manipulation tools of regulators and the exchanges (which are self-regulatory organizations) include position limits and contract design elements that make manipulation more costly for the would-be manipulator. Futures exchanges also conduct surveillance of their markets in order to deter manipulation before it occurs.

20. What does it mean for a futures market to be *transparent*?

The word *transparency* refers to the degree to which a futures exchange publicly disseminates real-time information on transaction prices, quotations, order flow, and other market variables. Much of the real-time information produced by futures exchanges is sold to data vendors, such as Bloomberg, who in turn make the information available to their subscribers. Other information, such as delayed quotes, is provided by futures exchanges for free. Transparency can be viewed as one dimension of competition between competing marketplaces.

21. What does it mean for a futures market to be *fungible*?

The word fungible mean interchangeable. Futures contracts are written with precise specifications to ensure that the quality of the underlying commodity is uniform. If the underlying good varies tremendously in quality, the delivery process will be impaired. A large fungible supply of the underlying commodity is a desirable feature for a futures contract.

22. What is churning?

Churning refers to the actions of brokers who execute trades on behalf of investors with the intent of generating commissions at the expense of the investors' interests. To establish a claim for churning in futures markets, an investor must be able to demonstrate three elements by a preponderance of the evidence: (1) that the broker (or advisor) controlled the level and frequency of trading in the account; (2) that the overall volume of the broker's trading was excessive in light of the investor's trading objective; and (3) that the broker acted with the intent to defraud the investor or acted with a reckless disregard for the investor's interests

FUTURES PRICES

1. Explain the function of the settlement committee. Why is the settlement price important in futures markets in a way that the day's final price in the stock market is not so important?

In futures markets, the settlement committee determines the settlement price for each contract each day. The settlement price estimates the true value of the contract at the end of the day's trading. In active markets, the settlement price will typically equal the last trade price. In inactive markets, the settlement price is the committee's estimate of the price at which the contract would have traded at the close, if it had traded. The settlement price is important, because it is used to calculate margin requirements and the cash flows associated with daily settlement. In the stock market, there is no practice comparable to daily settlement, so the closing price in the stock market lacks the special significance of the futures settlement price.

2. Open interest tends to be low when a new contract expiration is first listed for trading, and it tends to be small after the contract has traded for a long time. Explain.

When the contract is first listed for trading, open interest is necessarily zero. As traders take positions, the open interest builds. At expiration, open interest must again be zero. Every contract will have been fulfilled by offset, delivery, or an EFP. Therefore, as the contract approaches the expiration month, many traders will offset their positions to avoid delivery. This reduces open interest. In the expiration month, deliveries that occur further reduce open interest. Also, EFPs typically reduce open interest. This creates a pattern of very low open interest in the contract's early days of trading, followed by increases, diminution, and the contract's extinction.

3. Explain the distinction between a normal and an inverted market.

In a normal market, prices for more distant expirations are higher than prices for earlier expirations. In an inverted market, prices for more distant expirations are lower than prices for earlier expirations.

4. Explain why the futures price converges to the spot price and discuss what would happen if this convergence failed.

The explanation for convergence at expiration depends on whether the market features delivery or cash settlement, but in each case convergence depends on similar arbitrage arguments. We consider each type of contract in turn. For a contract with actual delivery, failure of convergence gives rise to an arbitrage opportunity at delivery. The cash price can be either above or below the futures price, if the two are not equal. If the cash price exceeds the futures price, the trader buys the future, accepts delivery, and sells the good in the cash market for the higher price. If the futures price exceeds the cash price, the trader buys the good on the cash market, sells futures, and delivers the cash good in fulfillment of the futures. To exclude both types of arbitrage simultaneously, the futures price must equal the cash price at expiration. Minor discrepancies can exist, however. These are due to transaction costs and the fact that the short trader owns the options associated with initiating the delivery sequence.

For a contract with cash settlement, failure of convergence also implies arbitrage. Just before delivery, if the futures price exceeds the cash price a trader can sell the futures, wait for expiration, and the futures price will be set equal to the cash price. This gives a profit equal to the difference between the cash and futures. Alternatively, if the cash price is above the futures price and expiration is imminent, the trader can buy the futures and wait for its price to be marked up to equal the cash price. Thus, no matter whether the futures price is above or below the cash price, a profit opportunity will be available immediately.

In short, the futures and cash price converge at expiration to exclude arbitrage, and failure of convergence implies the existence of arbitrage opportunities.

5. Is delivery, or the prospect of delivery, necessary to guarantee that the futures price will converge to the spot price? Explain.

No, delivery is not necessary. As explained in the answer to Question 4, cash settlement will also lead to convergence of the cash and the futures at expiration.

6. As we have defined the term, what are the two key elements of "academic arbitrage"?

The two elements are riskless profit and zero investment. Each condition is necessary for academic arbitrage, and the two conditions are jointly sufficient.

7. Assume that markets are perfect in the sense of being free from transaction costs and restrictions on short selling. The spot price of gold is \$370. Current interest rates are 10 percent, compounded monthly. According to the Cost-of-Carry Model, what should the price of a gold futures contract be if expiration is 6 months away?

In perfect markets, the Cost-of-Carry Model gives the futures price as:

$$F_{0,t} = S_0(1 + C)$$

The cost of carrying gold for six months is $(1 + 0.10/12)^6 - 1 = 0.051053$. Therefore, the futures price should be:

$$F_{0,t} = \$370(1.051053) = \$388.89$$

8. Consider the information in Question 7. Round trip futures trading costs are \$25/100 ounce gold contract, and buying or selling an ounce of gold incurs transaction costs of \$1.25. Gold can be stored for \$.15 per month per ounce. (Ignore interest on the storage fee and the transaction costs.) What futures prices are consistent with the Cost-of-Carry Model?

Answering this question requires finding the bounds imposed by the cash-and-carry and reverse cash-and-carry strategies. For convenience, we assume a transaction size of one 100-ounce contract. For the cash-and-carry the trader buys gold and sells the futures. This strategy requires the following cash outflows:

Buy gold	−\$370(100)
Pay transaction costs on the spot	−\$1.25(100)
Pay the storage cost	−\$0.15(100)(6)
Sell futures	0
Borrow to finance these outlays	+\$37,215
6 months later, the trader must	
Pay the transaction cost on one futures	−\$25
Repay the borrowing	−\$39,114.95
Deliver on futures	22

Net outlays at the outset were zero, and they were \$39,139.95 at the horizon. Therefore, the futures price must exceed \$391.40 an ounce for the cash-and-carry strategy to yield a profit.

The reverse cash-and-carry incurs the following cash flows. At the outset, the trader must:

Sell gold	+\$370(100)
Pay transaction costs on the spot	−\$1.25(100)
Invest funds	−\$36,875
Buy futures	0

These transactions provide a net zero initial cash flow. In 6 months, the trader has the following cash flows:

Collect on investment	$+\$36{,}875(1 + 0.10/12)^6 = \$38{,}757.59$
Pay futures transaction costs	−$25
Receive delivery on futures	?

The breakeven futures price is therefore $387.33 per ounce. Any lower price will generate a profit. From the cash-and-carry strategy, the futures price must be less than $391.40 to prevent arbitrage. From the reverse cash-and-carry strategy, the price must be at least $387.33. (Note that we assume there are no expenses associated with making or taking delivery.)

9. Consider the information in Questions 7 and 8. Restrictions on short selling effectively mean that the reverse cash-and-carry trader in the gold market receives the use of only 90 percent of the value of the gold that is sold short. Based on this new information, what is the permissible range of futures prices?

This new assumption does not affect the cash-and-carry strategy, but it does limit the profitability of the reverse cash-and-carry trade. Specifically, the trader sells 100 ounces short but realizes only $0.9(\$370)(100) = \$33{,}300$ of usable funds. After paying the $125 spot transaction cost, the trader has $33,175 to invest. Therefore, the investment proceeds at the horizon are: $\$33{,}175(1 + 0.10/12)^6 = \$34{,}868.69$. Thus, all of the cash flows are:

Sell gold	+$370(100)
Pay transaction costs on the spot	−$1.25(100)
Broker retains 10 percent	−$3,700
Invest funds	−$33,175
Buy futures	0

These transactions provide a net zero initial cash flow. In 6 months, the trader has the following cash flows:

Collect on investment	$34,868.69
Receive returns of deposit from broker	$3,700
Pay futures transaction costs	−$25
Receive delivery on futures	?

The breakeven futures price is therefore $385.44 per ounce. Any lower price will generate a profit. Thus, the no-arbitrage condition will be fulfilled if the futures price equals or exceeds $385.44 and equals or is less than $391.40.

10. Consider all of the information about gold in Questions 7–9. The interest rate in Question 7 is 10 percent per annum, with monthly compounding. This is the borrowing rate. Lending brings only 8 percent, compounded monthly. What is the permissible range of futures prices when we consider this imperfection as well?

The lower lending rate reduces the proceeds from the reverse cash-and-carry strategy. Now the trader has the following cash flows:

Sell gold	+\$370(100)
Pay transaction costs on the spot	−\$1.25(100)
Broker retains 10 percent	−\$3,700
Invest funds	−\$33,175
Buy futures	0

These transactions provide a net zero initial cash flow. Now the investment will yield only $\$33{,}175(1 + 0.08/12)^6 = \$34{,}524.31$. In 6 months, the trader has the following cash flows:

Collect on investment	\$34,524.31
Pay futures transaction costs	−\$25
Receive delivery on futures	?
Return gold to close short sale	0
Receive return of deposit from broker	\$ 3,700

Total proceeds on the 100 ounces are \$38,199.31. Therefore, the futures price per ounce must be less than \$381.99 for the reverse cash-and-carry strategy to profit. Because the borrowing rate has not changed, the bound from the cash-and-carry strategy remains at \$391.40. Therefore, the futures price must remain within the inclusive bounds of \$381.99 to \$391.40 to exclude arbitrage.

11. Consider all of the information about gold in Questions 7–10. The gold futures expiring in 6 months trades for \$375 per ounce. Explain how you would respond to this price, given all of the market imperfections we have considered. Show your transactions in a table similar to Tables 3.8 or 3.9. Answer the same question, assuming that gold trades for \$395.

If the futures price is \$395, it exceeds the bound imposed by the cash-and-carry strategy and it should be possible to trade as follows:

Cash-and-carry arbitrage

$t = 0$	
Borrow \$37,215 for 6 months at 10%	+\$37,215.00
Buy 100 ounces of spot gold	−37,000.00
Pay storage costs for 6 months	−90.00
Pay transaction costs on gold purchase	−125.00
Sell futures for \$395	0.00
Total cash flow	\$0
$t = 6$	
Remove gold from storage	\$0
Deliver gold on futures	+39,500.00
Pay futures transaction cost	−25.00
Repay debt	−39,114.95
Total cash flow	+\$360.05

If the futures price is $375, the reverse cash-and-carry strategy should generate a profit as follows:

Reverse cash-and-carry arbitrage

$t = 0$	
Sell 100 ounces of gold short	+ $37,000.00
Pay transaction costs	−125.00
Broker retains 10%	−3,700.00
Buy futures	0
Invest remaining funds for 6 months at 8%	−33,175.00
Total cash flow	$0
$t = 6$	
Collect on investment	+ $34,524.31
Receive delivery on futures	−37,500.00
Return gold to close short sale	0
Receive return of deposit from broker	+ 3,700.00
Pay futures transaction cost	−25.00
Total cash flow	+$699.31

12. Explain the difference between pure and quasi-arbitrage.

In a pure arbitrage transaction, the arbitrageur faces full transaction costs on each transaction comprising the arbitrage. For example, a retail customer with no initial position in the market, who attempts arbitrage, would be attempting pure arbitrage. By contrast, a quasi-arbitrage transaction occurs when a trader faces less than full transaction costs. The most common example arises in reverse cash-and-carry arbitrage, which requires short selling. For example, in stock index arbitrage, holding a large portfolio allows a trader to simulate a short sale by selling part of the portfolio from inventory. Therefore, this trader faces less than the full transaction costs due to the preexisting position in the market. By contrast, the pure arbitrage trade would require the actual short sale of the stocks, and short selling does not provide the full proceeds to earn interest in the reverse cash-and-carry transactions.

13. Assume that you are a gold merchant with an ample supply of deliverable gold. Explain how you can simulate short selling and compute the price of gold that will bring you into the market for reverse cash-and-carry arbitrage.

The breakeven price for reverse cash-and-carry arbitrage depends principally on the transaction costs the trader faces. With an existing inventory of gold, the trader can simulate short selling by selling a portion of the inventory. Further, because the trader already actually owns the gold, she can have full use of the proceeds of the sale. Therefore, the gold owner's reverse cash-and-carry transactions are similar to those in Problem 10:

Reverse cash-and-carry arbitrage

$t = 0$	
Sell 100 ounces of gold short	+ $37,000.00
Pay transaction costs	−125.00
Buy futures	0
Invest funds for 6 months at 8%	−36,875.00
Total cash flow	$0

$t = 6$	
Collect on investment	+ $38,374.80
Return gold to close short sale	0
Pay futures transaction cost	−25.00
Receive delivery on futures (Note: This is the futures price to give zero cash flow)	−38,349.80
Total cash flow	+$0

Therefore, if the futures price is $383.498 per ounce, the reverse cash-and-carry transactions give a zero cash flow. This is the breakeven price for reverse cash-and-carry. If the futures price is less that $383.498 per ounce, reverse cash-and-carry arbitrage will be possible for the trader who holds an initial inventory of gold. In Problem 10, the price of gold has to be less than $381.99 for reverse cash-and-carry arbitrage to work. The trader there faced full transaction costs, due to a lack of preexisting inventory.

14. Assume that silver trades in a full carry market. If the spot price is $5.90 per ounce and the futures that expires in 1 year trades for $6.55, what is the implied cost-of-carry? Under what conditions would it be appropriate to regard this implied cost-of-carry as an implied repo rate?

If the market is at full carry, then $F_{0,t} = S_0(1 + C)$ and $C = F_{0,t}/S_0 - 1$. With our values, $C = \$6.55/\$5.90 - 1 = 0.110169$. It would be appropriate to regard this implied cost-of-carry as an implied repo rate if the only carrying cost were the financing cost. This is approximately true for silver.

15. What is "normal backwardation"? What might give rise to normal backwardation?

Normal backwardation is the view that futures prices normally rise over their life. Thus, prices are expected to rise as expiration approaches. The classic argument for normal backwardation stems from Keynes. According to Keynes, hedgers are short in the aggregate, so speculators must be net long. Speculators provide their risk-bearing services for an expected profit. To have an expected profit, the futures price must be less than the expected future spot price at the time the speculators assume their long positions. Therefore, given unbiased expectations regarding future spot prices, we expect futures prices to rise over time to give the speculators their compensation. This leads directly to normal backwardation.

16. Assume that the CAPM beta of a futures contract is zero, but that the price of this commodity tends to rise over time very consistently. Interpret the implications of this evidence for normal backwardation and for the CAPM.

Because futures trading requires no investment, positive returns on long futures positions can be consistent with the CAPM only if futures have positive betas. With a zero beta (by our assumption) and a zero investment to acquire a long futures position (by the structure of the market), the CAPM implies zero expected returns. Therefore, a zero beta and positive returns is inconsistent with the CAPM. Even with zero beta, positive returns are consistent with normal backwardation resulting from speculators assuming long positions and being rewarded for their risk-bearing services.

17. Explain why futures and forward prices might differ. Assume that platinum prices are positively correlated with interest rates. What should be the relationship between platinum forward and futures prices? Explain.

Futures are subject to daily settlement cash flows, while forwards are not. If the price of the underlying good is not correlated with interest rates, futures and forward prices will be equal. If the price of the underlying good is positively correlated with interest rates, a long trader in futures will receive daily settlement cash inflows when interest rates are high and the trader can invest that cash flow at the higher rate from the time of receipt to the expiration of the futures. Because forwards have no daily settlement cash flows, they are unable to reap this benefit. Therefore, if a commodity's price is positively correlated with interest rates, there will be an advantage to a futures over a forward. Thus, for platinum in the question, the futures price of platinum should exceed the forward price. The opposite price relationship can occur if there is negative correlation. Generally, this price relationship is not sufficiently strong to be observed in the market.

18. Consider the life of a futures contract from inception to delivery. Explain two fundamental theories on why the futures prices might exhibit different volatility at different times over the life of the contract.

According to the Samuelson hypothesis, price volatility will be greater when more information about the price of the good is being revealed. According to this view, this tends to happen as the futures comes to expiration, particularly for agricultural goods. Therefore, the Samuelson hypothesis suggests that the volatility of futures prices should increase over the life of the contract.

There are several other theories that attempt to relate contract maturity and volatility. First, there seems to be some evidence for believing that volatility is higher for some commodities in certain seasons, particularly at times when information about the harvest of some good is reaching the market. With this view, volatility depends on the time of the year and not so much on the contract's expiration. Second, volatility also differs depending on the day of the week. Third, volatility is autocorrelated. High volatility in 1 month begets high volatility in the next month.

19. What is a limit order? How does placing a limit order provide an option to other traders?

A limit order is a conditioned order that requires that a trade not be executed unless the price reaches a predetermined level. For example, a corn producer may give instructions to their broker to sell March corn only if the price reaches $2.70/bushel. Or a buyer may give instructions not to buy March corn unless the price falls below $2.50/bushel. Limit orders are used by traders as a means of communicating their instructions to their broker.

Traders who submit limit orders give other traders the choice of whether to trade with the order or not. This choice can be characterized as an option to trade. For example, a limit order to sell gives other traders the option to buy at the limit price. The option to buy a set quantity at a set price is by definition a call option. A limit order to buy gives other traders the option to sell at the limit price. The option to sell a set quantity at a set price is by definition a put option.

20. What is market microstructure?

Market microstructure is a branch of financial economics that analyzes how trading technology influences the trading characteristics of a financial market. For example, market microstructure techniques can be applied to the futures market to determine whether the information content of prices formed in an open outcry trading environment differs from the information content of prices formed in an electronic trading environment. Trading technology defines what traders can do and what they can know. Broadly defined, trading technology includes such things as the physical layout of a market, trading protocols and rules, market governance, and information systems available to traders. Trading characteristics include the determinants of the market price, market liquidity, transaction costs, volatility, and trading profits.

21. What is the difference between fundamental volatility and transitory volatility?

Fundamental volatility is caused by the actions of informed traders who base their trades on the arrival of new fundamental information. As informed traders conduct their trades, market prices change to reflect the new information. Fundamental volatility can be good for the economy because it helps investors allocate scarce capital to its highest valued use.

Transitory volatility is caused by the trading decisions of speculators who are uninformed about market fundamentals and trade instead information they glean from the trading process itself.

22. What is the implied repo rate? What information does the implied repo provide about the relationship between cash and futures prices?

Most participants in the futures markets face a financing charge on a short-term basis that is equivalent to the repo rate, that is, the interest rate on repurchase agreements. In a repurchase agreement, a person sells securities at one time, with the understanding that they will be repurchased at a certain price at a later time. Most repurchase agreements are for 1 day only and are known, accordingly, as overnight repos. The repo rate is relatively low, exceeding the rate on Treasury bills by only a small amount.

In trading vernacular, the theoretical rate of return on a cost-of-carry strategy is the *implied repo rate*. An arbitrageur calculates the implied repo rate and compares it to his own financing cost (proxied by the actual repo rate) to determine whether or not an arbitrage opportunity exists. In a well–functioning market without arbitrage opportunities, the implied repo rate is equivalent to the actual repo rate. Deviations from this relationship lead to arbitrage opportunities in a perfect market.

USING FUTURES MARKETS

1. Explain how futures markets can benefit individuals in society who never trade futures.

One of the main benefits that the futures market provides is price discovery; futures markets provide information about the likely future price of commodities. This information is available to anyone in the economy, because the prices are publicly available. It is not necessary to trade futures to reap this benefit.

2. A "futures price" is a market quoted price today of the best estimate of the value of a commodity at the expiration of the futures contract. What do you think of this definition?

This claim is intriguing but controversial. If there is no risk premium imbedded in the futures price, the statement is likely to be true. The definition implies that random holding of futures positions should earn a zero profit. This seems to be approximately true, but studies, such as that by Bodie and

Rosansky, find positive returns to long futures positions. While the claim may not hold literally, it does seem to be close to correct. Further, those who reject the claim may have a difficult time in identifying futures prices that are above or below the future spot price.

3. Explain the concept of an unbiased predictor.

A predictor is unbiased if the average prediction error equals zero. This implies that errors in the prediction are distributed around zero, and that the prediction is equally likely to be high as well as low.

4. How are errors possible if a predictor is unbiased?

Saying that a predictor is unbiased merely claims that the predictions do not tend to be too high or too low. They can still be in error. For example, the futures price may provide an unbiased prediction of the future spot price of a commodity. Nonetheless, the errors in such a prediction are often large, because the futures price today can diverge radically from the spot price at the expiration of the futures.

5. Scalpers trade to capture profits from minute fluctuations in futures prices. Explain how this avaricious behavior benefits others.

Scalpers trade frequently, attempting to profit by a tick here or there. In pursuing their profit, the scalpers provide the market with liquidity. Thus, a trader who wishes to take or offset a position benefits from the presence of scalpers ready to take the opposite side of the transaction. With many scalpers competing for business, position traders will be able to trade at prices that closely approximate the true value of the commodity. Expressed another way, as scalpers compete for profits, they force the bid/asked spread to narrow, therefore contributing to the liquidity of the market.

6. Assume that scalping is made illegal. What would the consequences of such an action be for hedging activity in futures markets?

Without scalpers, the liquidity of the futures market would be greatly impaired. This would imply a widening of bid/asked spreads. The potential hedger would face having to accept a price that was distant from the true price. Faced with the higher transaction costs represented by wider bid/asked spreads, some hedgers might find that hedging is too expensive and they might not hedge. Thus, without scalpers, hedging would be more expensive, and we would observe a lower volume of hedging activity.

7. A trader anticipates rising corn prices and wants to take advantage of this insight by trading an intracommodity spread. Would you advise that she trade long nearby/short distant or the other way around? Explain.

The answer depends on the relative responsiveness to nearby and distant futures prices to a generally rising price level for corn. If the nearby contract price rises more than the price of the distant contract, the trader should go long nearby/short distant, for example. For most agricultural commodities there is no general rule to follow.

8. Assume that daily settlement prices in the futures market exhibit very strong first-order serial correlation. How would you trade to exploit this strategy? Explain how your answer would differ if the correlation is statistically significant but, nonetheless, small in magnitude.

With strong serial correlation, a price rise is likely to be followed by another price rise, and a price drop is likely to be followed by another price drop. Therefore, the trader should buy after a price rise and sell after a price fall. If the correlation is strong, the strategy should generate profits. However, the correlation must be very strong to generate profits sufficient to cover transaction costs. The correlation can be statistically significant, but still too small to be economically significant. To be economically significant, the correlation must be strong enough to generate trading profits that will cover the transaction costs. Studies typically find statistically significant first-order serial correlation in futures price changes, but they also find that these correlations are not economically significant.

9. Assume that you are a rabid efficient markets believer. A commodity fund uses 20 percent of its funds as margin payments. The remaining 80 percent are invested in risk-free securities. What investment performance would you expect from the fund?

For any efficient markets believer, rabid or calm, the expected return on the 80 percent of the funds is the risk-free rate. If there is no risk premium, the expected profit on the futures position is zero. Thus, we define a rabid efficient markets believer as one who denies the existence of a risk premium. Therefore, the rabid theorist expects returns from the funds that would be 80 percent of the risk-free rate.

10. Consider two traders. The first trader is an individual with his own seat who trades strictly for his own account. The other trader works for a brokerage firm actively engaged in retail futures brokerage. Which trader has a lower effective marginal trading cost? Relate this comparison in marginal trading costs to quasi-arbitrage.

This is a difficult question. The trader who owns a seat incurs the following costs to trade: the capital commitment to the seat, the opportunity cost of foregone alternative employment, and the exchange member's out-of-pocket transaction costs. These out-of-pocket costs are quite low. For the broker, the scale is much greater. Behind the broker in the pit stands the entire brokerage firm organization with the overhead it represents. Offsetting this overhead to some extent is the much greater scale associated with the brokerage firm. Also, for the trader associated with the brokerage firm, much of the overhead is associated with retail operations, and the marginal cost of trading an additional contract can be quite low. Thus, we judge that the brokerage firm has the lower marginal cost of trading. This difference in trading costs (whichever is really lower) can be important for quasi-arbitrage. Essentially, the fruits of quasi-arbitrage can be harvested by the trader with the lowest marginal transaction costs. If our assessment of these costs is correct, the brokerage firm should be able to squeeze out the market maker and capture these quasi-arbitrage profits.

11. Consider the classic hedging problems of the farmer who sells wheat in the futures market in anticipation of a harvest. Would the farmer be likely to deliver his harvested wheat against the futures? Explain. If he is unlikely to deliver, explain how he manages his futures position instead.

Most farmers that hedge would not deliver against the futures. Often the wheat would not be deliverable, due to differences in grade or type of wheat. Also, the wheat is probably distant from an approved delivery point, and trying to deliver the wheat would involve prohibitively high transportation costs. Instead of actually delivering, the farmer would be much more likely to sell the harvested wheat to the local grain elevator and offset the futures position.

12. A cocoa merchant holds a current inventory of cocoa worth $10 million at present prices of $1,250/metric ton. The standard deviation of returns for the inventory is 0.27. She is considering

a risk-minimization hedge of her inventory using the cocoa contract of the Coffee, Cocoa and Sugar Exchange. The contract size is 10 metric tons. The volatility of the futures is 0.33. For the particular grade of cocoa in her inventory, the correlation between the futures and spot cocoa is 0.85. Compute the risk-minimization hedge ratio and determine how many contracts she should trade.

We know that the hedge ratio is:

$$HR = \frac{\rho_{SF}\,\sigma_S\,\sigma_F}{\sigma_F^2}$$

where S and F indicate the spot and futures, respectively. Therefore, with our data, the hedge ratio is:

$$HR = \frac{\rho_{SF}\,\sigma_S\,\sigma_F}{\sigma_F^2} = \frac{0.85\,(0.27)\,(0.33)}{(0.33)\,(0.33)} = 0.6955$$

Currently, the merchant holds \$10,000,000/\$1,250 = 8,000 metric tons. The hedge ratio indicates trading 0.6955 of the futures for each unit of the spot. This implies a futures position of 8,000(0.6955) = 5,563.64 metric tons. With the futures consisting of 10 tons per contract, the correct futures quantity is 5,564/10 ≈ 56 contracts. Because she is long the physical cocoa, she should sell 56 futures contracts.

13. A service station operator read this book. He wants to hedge his risk exposure for gasoline. Every week, he pumps 50,000 gallons of gasoline, and he is confident that this pattern will hold through thick and thin. What advice would you offer?

The operator should probably not hedge. By construction, the operator faces a fairly small and recurring risk. If the futures price equals the expected future spot price, the expected gains from hedging are zero, ignoring transaction costs. If we consider transaction costs, the hedging program is almost certain to cost money over the long run. Futures hedging is better designed for large risks or special applications. Persistent hedging of repeated small and independent risks will lead to losses equal to the transaction costs the more often the hedge is attempted (assuming the futures price equals the expected future spot price).

14. Describe the difference between a stack hedge and a strip hedge. What are the advantages and disadvantages of each?

A hedge implemented by establishing futures positions in a series of futures contracts of successively longer expirations is called a strip hedge. A hedge implemented by stacking the entire futures position in the front month and then rolling the position forward (less the portion of the hedge that is no longer needed) into the next front month contract is called a stack hedge.

Each strategy involves tradeoffs. The strip hedge has a higher correlation with the underlying risks than the stack hedge (i.e. has lower tracking error), but may have higher liquidity costs because the more distant contracts may be very thinly traded and may have high bid/ask spreads accompanied by high trade-execution risk. The stack hedge has lower liquidity costs but has higher tracking error.

15. Why might it be inappropriate for a corporation to hedge?

First, reducing risk also means reducing expected return. Whether hedging improves the trade-off between risk and expected return depends on the risk preferences of individual traders. Second, when applied to publicly held corporations, we find that hedging may not add to shareholder value. These companies are organized using the corporate form specifically for the purpose of spreading the risk of corporate investments across many shareholders who further spread the risk through their individual ownership of diversified portfolios of stocks from many corporations. In a sense, a publicly held corporation is hedged naturally through its ownership structure. Shareholders are therefore likely to be at best indifferent to hedges constructed at the corporate level since such hedges can be replicated or undone by the portfolio composition of individual shareholders. The shareholder's indifference means that they are unwilling to pay a premium for shares of stock, where earnings are hedged at the corporate level. Yet in spite of this indifference, many publicly held corporations are observed to hedge. We must assume that since capital market discipline creates powerful incentives for corporations to make value-maximizing decisions, not all observed hedging is being done over the objections of shareholders.

16. What is a hedge fund? Do hedge funds actually hedge?

A hedge fund is a term used to describe a wide range of pooled investment vehicles that are privately organized and not widely available to the public. Hedge funds are accessible only to wealthy individuals and institutional investors. Hedge funds can employ any trading strategy they choose, including highly risky strategies, hence the term "hedge" is misleading in describing the risk appetite of the fund's investors. Many investors are attracted to certain hedge funds because they represent a separate asset class that fits well into an overall portfolio diversification strategy.

AGRICULTURAL, ENERGY, AND METALLURGICAL FUTURES CONTRACTS

1. Some futures market experts have proposed a futures contract on computer memory chips. Do you think that prices on such a contract would follow the Cost-of-Carry Model closely? In other words, would the identifiable elements of the cost-of-carry explain most of the difference between spot and futures prices? What factors would be important in determining whether the contract adhered to the Cost-of-Carry Model?

The Cost-of-Carry Model would probably work well for a standardized computer chip. The production of computer chips is almost continuous, so there is no production cycle to induce shortages. In general, one would expect it to be possible to create a fairly large stock of chips, and this large supply would reduce the convenience yield and bring prices more in line with the Cost-of-Carry Model. For computer chips, the chance of obsolescence is also potentially important. If anticipated innovation keeps chip-makers from building a large inventory, then shortages are more likely to develop and these shortages will drive prices away from the price relationships given by a simple Cost-of-Carry Model.

2. Consider a futures contract on common sand. What are the prospects for success for such a contract? Explain.

Probably, the world does not need this kind of futures contract. Sand is cheap and plentiful. Perhaps, even more important, the price of sand is not highly variable. Instead, the main costs associated with acquiring sand are labor costs and capital inputs. These costs rise with general inflation and would not be associated particularly strongly with factors specific to sand.

3. If there were a futures contract on sand, what would be the likely convenience yield of sand? Explain.

The convenience yield would probably be near zero. Convenience yield arises from shortages or the prospects of shortage. Sand does not seem to be the kind of commodity that is likely to come into shortage, so the convenience yield is likely to be near zero.

4. The Miami Fictional Futures Exchange contemplates a futures contract on poinciana blossoms, a beautiful but perishable flower found in south Florida. What factors are likely to determine the prices on such a futures and the relationship between spot and futures price?

The poinciana blossom is similar to the tropical fruit discussed in the chapter. The blossom is, we assume, difficult to store. Therefore, the Cost-of-Carry Model provides little insight into blossom futures pricing. Instead, the futures price of the blossom is more likely to depend on the expected price that will prevail at the blossom harvest.

5. Assume that a research lab announces that palladium can serve as a critical ingredient in nuclear fusion. Thus, palladium can be a key element in creating energy. If palladium futures follow the Cost-of-Carry Model normally, what reaction might you expect for prices of different futures expirations?

Palladium normally follows the Cost-of-Carry Model fairly closely. Before the fusion announcement, we might expect prices to be normal, with more distant contract expirations having higher prices than nearby contracts. The announcement of the fusion principle represents an important new use for palladium, so we might expect the demand curve to shift. Further, the potential for a radical new use for palladium could easily cause a demand for the immediate holding of palladium. In other words, palladium might acquire a convenience yield it did not previously possess. In this situation, we would expect all contract prices to rise, due to the shift in demand. We might also expect nearby prices to rise more than distant prices, due to the demand for immediate supplies. In other words, the announcement could cause palladium to acquire a significant convenience yield.

6. Some people believed that high oil prices following the 1990 Iraqi invasion of Kuwait were due to speculative fever in the futures market. Assume that you had been retained as an apologist for the futures industry. What answer would you have given? In other words, support the argument that such price jumps can be due to rational economic forces.

Oil is a key input for virtually the entire economy. In spite of this fact, oil does not always have a convenience yield, because oil production is a continuous process and supplies are typically ample in normal conditions. (Even in normal conditions, oil products do not always follow the simple Cost-of-Carry Model and they can have a convenience yield.) The Iraqi invasion of Kuwait affected the potential world supply of oil very dramatically. Surely this change in the supply picture justified a substantial increase in oil prices across the board. Further, the threat to the continuous supply of oil naturally gives a premium to cash market oil. Firms need oil as an input, and when their normal supplies are threatened, the firms will be willing to pay a premium for the immediate acquisition of oil. This natural tendency can be expected to give an additional boost to cash oil prices and nearby oil futures prices.

Further, the argument that some of the oil price increase is due to speculators does not seem particularly plausible. Surely, some long speculators believed that prices would rise even more and their buying surely helped prices increase. Moreover, the argument appears to ignore the presence of short (or at least potentially short) speculators in the market. If prices were excessively high, the high prices would attract short speculators who would willingly sell to the speculators. But their selling would surely help force prices down. Instead of high prices due to avaricious speculators, it appears more plausible that the jump in oil prices stemmed from rational fears about the future supply of oil and from the demand for immediate supplies of oil as an input to production processes.

7. Explain the conditions under which the cash price of a commodity can exhibit seasonal patterns while the futures prices for the same commodity does not.

For a good with a strong harvest pattern, the cash price will most likely be seasonal. As an example, wheat has a strong harvest pattern, so the supply is highly seasonal. However, demand is fairly steady. With seasonal supply and steady demand, we might reasonably expect a seasonal cash price. This does not necessarily translate into a seasonal pattern of futures prices, however. The harvest period is known in advance, so futures prices can reflect this information. We would expect the futures price for a contract expiring just before harvest to be higher than the futures price for a contract expiring just after the completion of the harvest. For both of the futures expirations, we need not expect the prices to fluctuate over the harvest cycle, because both prices already reflect the differences in anticipated supply of cash wheat. Yes, the two futures prices will differ because of the time of their expiration, but that does not mean that the futures prices will fluctuate over time. Therefore, the futures price of a given contract can be stable, even when the cash price fluctuates.

8. For an intracommodity spread in a full carry good, explain why the bull spread is to be long the distant expiration and short the nearby expiration.

The perfect markets version of the Cost-of-Carry Model simply asserts:

$$F_{0,d} = F_{0,n}(1 + C)$$

where $F_{0,d}$ and $F_{0,n}$ are the distant and nearby futures prices and C is the cost-of-carry between the two expirations. A bull spread is designed to profit from generally rising prices. If$C > 0$, the distant futures price must rise more than the nearby futures price. Therefore, the bullish spread trader must buy the distant futures and sell the nearby futures.

9. Consider the crack spread. Assume that you believe a major technological breakthrough will be announced soon that will show how to radically reduce the cost of oil refining. How would you trade energy futures to exploit this expectation?

By saying that the breakthrough will be announced soon, we imply that the market is unaware of the change in technology, and market prices do not yet reflect the new technology. When the new technology is announced, the difference in price between crude oil and the refined product should narrow, because the difference is essentially due to the cost of refining or cracking the crude oil. To profit from this development, we would need to sell the crack (sell the refined product and buy the crude). When the breakthrough becomes public, the difference will narrow, and the trade will show a profit. (Note: This example might provide a good opportunity to discuss the law and ethics of trading on inside information.)

10. Explain how to measure the hedging effectiveness of a risk-minimization hedging strategy. Be sure to contrast in-sample and out-of-sample measures of effectiveness.

For risk-minimization hedging strategies, hedging effectiveness is measured by the R^2 from regressing the price change of the spot good on the price change of the futures contract. For the in-sample data used to estimate the hedge ratio, the R^2 shows the percentage of the variability in the spot instrument price changes that can be related to changes in the futures price. As such, the R^2 shows the percentage of price variability in the spot instrument that could have been eliminated by holding the futures position given by the hedge ratio. However, all of this information is about what could have happened in history. Risk-minimization hedging then applies this hedge ratio estimated on historical data to future price in the actual market. The R^2 gives an excellent guide to the risk reduction that could have been achieved in the past. If the same price relationships prevail over the future hedging period, then the same percentage of risk reduction can be achieved. However, this is unlikely to be the case. After all, in the estimation period, the hedge ratio is chosen to maximize R^2, so in the future period it is unlikely that the same hedge ratio will maximize hedging effectiveness. Therefore, we should expect the percentage risk reduction in the estimation period to exceed the percentage risk reduction actually achieved in the out-of-sample future hedging period.

11. The Mesa Rosa Tortilla Company is a large producer of tortilla chips whose main ingredient is corn. The demand for Mesa Rosa corn chips is seasonal with the largest demand occurring mid-November through the end of December. Production schedules require acquisition of 25,000 bushels of corn in late September to meet the holiday season demand. Mesa Rosa management is concerned about the possibility that a rise in the price of corn between now and September could hurt profitability. Corn must be acquired at a price of $2.25/bushel or less to ensure profitability. The September corn futures contract (5,000 bushel quantity) is selling for $2.11/bushel.

(a) What can Mesa Rosa do to ensure its profitability?

Mesa can acquire corn today and store it until September, or Mesa can acquire corn using the September corn futures contract. Using the futures contract, they would buy 5 September contracts at $2.11/bushel.

(b) What risks does Mesa Rosa face in acquiring corn by their taking delivery of the futures contract? How should Mesa Rosa acquire the corn they need?

When September arrives Mesa can acquire the corn in one of two ways. First, they can take delivery of the corn via the futures contract. Unfortunately, the short side of the contract chooses the delivery location. This location may or may not be convenient for Mesa. The second alternative for acquiring corn eliminates this risk. In this method, Mesa acquires corn in the spot market and enters a reversing trade in the futures market. If the futures price has moved since the initiation of the hedge, any gains (losses) on the futures contract offset any losses (gains) in the cash market so that the effective price they pay for corn is $2.11.

(c) If the September spot price turns out to be $3.15/bushel, show Mesa Rosa's transactions in the corn cash and futures markets and calculate their net wealth change.

Mesa's long hedge:

Date	Cash Market	Futures Market
Today	Anticipate need for 25,000 bushels of corn in September Wants to pay $2.11/bushel or $52,750 total	Buy five 5,000 bushel September corn futures at $2.11/bushel

September	Spot price of corn is \$3.15/bushel. Mesa buys 25,000 bushels for \$78,750 Opportunity loss = \$52,750 − \$78,750 = − \$26,000	At maturity, the futures price equals the spot price. Sell five futures contracts at \$3.15/bushel Futures profit = 25,000 (\$3.15 − \$2.11) = \$26,000
	Net wealth change = 0	

12. It is August 10 and Farmer John is making final estimates of this year's wheat crop. His production is turning out to be much better than expected. This causes concern because if his production is better than expected, other farmers must be experiencing the same situation. The current spot price is \$2.25/bushel and the September wheat futures (5,000 bushels per contract) price is \$2.52/bushel. At the current spot price Farmer John would just break even with his anticipated 60,000 bushels. His wheat will not be ready to harvest until September.

(a) What can Farmer John do to ensure his profitability? Is this a long or a short hedge? Why?

Farmer John can sell his anticipated wheat production in the futures market. Any opportunity gains (losses) resulting from changing wheat prices will be offset by losses (gains) in the futures market. This is a short hedge because Farmer John is selling his production forward. The counterparty to his contract might be a producer acquiring wheat forward for whom the transaction would be a long hedge.

(b) At harvest time in September, Farmer John's concerns are realized in that the cash price has dropped to \$1.70/bushel. Compute Farmer John's net wealth change due to the drop in corn prices, assuming he hedged his anticipated production and his final yield was 60,000 bushels.

Date	**Cash Market**	**Futures Market**
August 10	Farmer John anticipates the production of 60,000 bushels of wheat, which he wishes to sell at \$2.52/bushel for a total of \$151,200	Sell 12 5,000-bushel September wheat futures contracts at \$2.52/bushel
September	Farmer John sells 60,000 bushels in the spot market at \$1.70/bushel for a total of \$102,000 Opportunity loss = (\$102,000 − \$151,200) = − \$49,200	At maturity, the futures price will equal the spot price. Farmer John buys 12 contracts at \$1.70/bushel. Futures profit = 60,000 (\$2.52 − \$1.70) = \$49,200
	Net wealth change = \$0	

(c) Suppose Farmer John's production turned out to be only 50,000 bushels. Compute his net wealth change.

Date	**Cash Market**	**Futures Market**
August 10	Farmer John anticipates the production of 60,000 bushels of wheat which he wishes to sell at \$2.52/bushel for a total of \$151,200	Sell 12 5,000 bushel September wheat futures contracts at \$2.52/bushel

September	Farmer John sells 50,000 bushels in the spot market at \$1.70/bushel for a total of \$85,000 Opportunity losses Price change = 60,000 (\$1.70 −\$2.52) = − \$49,200 Production variation = 10,000 × \$1.70 = − \$17,000 Total = −\$66,200	At maturity, the futures price will equal the spot price. Farmer John buys 12 contracts at \$1.70/bushel Futures profit = \$49,200
	Net wealth change = − \$17,000	

Farmer John's net wealth change is negative because he had anticipated 60,000 bushels of wheat production, but his final production was 10,000 bushels less. He could have sold those 10,000 bushels at \$1.70/bushel if he had them. This results in \$17,000 opportunity loss attributable to production variation.

13. Ace Trucking Lines has a fleet of 10,000 trucks that carry a variety of commodities throughout North America. One of its major costs of operation is diesel fuel. There is no futures contract traded on diesel fuel, but there are futures contracts traded on No. 2 heating oil, closely related to diesel, and unleaded regular gasoline. Both of these contracts are traded in quantities of 42,000 gallons/contract. Identify three factors related to Ace Trucking Line's situation that would make any hedging activity to be characterized as cross-hedging.

A cross-hedge is a hedge in which the commodity being hedged and the hedging instrument have dissimilar characteristics. These characteristics can relate to: the time span of the hedge and delivery data on the hedging instrument; the quantity hedged and the size of the underlying instrument; and the commodity being hedged and the commodity deliverable on the hedging instrument.

In Ace Trucking's case, most likely, all three of these characteristics apply. First, Ace Trucking would not maintain much inventory of fuel. Most fuel would be purchased through retailers. This makes it difficult to identify a delivery date as fuel is purchased on a continual basis. Second, Ace's demand for diesel fuel is likely to vary from exact multiples of 42,000 gallons. Hence the quantity being hedged and the quantity deliverable on the underlying contracts are likely to vary. Finally, there is no diesel contract. The closest contract would be the No. 2 heating oil. Hence the commodity being hedged is not the same as that deliverable on the hedging instrument.

Ace Trucking, if they would be interested in hedging their fuel price risk, might investigate risk-minimization hedging techniques. In this case they would find the futures contract whose price is most highly correlated with the price of diesel fuel and use that contract for hedging.

14. QT has a network of 150 gasoline outlets throughout the central United States. At any one time, the company has 1.125 million gallons of gasoline inventory. Derek Larkin has suggested that QT hedge the risk of their gasoline inventories. He says that the appropriate hedging technique would be risk minimization.

    (a) What is risk-minimization hedging?

In risk-minimization hedging, one trade futures contracts in the amount that will minimize the variation of the value of a portfolio composed of the cash position and the futures position. To determine the risk-minimizing hedge ratio, one regresses the price changes of the spot price against the hedging instrument's price changes over the same time period. The slope coefficient from the regression is the risk-minimizing hedge ratio. The regression result indicates the units of the hedging commodity to trade for each unit of the spot commodity.

(b) Derek estimates the following relationship between spot, S_t, and futures, F_t, prices using the nearby 42,000 gallon unleaded regular gasoline contract:

$$\Delta S_t = \alpha + \beta \Delta F_t + \epsilon_t$$

Derek's estimation gives the following results:

$$\alpha = 0.5231$$
$$\beta = 0.9217$$
$$R^2 = 0.88$$

Based on these results, what should QT do to hedge their inventory price risk?

QT has 1.125 million gallons of gasoline in inventory. Derek's results suggest a hedge ratio of 0.9217 gallons of futures for each gallon of inventory. Computing the number of contracts:
No. of contracts = 0.9217 (1,125,000 / 42,000) = 24.7
Derek's recommendation would be to sell 25 contracts to hedge QT's price risk.

(c) Derek also estimated the same relationship using the nearby 42,000 gallon No. 2 heating oil futures contract with the following results:

$$\alpha = 0.7261$$
$$\beta = 0.6378$$
$$R^2 = 0.55$$

Compare the results from the two regressions and comment on which contract would be most appropriate for hedging purposes.

Comparing the results of the two regressions, the most important consideration is the R^2. The R^2 tells the percentage of spot price change variation explained by changes in the futures price. A perfect hedging instrument would explain 100 percent of the price change variation. Failing that, Derek should recommend the hedging instrument with the highest R^2. In this case, the unleaded gasoline contract with its R^2 of 88 percent dominates the No. 2 heating oil contract with its R^2 of 55 percent.

15. Anton Beneke & Son, LLP is a medium-sized family farming operation located in North Central Iowa. The operators are eager to expand the portion of their operation devoted to producing improved varieties of certified soybean seed. To achieve their objective, Beneke & Son sell certified seed to local producers under the condition that, at harvest time, the crop will be sold back to Beneke & Son. As part of the commitment, Beneke & Son agree to pay a 50 cents/bushel premium over the local market price for the crop. The operators expect that the seeds will yield 15,000 bushels of soybeans. Because of their commitment to buy this quantity, the operators are exposed to rising soybean prices. How can Beneke & Son use the futures market to hedge their exposure to rising soybean prices? What factors can complicate their hedging strategy?

To lock in the cost of their anticipated purchase, Beneke & Son can establish a long futures position in October soybean futures. Since each contract calls for the delivery of 5,000 bushels, they go long three contracts. Beneke & Son need to be aware of differences in quality between the improved variety of soybeans in the field and the quality of soybeans specified in the futures contract. Quality differences mean that the price for the crop may not be perfectly correlated with the futures price. Also, their hedge construction assumes that the actual quantity produced will equal 15,000 bushels, the amount expected at the time the commitment is made.

A BOND PRIMER

1. Assume that you are a money manager with a large stock of cash that you will be investing in bonds. You anticipate a strong upward movement in interest rates in general. What do your beliefs imply about the kinds of bonds you will select for investment, particularly with respect to maturity and coupon?

If interest rates move up as you anticipate, all bond prices will fall. As a consequence, you would prefer bonds that will fall as little as possible. Bonds with short maturities and large coupon rates will move less than other kinds of bonds, so you can minimize the fall in the portfolio's value by buying short maturity large coupon bonds.

2. Again, assume that you expect sharply rising interest rates and you will buy one of the following two bonds: a 20-year 8 percent coupon bond or a 15-year $9\frac{3}{4}$ percent coupon bond. Does this give you enough information to make a decision? What else might you need to know?

Because these bonds are similar in their overall sensitivity to interest rates, it is difficult to choose without more information. If you expect rates to rise, you will prefer a bond with a low sensitivity to rate changes or a low duration bond. To choose between these two bonds, you would probably like to know the duration of the two bonds.

3. As an investor, you are trying to decide between two bonds as investments for a 4-year investment horizon. The first has a 12 percent coupon and matures in 3 years. The other is a zero coupon bond maturing in 5 years. Compare and contrast the risks associated with each bond and their suitability for your investment horizon.

With the short-maturity bond, the investor faces the risk of reinvesting cash throw-offs from the bond up to the horizon in 4 years. These reinvestment rates may be unattractively low when funds become available for investment. The zero coupon bond must be sold at the horizon date, and the price will depend on interest rates prevailing at that time. If rates are high, the receipts will be low.

4. Consider a 5-year pure discount bond with a face value of \$1,000 that yields 10 percent compounded annually. What is its price? What will its price be if interest rates suddenly rise to 11 percent? What will its price be if interest rates suddenly fall to 9 percent? Are the capital gain and loss the same?

$$P = \$1000/(1 + r)^t$$

$$\text{If } r = 0.10, \qquad P = \$1000/(1.1)^5 = \$620.92$$

$$\text{If } r = 0.11, \qquad P = \$1000/(1.11)^5 = \$593.45$$

$$\text{If } r = 0.09, \qquad P = \$1000/(1.09)^5 = \$649.93$$

Capital gain when rates go from 10 to 9 percent: \$649.93 – \$620.92 = \$29.01
Capital loss when rates go from 10 to 11 percent: \$593.45 – \$620.92 = −\$27.47
The capital loss is smaller, as stated in bond pricing principle number 5.

5. What is the price of a 3-year 8 percent annual coupon bond yielding 11 percent and having a face value of \$1,000? Assume annual compounding. What is the duration of the bond?

$$\begin{aligned} P &= 80/1.11 + 80/(1.11)^2 + 1080/(1.11)^3 \\ &= 72.07 + 64.93 + 789.69 = \$926.69 \\ D &= [1(72.07) + 2(64.93) + 3(789.69)]/\$926.69 = 2.7745 \end{aligned}$$

6. Consider again the 3-year 8 percent annual coupon bond yielding 11 percent and having a face value of \$1,000. Assume that the interest rates suddenly rise to 13 percent. Compute the new price of the bond by discounting the cash flows at the new rate and by using the duration price change formula. Are the two answers the same? Why or why not?

$$\begin{aligned} P &= 80/1.13 + 80/(1.13)^2 + 1080/(1.13)^3 \\ &= 70.80 + 62.65 + 748.49 \\ &= \$881.94 \end{aligned}$$

By the duration price change formula:

$$dP_i = -\,D_i\left(\frac{d\,(1 + r_i)}{1 + ri}\right)P_i$$

Therefore, $dP_i = -2.7745\ (0.02/1.11)\$925.69 = -\$46.28$. The new price is \$880.41. The two answers differ, because the duration price change formula holds only for infinitesimal changes in yields and the 2 percent jump is a discrete change. Expressed differently, the bond price is convex with respect to changes in interest rates, but the duration price change formula uses the derivative measured at the original yield and corresponding price. The discrepancy between \$881.94 and \$880.41 is due to convexity.

7. What is the duration of a pure discount bond yielding 8 percent and maturing in 3 years and having a face value of \$1,000?

The duration is 3 years. For any pure discount bond, the duration equals the term to maturity.

8. What is the duration of a 12 percent annual coupon bond maturing in 5 years and yielding 11 percent? Assume a \$1,000 face value.

First, we find the price and the present values of the individual payments:

$$\begin{aligned} P &= 120/1.11 + 120/(1.11)^2 + 120/(1.11)^3 + 120/(1.11)^4 + 1120/(1.11)^5 \\ &= 108.11 + 97.39 + 87.74 + 79.05 + 664.67 = \$1,036.96 \end{aligned}$$

The duration is:

$$\begin{aligned} D &= [1(108.11) + 2(97.39) + 3(87.74) + 4(79.05) + 5(664.67)]/1,036.96 \\ &= 4,205.66/1,035.96 = 4.06 \end{aligned}$$

9. Today you purchase a $1,000 face value bond paying an 8 percent annual coupon bond and maturing in 5 years for a purchase price of $930. Assuming that you are able to reinvest all coupons at 11 percent, what is your terminal wealth after 5 years? What is your RCYTM?

$$\begin{aligned}\text{Terminal wealth} &= \$80(1.11)^4 + \$80(1.11)^3 + \$80(1.11)^2 + \$80(1.11) + \$1080 \\ &= \$121.45 + \$109.41 + \$98.57 + \$88.80 + \$1080 = \$1{,}498.23\end{aligned}$$

$$\text{RCYTM} = 5\sqrt{\frac{\$1{,}498.23}{\$930}} - 1 = 0.100067$$

10. Assume you have an investment horizon of 5 years. You purchase a pure discount bond with a 5 year maturity and a face value of $1,000 for a purchase price of $621. Your friend purchases a 10 percent annual coupon bond with a face value of $1,000 at par. What is the current interest rate on each bond? Assuming that rates do not change, what will be the terminal wealth and RCYTM for each bond, assuming that your friend reinvests all coupons? Assume that immediately after, the purchase interest rates drop to 8 percent. What will be the terminal wealth and RCYTM for the two investments?

For the pure discount bond the yield is given by X in the following equation:

$$\begin{aligned}\$631(1 + X)^5 &= \$1{,}000 \\ X &= 0.09646\end{aligned}$$

For the coupon bond, the yield is 10 percent because the bond has a 10 percent coupon and trades at par. For the pure discount bond, the terminal wealth will be $1,000, the face value of the bond, and the RCYTM will be 0.09646. For the coupon bond, the terminal wealth will be $1,610.51 because all proceeds will be invested at the currently prevailing rate of 10 percent and $\$1{,}000(1.1)^5 = \$1{,}610.51$. Because all coupons are to be invested at the presently prevailing yield to maturity, the RCYTM also equals the same yield to maturity of 10 percent.

If reinvestment rates drop to 8 percent, the terminal wealth and RCYTM of the pure discount bond are unaffected. The maturity of the pure discount bond matches the investment horizon, so the bond is immunized, via Planning Period Immunization. For the coupon bond, the new terminal wealth will be $\$1{,}000(1.08)^5 = \$1{,}469.33$ and the RCYTM is given as:

$$\text{RCYTM} = 5\sqrt{\frac{\$1{,}469.33}{\$1{,}000}} - 1 = 0.08$$

The calculation is superfluous, because we know that the RCYTM must equal the reinvestment rate if the rate prevails over the entire holding period.

11. A bank has hired you as a consultant to advise it on its interest rate exposure. The bank has an asset portfolio of $1 million with a duration of 5 years, and the portfolio is currently yielding 12 percent. This asset portfolio is funded by a liability portfolio, also worth $1 million, with a duration of 1.5 years and yielding 10.5 percent. Your problem is to advise the bank on its risk exposure in case interest rates change by 1 percent in either direction. Analyze the resulting position of the bank for both of these cases.

We use the duration price change formula and we first consider a rise in rates:

$$dP_i = -D_i\left(\frac{d(1+r)}{1+r}\right)P_i$$

Let a denote the assets and l the liabilities. If rates rise, the change in the portfolio values will be:

$$dP_{\mathrm{a}} = -5(0.01/1.12)\ \$1{,}000{,}000 = -\$44{,}642.86$$
$$dP_l = -1.5(0.01/1.105)\ \$1{,}000{,}000 = -\$13{,}574.66$$

For a drop in rates of 1 percent we would have:

$$dP_a = -5(-0.01/1.12)\ \$1{,}000{,}000 = \$44{,}642.86$$
$$dP_l = -1.5(-0.01/1.105)\ \$1{,}000{,}000 = \$13{,}574.66$$

Thus, the value of the assets will change 3.29 times as much as the value of the liabilities. If rates rise, the value of the assets will fall, much more than the value of the liabilities, thereby eroding the equity value of the bank. By the same token, the bank could reap a speculative windfall if rates fall, instead of rise.

12. Assume that you are asked to manage a \$1 million immunized portfolio with a horizon date of 3 years. Two bonds are available to you: a 5-year pure discount bond yielding 10 percent and a 2-year 12 percent annual coupon bond yielding 12 percent. How would you make up the immunized portfolio from these two bonds?

To immunize with these two bonds, you need to commit funds to them in the proportion that will give a portfolio duration of 3 years. Let D_2 and D_5 equal the durations of the 2-year and 5-year bonds, respectively. Then:

$$D_2 = [1(\$120/1.12) + 2(\$1{,}120/1.12^2)]/\$1{,}000 = 1.8929$$

$D_5 = 5$, because the bond is a pure discount bond.

Let W be the weight given to the 5-year bond, and let $(1 - W)$ be the weight given to the 2-year bond. The problem is to choose W, such that:

$$D_p = WD_2 + (1 - W)D_5 = 3$$
$$1.8929W + 5(1 - W) = -3.1071W + 5 = 3$$
$$W = -2/-3.1071 = 0.6437$$

Therefore, to immunize, invest 64.37 percent (\$643,678) of the funds in the 2-year bond, and invest 35.63 percent (\$356,322) in the 5-year bond.

13. For the preceding problem, what will be the terminal wealth of the portfolio assuming that interest rates do not change for 3 years? What is the RCYTM over this period? Now assume that interest rates drop by 1 percent on both bonds. Calculate the terminal wealth at the end of 3 years. Do the same assuming a 2 percent rise in rates.

We assume that cash flows from each bond can be invested at that bond's yield to maturity. Therefore, the terminal wealth stemming from the 2-year bonds will be:

$$\begin{aligned}\$643{,}678(0.12)(1.12)^2 &+ \$643{,}678(1.12)(1.12)\\ &= \$96{,}891.56 + \$807{,}429.68\\ &= \$904{,}321.24\end{aligned}$$

At year 3, the investment in the 5-year bond will have grown for 3 years at 10 percent and it will be worth $\$356{,}322(1.10)^3 = \$474{,}264.58$. Assuming no change in rates, the portfolio will be worth:

$$\$904{,}321.24 + \$474{,}264.58 = \$1{,}378{,}585.82$$

The RCYTM $= (\$1{,}378{,}585.82/\$1{,}000{,}000)^{(1/3)} - 1 = 0.112956$
If rates dropped by 1 percent on both bonds immediately after purchase, the terminal value for the coupon bond's cash flows would be:

$$\begin{aligned}[\$643{,}678(0.12)](1.11)^2 &+ [\$643{,}678(1.12)](1.11)\\ &= \$95{,}169.08 + \$800{,}220.49\\ &= \$895{,}389.57\end{aligned}$$

The 5-year bond originally sold for $\$1{,}000/1.10^5 = 62.0921$ percent of par. Therefore, the original investment of \$356,322 implies a par value of \$573,860.14, if rates are 10 percent. With rates dropping by 1 percent, this par value to be received at year 5 will be worth $\$573{,}860.14/1.09^2 = \$483{,}006.60$ at year 3. Therefore, the terminal value of the entire portfolio will be \$895,389.57 + \$483,006.60 = \$1,378,396.17. This is virtually the same as the terminal value with no change in rates.

If rates rise by 2 percent, we have a similar result. The coupon bond will have a terminal value of:

$$\begin{aligned}[\$643{,}678(0.12)](1.14)^2 &+ [\$643{,}678(1.12)](1.14)\\ &= \$100{,}328.87 + \$821{,}848.07\\ &= \$922{,}230.94\end{aligned}$$

With rates at 12 percent on the pure discount bond, its value will be $\$573{,}860.14/1.12^2 = \$457{,}477.68$. The terminal value of the entire portfolio will be \$922,230.94 + \$457,477.68 = \$1,379,708.62. Again, this is virtually the same as the portfolio's value with no change in rates.

INTEREST RATE FUTURES: INTRODUCTION

1. A 90-day T-bill has a discount yield of 8.75 percent. What is the price of a \$1 million face value bill?

Applying the equation for the value of a T-bill, the price of a \$1 million face value T-bill is \$1,000,000 − DY(\$1,000,000)(DTM)/360, where DY is the discount yield and DTM = days until maturity. Therefore, if DY = 0.0875 the bill price is:

$$\text{Bill price} = \$1{,}000{,}000 - \frac{0.0875(\$1{,}000{,}000)(90)}{360} = \$978{,}125$$

2. The IMM Index stands as 88.70. What is the discount yield? If you buy a T-bill futures at that index value and the index becomes 88.90, what is your gain or loss?

The discount yield = 100.00 − IMM Index = 100.00 − 88.70 = 11.30 percent. If the IMM index moves to 88.90, it has gained 20 basis points, and each point is worth \$25. Because the price has risen and the yield has fallen, the long position has a profit of \$25(20) = \$500.

3. What is the difference between Position Day and First Position Day?

First Position Day is the first day on which a trader can initiate the delivery sequence on CBOT futures contracts. With the 3-day delivery sequence characteristics of T-bond futures, for example, First Position Day is the second to last business day of the month preceding the contract's expiration month. For example, May 30 is the first position day for the JUN contract, assuming that May 30–June 1 are all business days. Position Day is functionally the same, but it is not the first day on which a trader can initiate the sequence. For example, assuming June 10–12 are all business days, the Position Day could be June 10, with actual delivery occurring on June 12.

4. A \$100,000 face value T-bond has an annual coupon rate of 9.5 percent and paid its last coupon 48 days ago. What is the accrued interest on the bond?

$$\text{Accrued interest} = \$100{,}000\,(0.095/2)(48/182.5) = \$1{,}249.32.$$

Note that we assume that the half-year has 182.5 days. There are specific rules for determining the number of days in a half-year.

5. What conditions are necessary for the conversion factors on the CBOT T-bond contract to create favorable conditions for delivering one bond instead of another?

There is one market condition under which the conversion factor method creates no bias: the yield curve is flat and all rates are 6 percent. Under any other circumstance, the conversion factor method will give incentives to deliver some bonds in preference to others.

6. The JUN T-bill futures IMM Index value is 92.80, while the SEP T-bill has a value of 93.00. What is the implied percentage cost-of-carry to cover the period from June to September?

For the JUN contract the implied invoice amount is:

$$\text{Bill price} = \$1{,}000{,}000 - 0.0720(\$1{,}000{,}000)(90)/360 = \$982{,}000$$

Paying this amount in June will yield \$1,000,000 in September when the delivered T-bill matures. Therefore, the implied interest rate is:

$$\text{Implied cost-of-carry} = \frac{\$1{,}000{,}000}{\$982{,}000} - 1 = 0.018330$$

Therefore, the implied interest rate to cover the June–September period is 1.8330 percent. (The information about the SEP futures is just a distraction.)

7. A spot 180-day T-bill has a discount yield of 9.5 percent. If the implied bank discount rate for the next 3 months is 9.2 percent, what is the price of a futures that expires in 3 months?

To exclude arbitrage, the strategy of holding the 180-day T-bill must give the same return as investing for the first 3 months at the repo rate and taking delivery on the futures to cover the second three month period to make up the 180-day holding period.

Assuming $1,000,000 face values, the price of the 180-day bill must be:

$$\text{Bill price} = \$1,000,000 - 0.095(\$1,000,000)\,(180)/360 = \$952,500$$

This is a ratio of face value to price of 1.049869. With a bank discount yield of 9.2 percent, a bill that pays $1,000,000 in 90 days must have a price of:

$$\text{Bill price} = \$1,000,000 - 0.092(\$1,000,000)\,(90)/360 = \$977,000$$

giving a ratio of face value to price of 1.023541. Therefore, the ratio of the $1,000,000 face value to the price of the futures, *X*, must satisfy the following equation:

$$1.049869 = 1.023541\,X$$

$X = 1.025722$. Therefore, the futures price must be $1,000,000/1.025722 = $974,923, or $974,925 rounded to the nearest $25 tick.

7. For the next three futures expirations, you observe the following Eurodollar quotations:

MAR	92.00
JUN	91.80
SEP	91.65

What shape does the yield curve have? Explain.

These IMM Index values imply Eurodollar add-on yields of 8 percent, 8.2 percent, and 8.35 percent, respectively. These rates apply to the following periods: March–June, June–September, and September–December, respectively. Essentially, we may regard these futures rates as forward rates. If forward rates increase with futurity, the yield curve must be upward sloping.

9. Assume that the prices in the preceding problem pertain to T-bill futures and the MAR contract expires today. What should be the spot price of a 180-day T-bill?

To avoid arbitrage, the spot price of a 180-day T-bill must give the same return as taking delivery on the futures today and taking a long position in the JUN contract with the intention of taking delivery of it as well. For convenience, we assume a T-bill with a face value of $1,000,000.

With the strategy of two 90-day positions, a trader would need to take delivery of both one full JUN contract and enough bills on the MAR contract to pay the invoice amount on the JUN contract. For the JUN contract, the IMM Index value implies a delivery price of $1,000,000 − 0.0820($1,000,000)(90)/360 = $979,500. For the MAR contract, the delivery price is $1,000,000 − 0.08($1,000,000)(90)/360 = $980,000. But the trader requires only $979,500 (or 97.95 percent) of the JUN contract. Therefore, for the short-term strategy, the current price of \$1,000,000 in September is 0.9795(\$980,000) = \$959,910. To avoid arbitrage, the 180-day bill must also cost \$959,910, implying a discount yield of 0.08018.

10. The cheapest-to-deliver T-bond is a 10 percent bond that paid its coupon 87 days ago and is priced at 105–16. The conversion factor of the bond is 1.0900. The nearby T-bond futures expires in 50 days and the current price is 98–00. If you can borrow or lend to finance a T-bond for a total interest outlay of 2 percent over this period, how would you transact? What if you could borrow or lend for the period at a total interest cost of 3 percent? What if you could borrow for the period at a total interest cost of 3 percent and earn 2 percent on an investment over the whole period? Explain.

To know how to respond to these quotations requires knowing the invoice amount that can be obtained for the bond and comparing this with the cost-of-carrying the bond to delivery on the futures. For convenience, we assume a face value that equals the contract size of \$100,000. First, the accrued interest (assuming a 182.5-day half-year) is:

$$AI = (87/182.5)(0.5)(0.10)\$100{,}000 = \$2{,}383.56$$

At expiration, the accrued interest will be:

$$AI = (137/182.5)(0.5)(0.10)\$100{,}000 = \$3{,}753.42$$

For this bond and the futures price of 98–00, the invoice amount will be:

$$\text{Invoice amount} = 0.9800(\$100{,}000)(1.09) + \$3{,}753.42 = \$110{,}573.42$$

Buying the bond and carrying it to delivery (at 2 percent interest for the period) costs:

$$(\$105{,}500 + \$2{,}383.56)(1.02) = \$110{,}041.23$$

Because the cost of acquiring and carrying the bond to delivery is less than the expected invoice amount, the trader could engage in a cash-and-carry arbitrage. Buying the bond and carrying it to delivery costs \$110,041.23 and nets a cash inflow of \$110,573.42. This gives an arbitrage profit. (Notice that the actual invoice amount is unknown, but transacting at the futures price of 98–00 guarantees the profit we have computed. This profit may be realized earlier depending upon the daily settlement cash flows.)

If the cost of carrying the bond for the next 50 days is 3 percent instead of 2 percent, the total cost of acquiring and carrying the bond will be:

$$(\$105{,}500 + \$2{,}383.56)(1.03) = \$111{,}120.07$$

This cost exceeds the expected invoice amount, so the cash-and-carry trade will not work for a 3 percent total cost-of-carry over the period.

Ignoring the short seller's options to choose the deliverable bond and the delivery date within the delivery month, the following reverse cash-and-carry strategy will be available with the 3 percent financing rate. The trader can buy the futures, borrow the bond and sell it short, and invest the proceeds

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to earn $111,120.07 by delivery. The short, we assume, obligingly delivers the same bond on the right day for the invoice amount of $110,573.42 and the profit is: $111,120.07 − $110,573.42 = $546.65

If the trader can borrow at 3 percent and lend at 2 percent, these prices create no arbitrage opportunities. The cash-and-carry strategy is too expensive, because buying and carrying the bond costs $111,120.07, more than the invoice amount of $110,573.42. The reverse cash-and-carry strategy is also impractical, because it nets only $110,041.23, less than the invoice amount of $110,573.42.

11. You expect a steepening yield curve over the next few months, but you are not sure whether the level of rates will increase or decrease. Explain two different ways you can trade to profit if you are correct.

If the yield curve is to steepen, distant rates must rise relative to nearby rates. If this happens, we can exploit the event by trading just short-term instruments. The yield on distant expiration short-term instruments must rise relative to the yield on nearby expiration short-term instruments. Therefore, one should sell the distant expiration and buy the nearby expiration. This strategy could be implemented by trading Eurodollar or T-bill futures.

As a second basic technique, one could trade longer-term T-bonds against shorter maturity T-notes. Here the trader expects yields on T-bonds to rise relative to yields on T-notes. Therefore, the trader should sell T-bond futures and buy T-note futures. Here the two different contracts can have the same expiration month.

12. The Iraqi invasion of Alaska has financial markets in turmoil. You expect the crisis to worsen more than other traders suspect. How could you trade short-term interest rate futures to profit if you are correct? Explain.

Greater than expected, the turmoil might result in rising yields on interest rate futures. To exploit this event, a trader could sell futures outright. A second result might be an increasing risk premium on short-term instruments. In this case, the yield differential between Eurodollar and T-bill futures might increase. To exploit this event, the trader could sell Eurodollar futures and buy T-bill futures of the same maturity.

13. You believe that the yield curve is strongly upward sloping and that yields are at very high levels. How would you use interest rate futures to hedge a prospective investment of funds that you will receive in 9 months? If you faced a major borrowing in 9 months, how would you use futures?

If you think yields are near their peak, you will want to lock-in these favorable rates for the investment of funds that you will receive. Therefore, you should buy futures that will expire at about the time you will receive your funds. The question does not suggest whether you will be investing long term or short term. However, if the yield curve is strongly upward sloping, it might favor longer-term investment. Consequently, you might buy T-bond futures expiring in about 9 months.

If you expect to borrow funds in 9 months you may not want to use the futures market at all. In the question, we assume that you believe rates are unsustainably high. Trading to lock-in these rates only ensures that your borrowing takes place at the currently very high effective rates. Given your beliefs, it might be better to speculate on falling rates.

14. The spot rate of interest on a corporate bond is 11 percent, and the yield curve is sharply upward sloping. The yield on the T-bond futures that is just about to expire is 8 percent, but the yield for

the futures contract that expires in 6 months is 8.75 percent. (You are convinced that this difference is independent of any difference in the cheapest-to-deliver bonds for the two contracts.) In these circumstances, a corporate finance officer wants to lock-in the current spot rate of 11 percent on a corporate bond that her firm plans to offer in 6 months. What advice would you give her?

Reform your desires to conform to reality. The yield curve is upward sloping and the spot corporate rate is 11 percent. Therefore, the forward corporate rate implied by the yield curve must exceed 11 percent. Trading futures now to lock-in a rate for the future locks in the rate implied by the yield curve, and that rate will exceed 11 percent. Consequently, she must expect to lock in a rate above the current spot rate of 11 percent.

15. Helen Jaspers was sitting at her trading desk watching the T-bill spot and futures market prices. Her firm was very active in the T-bill market, and she was eager to make a trade. The quote on the T-bill having 120 days from settlement to maturity was 4.90 percent discount yield. This bill could be used for the September 20 delivery on the September T-bill futures contract which was trading at 95.15. The quote on the T-bill maturing on September 20, having 29 days between settlement and maturity, was 4.70 percent discount yield.

    (a) Compute the T-bill and futures prices per dollar of face value.

T-bills are quoted in bank discount yield (DY), and T-bill futures are quoted in 100 – DY. These must be converted to dollars using the following formula:

$$\mathrm{P} = \left(1 - \frac{\mathrm{DY} \times n}{360}\right)\mathrm{FV}$$

where n is the number of days to settlement to maturity and FV is the face value.

Compute prices per dollar of face value as shown.
120-day T-bill quoted at 4.90 percent DY:

$$\mathrm{P}_{120} = \left(1 - \frac{0.049 \times 120}{360}\right)\$1 = \$0.9837$$

29-day T-bill quoted at 4.70 percent DY:

$$\mathrm{P}_{29} = \left(1 - \frac{0.047 \times 29}{360}\right)\$1 = \$0.9962$$

The futures contract on its delivery date will call for a T-bill with 91 days from settlement to maturity. The discount yield on this T-bill is currently:

$$\mathrm{DY} = 100 - 95.15 = 4.85 \text{ percent}$$

The dollar price of the T-bill is:

$$F_{0,t} = \left(1 - \frac{0.0485 \times 91}{360}\right)\$1 = \$0.9877$$

(b) Compute the implied repo rate. Could the implied repo rate be used to tell Helen where arbitrage profits are possible? If so, how?

The implied repo rate, C, is:

$$C = (F_{0,t}/S_0) - 1$$

In this case, C is:

$$C = (0.9877/0.9837) - 1 = 0.4066 \text{ percent}$$

What does this tell us? It tells us the return we get over the next 29 days if we buy the 120-day T-bill and deliver it against the September futures contract. This implied repo rate is compared with the 29-day borrowing/lending rate to point out opportunities for arbitrage. If C is greater than the 29-day financing rate, then the appropriate arbitrage is a cash-and-carry. If C is less than the 29-day financing rate, then a reverse cash-and-carry would be appropriate. The borrowing/lending rate over this 29-day time period is:

$$(\text{FV}/P_{29}) - 1 = (1/0.9962) - 1 = 0.3814 \text{ percent}$$

Since the implied repo rate is greater than the cost of financing, we have the possibility of cash-and-carry arbitrage.

(c) What would be the arbitrage profit from a $1 million transaction?

A cash-and-carry arbitrage would call for borrowing for 29 days, buying the 120-day bill and selling the 120-day bill forward using the futures contract. On the delivery date, the 120-day bill, then having 91 days to maturity would be delivered against the futures contract. The proceeds would be used to payoff the 29-day borrowing.

Date	**Cash Market**	**Futures Market**
Today	Borrow $0.9837 million for 29 days at 4.70% DY Buy $1 million FV 120-day T-bill at a price of $0.9837 million Net investment = $0	Sell $1 million 91-day T-bills for September 10 delivery
September 10	Payoff borrowing; amount due = 0.9837/0.9962 = $0.9874 million	Deliver 91-day T-bill against the futures contract; receive $0.9877 million
	Net profit = ($0.9877 – $0.9874) million = $300	

16. Angela Vickers has the responsibility of managing Seminole Industries' short-term capital position. In 3 weeks, Seminole will have a cash inflow that will be rolled over into a $10 million 90-day T-bill. There is a T-bill futures contract that calls for delivery at the same time as the anticipated cash inflow. It is trading at 94.75. There have been signs that the financial markets are calming and that interest rates might be falling.

(a) What type of hedge might Angela employ?

Angela might employ a long hedge of the anticipated $10 million cash inflow. This would be accomplished by buying 10 T-bill futures contracts with a delivery date matching the anticipated cash inflow. The rate she would be locking in is:

$$100 - 94.75 = 5.25 \text{ percent discount yield.}$$

(b) Three weeks in the future, interest rates are actually higher. The 90-day T-bill discount yield is 6.00 percent. Calculate Seminole's net wealth change if the position is left unhedged.

Three weeks before the cash inflow, Ms Vickers would have been able to lock in a 5.25 percent discount yield using the futures contract. This would have allowed her to purchase $10 million of face value for:

$$\text{Anticipated price} = \left(1 - \frac{0.0525 \times 90}{360}\right)\$10{,}000{,}000 = \$9{,}868{,}750$$

When it is time to actually invest, interest rates have risen to 6 percent discount yield. Then the price of $10 million of face value costs:

$$\text{Realized price} = \left(1 - \frac{0.06 \times 90}{360}\right)\$10{,}000{,}000 = \$9{,}850{,}000$$

Seminole had an opportunity gain of $18,750.

(c) Calculate Seminole's net wealth change if the position is hedged.

If the position had been hedged, Seminole would have been long futures that were bought at 5.25 percent discount yield, or $9,868,750. This results in a loss of $18,750. The loss in the futures market is offset by the opportunity gain in the cash market so that the net wealth change is $0.

Date	**Cash Market**	**Futures Market**
Today	Anticipate the purchase of $10 million in T-bills for $9,868,750	Buy $10 million of T-bill futures at $9,868,750
3 weeks	Buy $10 million in T-bills for $9850,000	Sell $10 million T-bill futures contracts at $9,850,000
	Opportunity gain = $18,750	Futures loss = -$18,750
	Net wealth change = $0	

(d) Was the hedge a mistake?

In hindsight, Seminole would have been better-off unhedged, but hindsight is 20/20. *Ex ante* the concern was the risk of falling interest rates. Seminole viewed the 5.25 percent discount yield as acceptable for the investment and wanted to guarantee it. Therefore, *ex ante*, the hedge was not a mistake.

17. Fred Ferrell works for ABC Investments. As part of ABC's investment strategy, Fred is charged with liquidating $20 million of ABC's T-bill portfolio in 2 months. Fred has identified $20 million of T-bills that would be deliverable against the March T-bill futures contract at the time of liquidation.

The price of the futures contract is 94.50. Fred is losing sleep at night over concerns about future economic uncertainty that could lead to a rise in interest rates.

(a) What action can Fred take to reduce ABC's exposure to interest rate risk?

Fred could enter a short hedge using 20 March T-bill futures contracts. Since ABC anticipates liquidating \$20 million in T-bills, ABC can lock in a liquidation price based on the $100 - 94.50 = 5.5$ percent discount yield.

(b) At the time of liquidation, the price of the 90-day T-bill has risen to 5.25 percent discount yield. Compute the change in ABC's net wealth that would have occurred if Fred failed to hedge the position.

If Fred does not hedge the position, ABC's net wealth change will be an opportunity gain or loss. ABC had the opportunity to lock in a 5.5 percent discount yield but did not. The proceeds of \$20 million face value at 5.5 percent is:

$$\text{Anticipated proceeds} = \left[1 - \frac{0.055 \times 90}{360}\right] \times \$20{,}000{,}000 = \$19{,}275{,}000$$

The realized price is calculated using 5.25 percent discount yield:

$$\text{Realized proceeds} = \left(1 - \frac{0.0525 \times 90}{360}\right)\$20{,}000{,}000 = \$19{,}737{,}500$$

The difference is the opportunity gain/loss:

$$\begin{aligned}\text{Realized proceeds} - \text{anticipated proceeds} &= \$19{,}737{,}500 - \$19{,}725{,}000 \\ &= 12{,}500 \text{ gain}\end{aligned}$$

(c) Compute the change in ABC's net wealth that would have occurred, assuming Fred hedged the position.

If Fred had hedged the position, he would have sold \$20 million of face value using the futures contracts at a price of \$19,725,000, and he would have closed that position at a price of \$19,737,500. There would have been a \$12,500 loss in the futures market. The net wealth change of ABC would then be zero because the loss in the future market would have exactly offset the opportunity gain in the cash market.

Date	Cash Market	Futures Market
Today	Anticipate selling \$20 million of T-bills for \$19,725,000	Sell \$20 million T-bill futures at \$19,725,000
March	Sell \$20 million T-bills for \$19,737,500. Opportunity gain = \$12,500	Buy \$20 million T-bill futures at \$19,737,500. Loss = -\$12,500
	Net wealth change = \$0	

18. Alex Brown is a financial analyst for B.I.G. Industries. He has been given responsibility for handling the details of refinancing a \$500 million long-term debt issue that will be rolled over in

May (5 months from today). The new 8 percent, 30-year debt, with a face value of $500 million, is anticipated to have a 75 basis-point default risk premium over the yield on the 30-year T-bond. The 30-year T-bond is currently trading at 5.62 percent. Alex sees this risk premium as typical for corporate debt of a quality similar to B.I.G.'s debt. Alex looks at the June T-bond and notices that it is trading at 123–25. He is concerned that changing interest rates between now and May could have a negative impact on the refinancing cash flow.

(a) Assuming no interest rate changes, what are B.I.G.'s anticipated proceeds from refinancing?

If interest rates do not change over the next 5 months, B.I.G. Industries can expect to price their bonds at:

$$\text{YTM}_{\text{B.I.G.}} = 5.62 + 0.75\% = 6.37\%$$

The proceeds of $500 million face with an 8 percent coupon, 30 years to maturity and a yield to maturity of 6.37 percent would be:

$$\text{Anticipated proceeds} = \sum_{t=1}^{60} \frac{0.04(500{,}000{,}000)}{(1 + 0.0637/2)^t} + \frac{500{,}000{,}000}{(1 + 0.0637/2)^{60}} = \$608{,}443{,}959$$

(b) What can Alex do to reduce the refinancing risks faced by B.I.G. Industries?

Alex could reduce the risk by selling T-bond futures. Alex's concern is rising interest rates. As interest rates rise, the proceeds from the debt issue are diminished. These diminished proceeds could be offset by profits made on the short position in the futures market because as rates rise, the futures prices fall and the short position makes money. Naively, Alex could sell $500 million in the T-bond futures contract to hedge the risk.

(c) At the time of refinancing, the 30-year T-bond yield is 5.80 percent, the T-bond futures price is 121-09, and B.I.G.'s new debt issue is priced to yield 6.75 percent. Compute the realized proceeds from the refinancing.

When the debt is issued, it is issued with a yield to maturity of 6.75 percent. Interest rates have risen. The proceeds are given by:

$$\text{Realized proceeds} = \sum_{t=1}^{60} \frac{0.04(500{,}000{,}000)}{(1 + 0.0675/2)^t} + \frac{500{,}000{,}000}{(1 + 0.0675/2)^{60}} = \$579{,}955{,}566$$

This is an opportunity loss of $28,488,392.

(d) Assuming Alex sold $500 million in T-bond futures at 123–25 to hedge the refinancing and liquidated the futures position when the refinancing took place, find the profit from the futures trade, and evaluate the net wealth change due to the change in the refinancing rate and the futures trade.

The futures profit is:

$$\text{Futures profit} = \$5{,}000{,}000{,}000\left[\left(123 + \frac{25}{32}\right)\% - \left(121 + \frac{9}{32}\right)\%\right] = \$12{,}500{,}000$$

This profit only partially offsets the opportunity loss in the cash market. The net wealth change is:

$$\text{Net wealth change} = -\ \$28{,}488{,}392 + \$12{,}500{,}000 = -\$15{,}998{,}392$$

Date	Cash Market	Futures Market
Today	Anticipate issuing debt at 6.37% for proceeds of $608,443,959	Sell $500 million in T-bond futures at nominal price of $618,906,250
May	Issue debt with yield of 6.75% for proceeds of $579,955,566	Buy $500 million in T-bond futures at nominal price of $606,406,250
	Opportunity loss = −$28,488,392	Gain = $12,500,000
	Net wealth change = −$15,998,392	

(e) Discuss possible reasons why the net wealth change is not zero.

There are at least two reasons that hedge did not do a better job of offsetting the cash market risk:

Cross-hedge. This was a cross-hedge for several reasons. First, Alex was hedging the interest rates of a corporate bond, but the hedging instrument was a T-bond. Second, the cheapest-to-deliver T-bond may not be a 30-year bond. It may have as little as 15 years to maturity. This will make the price sensitivity of the futures contract and the corporate bond differ. Third, the delivery date on the T-bond futures contract is June, but the refinancing is taking place in May.

Faulty expectations. First Alex expected the default yield spread to stay fixed at 75 basis points, but it increased with the rise in interest rates to 6.75 percent giving a rise of 95 basis points. Second, Alex expected the pricing of the T-bond futures contract to react to interest rates in the same manner as the 30-year T-bond, but in reality the reactions of the T-bond futures price was not as strong as the reaction of the 30-year T-bond price.

While the hedge was not perfect, it did offset some of the refinancing price risk.

INTEREST RATE FUTURES: REFINEMENTS

1. Explain the risks inherent in a reverse cash-and-carry strategy in the T-bond futures market.

The reverse cash-and-carry strategy requires waiting to receive delivery. However, the delivery options all rest with the short trader. The short trader will initiate delivery at his or her convenience. In the T-bond market, this exposes the reverse cash-and-carry trader to receiving delivery at some time other than the date planned. Also, with so many different deliverable bonds, the reverse cash-and-carry trader is unlikely to receive the bond he or she desires. (These factors are fairly common for other commodities as well.) In the T-bond futures market, the short trader holds some special options, such as the wildcard and end-of-month options. The reverse cash-and-carry trader suffers the risk that the short trader will find it advantageous to exploit the wildcard play or exercise the end-of-month option.

2. Explain how the concepts of quasi-arbitrage help to overcome the risks inherent in reverse cash-and-carry trading in T-bond futures.

In pure reverse cash-and-carry arbitrage, the trader sells the bond short and buys the future. The trader thereby suffers risk about which bond will be delivered and the time at which it will be delivered. If the trader holds a large portfolio of bonds and sells some bond from inventory to simulate the short sale, these risks are mitigated. Receiving a particular bond on delivery is no longer so crucial to the trader's cash flows; after all, whichever bond is delivered will merely supplement the trader's portfolio. Further, the timing of delivery presents fewer problems to the quasi-arbitrage trader. In selling a bond from inventory, as opposed to an actual short sale, the trader did not need to worry about financing the short sale for a particular time. Therefore, the selection of a particular delivery date by the short futures trader is less critical. While quasi-arbitrage helps to mitigate the risks associated with the reverse cash-and-carry trade, risks still remain, particularly the risks associated with the short trader's options.

3. Assume that economic and political conditions are extremely turbulent. How would this affect the value of the seller's options on the T-bond futures contract? If they have any effect on price, would they cause the futures price to be higher or lower than it otherwise would be?

Generally, options are more valuable the greater the price risk inherent in the underlying good. This is certainly true for the seller's options on the T-bond futures contract. To see this most clearly, we focus on the wildcard option. Exploitation of the wildcard option depends on a favorable price development on any position day between the close of futures trading and the end of the period to announce delivery at 8 p.m., Chicago time. If markets are turbulent, there is a greater chance that something useful will occur in that time window on some day in the delivery month. The greater value of the seller's options in this circumstance would cause the futures price to be lower than it otherwise would be.

4. Explain the difference between the wildcard option and the end-of-the-month option.

The wildcard option is the seller's option to initiate the delivery sequence based on information generated between the close of futures trading and 8 p.m., Chicago time, the time by which the seller must initiate the delivery sequence for a given day. The settlement price determined at the close of trading is the price that will be used for computing the invoice amount. Trading of the T-bond futures contract ceases on the eighth to last business day of the expiration month, and the settlement price on that day is used to determine the invoice amount for all deliveries. Any contracts not closed by the end of the trading period must be fulfilled by delivery. Even though the short trader must make delivery in this circumstance, the short trader still possesses an end-of-the-month option. The short trader can choose which day to deliver and can choose which bond to deliver. The short trader will deliver late in the month if the rate of accrual on the bond planned for delivery exceeds the short-term financing rate at which the bond is carried. Also, changing market conditions can change which bond will be cheapest-to-deliver, and the right to wait and choose a later delivery date has value to the short trader.

5. Some studies find that interest rate futures markets were not very efficient when they first began but that they became efficient after a few years. How can you explain this transition?

The growing efficiency of these markets seems to be due to a market seasoning or maturation process. When these contracts were first initiated, it appears that some of their nuances were not fully appreciated. In particular, the complete understanding of the importance of the seller's options seems to have emerged only slowly.

6. Assume you hold a T-bill that matures in 90 days, when the T-bill futures expires. Explain how you could transact to effectively lengthen the maturity of the bill.

Buy the T-bill futures that expires in 90 days. After this transaction, you will be long a spot 90-day bill, and you will hold (effectively) a spot position in a 90-day bill to begin in 90 days. The combination replicates a 180-day bill.

7. Assume that you will borrow on a short-term loan in 6 months, but you do not know whether you will be offered a fixed rate or a floating rate loan. Explain how you can use futures to convert a fixed to a floating rate loan and to convert a floating rate to a fixed rate loan.

For convenience, we assume that the loan will be a 90-day loan. If the loan is to be structured as a floating rate loan, you can convert it to a fixed rate loan by selling a short-term interest rate futures contract (Eurodollar or T-bill) that expires at the time the loan is to begin. The rate you must pay will depend on rates prevailing at the time of the loan. If rates have risen, you must pay more than anticipated. However, if rates have risen, your short position in the futures will have generated a profit that will offset the higher interest you must pay on the loan.

Now assume that you contract today for a fixed interest rate on the loan. If rates fall, you will be stuck paying a higher rate than the market rate that will prevail at the time the loan begins. To convert this fixed rate loan to a floating rate loan, buy an interest rate futures that expires at the time the loan is to begin. Then, if rates fall, you will profit on the futures position, and these profits will offset the higher than market rates you are forced to pay on your fixed rate loan.

8. You fear that the yield curve may change shape. Explain how this belief would affect your preference for a strip or a stack hedge.

If the yield curve is to change shape, rates on different futures expirations for the same interest rate futures contract may change by different amounts. In this case, it is important to structure the futures hedge so that the futures cash flows match the exposure of the underlying risk more closely. Thus, if the cash market exposure involves the same amount at regular intervals over the future, a strip hedge will be more effective against changing yield curve shapes.

9. A futures guru says that tailing a hedge is extremely important because it can change the desired number of contracts by 30 percent. Explain why the guru is nuts. How much can the tailing factor reasonably change the hedge ratio?

To tail a hedge, one simply reduces the computed hedge ratio by discounting it at the risk-free rate for the time of the hedge. For convenience, we assume that the untailed computed hedge ratio is 1.0. If the hedging period is 1 year, a 30 percent effect would require an interest rate of 43 percent, because $0.7 = (1/1.43)$. If the hedging horizon is long, say a full 2 years, the interest rate would still have to be 19.52 percent to generate the 30 percent effect, because $(1.1952)^2 = 1/0.7$. Thus, it seems extremely improbable that the tailing effect could be so large.

10. We have seen in Chapter 4 that regression-based hedging strategies are extremely popular. Explain their weaknesses for interest rate futures hedging.

First, regression-based hedging (the RGR Model) involves statistical estimation, so the technique requires a dataset for both cash and futures prices. This data may sometimes be difficult to acquire, particularly for an attempt to hedge a new security. Second, the RGR Model does not explicitly consider the differences in the sensitivity of different bond prices to changes in interest rates, and this can be a very important factor. The regression approach does include the different price sensitivities indirectly, however, since their differential sensitivities will be reflected in the estimation of the hedge ratio. Third, any cash bond will have a predictable price movement over time, and the RGR Model does not consider this change in the cash bond's price explicitly. However, the sample data used to estimate the hedge ratio will reflect this feature to some extent. Fourth, the RGR hedge ratio is chosen to minimize the variability in the combined futures-cash position over the life of the hedge. Since the RGR hedge ratio depends crucially on the planned hedge length, one might reasonably prefer a hedging technique focusing on the wealth position of the hedge when the hedge ends. After all, the wealth change from the hedge depends on the gain or loss when the hedge is terminated, not on the variability of the cash-futures position over the life of the hedge.

11. You estimate that the cheapest-to-deliver bond on the T-bond futures contract has a duration of 6.5 years. You want to hedge your medium-term Treasury portfolio that has a duration of 4.0 years. Yields are 9.5 percent on the futures and on your portfolio. Your portfolio is worth $120,000,000, and the futures price is 98–04. Using the PS Model, how would you hedge?

From the text, the PS hedge ratio is:

$$N = -\left(\frac{\mathrm{P_i\,MD_i}}{FP_\mathrm{F}\,\mathrm{MD_F}}\right)\mathrm{RYC}$$

For this problem, we are entitled to assume that RYC = 1.0 since no other value is specified. Applying this equation to our data gives:

$$N = -\frac{\$120{,}000{,}000 \times 3.652968}{\$98{,}125 \times 5.936073} = -752.5723$$

Therefore, the PS hedge would require selling about 753 T-bond futures.

12. Explain the relationship between the Bank Immunization Case and hedging with the PS Model.

Both bank immunization and the PS Model rely essentially on the concept of duration. A PS hedge finds the futures position to make the combined cash/futures position have a duration of zero. Similarly, in bank immunization with equal asset and liability amounts, the asset duration is set equal to the liability duration. For the combined balance sheet, the overall duration is effectively zero as well. Therefore, the two techniques are quite similar in approach, even if they use different instruments to achieve the risk reduction.

13. Compare and contrast the BP Model and the RGR Model for immunizing a bond portfolio.

The BP Model essentially is an immunization model that is suitable for the Bank Immunization Case. The BP hedge ratio is found empirically, but it is the hedge ratio that gives a price movement on the futures position that offsets the price movement on the cash position. As such, it is effectively reflecting the duration of the two instruments. (Notice that the BP Model does not really help with the planning period case, because it considers only the effect of a current change in rates, not a change over some hedging horizon.) The RGR Model does not really take duration into account in any direct fashion, so it is not oriented toward immunizing at all.

14. It was a hot day in August, and William had just completed the purchase of $20 million of T-Bills maturing next March and $10 million of T-Bills maturing in 1 month. The phone rang, and William was informed that the firm had just made a commitment to provide $30 million in

capital to a client in mid-December. If William had known this 20 min earlier, he would have invested differently.

(a) What risks does William face by using his present investments to meet the December commitment?

William faces several interest rate risks using his present investments to fund the anticipated cash outflow. These risks stem from the fact that the maturity of his present investments do not match the timing of the cash outflow. The proceeds of the $10 million of September T-Bills must be reinvested from September to December. William faces the risk that interest rates may fall between now and September, which would reduce the December proceeds from the reinvestment. The $20 million of March T-Bills are subject to price risk. If interest rates rise between now and December, the proceeds from the sale of the March T-Bills will be less than anticipated. Had he known about the firm's commitment earlier, he could have invested in $30 million of December maturity T-Bills.

(b) Using the futures markets, how can William reduce the risks of the December commitment? Show what transactions would be made.

William would like to lock in a reinvestment rate for September and a selling price for December. He can accomplish this by buying $10 million September futures contracts and selling $20 million December futures contracts. He is in effect lengthening the maturity of his September bills and shortening the maturity of his March bills. His transactions would be:

Date	Cash Market	Futures Market
Today		Buy $10 million September T-Bill futures; sell $20 million December T-Bill futures
September	Receive proceeds from $10 million September T-Bills; reinvest $10 million into 90-day T-Bills	Reverse September T-Bill futures position by selling $10 million September T-Bill futures
December	Receive proceeds from maturing December T-Bills; sell March T-Bills in cash market	Reverse December T-Bill futures position by buying $20 million December T-Bill futures

Gains and losses in the futures market will offset losses and gains in the cash market so that the total proceeds available to meet the December commitment will be as anticipated.

15. Handcraft Ale, Ltd. has decided to build additional production capacity in the United States to meet increasing demand in North America. Uma Peele has been given the responsibility of obtaining financing for the project. Handcraft Ale will need $10 million to carry the firm through the construction phase. This phase will last 2 years, at which time the $10 million debt will be repaid using the proceeds of a long-term debt issue.

Ms Peele gets rate quotes from several different London banks. The best quote is:

Variable:	Fixed:
LIBOR + 150 bp.	8.5 percent.

Each of these loans would require quarterly interest payments on the outstanding loan amount. Ms Peele looks up the current LIBOR rate and finds that it is 5.60 percent. The variable rate of 7.1

percent (5.60 + 1.5) looks very attractive, but Ms Peele is concerned about interest rate risk over the next 2 years.

(a) What could Ms Peele do to take advantage of the lower variable rate while at the same time have the comfort of fixed rate financing?

Ms Peele could hedge each of the anticipated interest rate adjustments using the Eurodollar futures contract. This would be a strip hedge. She would sell $10 million 3-month Eurodollar futures contracts with expirations in the months when interest rates are adjusted. For longer term loans this can present a problem. Contracts may not be traded with the proper expiration date, or the market may be very thin making it difficult to find a counterparty. Alternatively, Ms Peele could employ a stack hedge. In this case, she would stack the hedges for all interest rate adjustments on a single contract whose expiration is not so far into the future. For example, she could sell $70 million Eurodollars futures contracts with expiration in the month of the first interest rate adjustment.

(b) Consider the following 3-month Eurodollar quotes:

Delivery Month	**Rate**
AUG× 0	94.32
SEP	94.34
OCT	94.31
NOV	94.33
DEC	94.35
JAN× 1	94.43
MAR	94.40
JUN	94.43
SEP	94.40
DEC	94.25
MAR× 2	94.30
JUN	94.27
SEP	94.23
DEC	94.15

Handcraft Ale takes out a floating rate note with the first interest rate adjustment coming in December. LIBOR at the time of loan initiation is 5.70 percent. Design a strip hedge to convert the Handcraft Ale floating rate note to a fixed rate note. What is the anticipated fixed rate?

Peele would hedge each of her interest payments by selling $10 million of the 3-month Eurodollars in the following months:

Expiration	**Price**	**Anticipated Loan Rate**
DEC	94.35	7.15
MAR× 1	94.40	7.10
JUN	94.43	7.07
SEP	94.40	7.10
DEC	94.25	7.25
MAR× 2	94.30	7.20
JUN	94.27	7.23

The anticipated loan rate for each adjustment date is computed as:

$$\text{Anticipated rate} = 100 - \frac{\text{Price}}{100} + 1.5$$

While the hedge is not the same as pure fixed rate financing, there is very little variation anticipated. The range of anticipated rates are from 7.07 to 7.25 percent. The average rate over the life of the loan is 7.15 percent, including the initial period.

(c) Suppose that Handcraft Ale's quarterly interest payments were in November, February, May, and August. Would a strip hedge be possible? Design a hedge that Ms Peele could use in this case.

If Handcraft's interest payment cycle were November, February, May, and August, Ms Peele would have a problem. It would only be possible to match the timing of the nearest interest rate adjustment. This is a situation in which Ms Peele could use a stack hedge. She would stack hedge all seven interest rate adjustments by selling $70 million in Eurodollar futures with November expiration. In November, Handcraft would make its interest payment. At that time there will be a market for February Eurodollars, and Ms Peele would stack the six remaining interest rate adjustments on the February contract. This process would continue until maturity with each interest rate adjustment reducing the size of the subsequent stack hedge by $10 million. Alternatively, Ms Peele could hedge each rate adjustment using the Eurodollar futures contract that expires in the month just following the date the interest rate adjustment is made.

16. Jim Hunter is preparing to hedge his investment firm's decision to purchase $100 million of 90-day T-Bills 60 days from now in June. The discount yield on the 60-day T-Bill is 6.1 percent and the June T-Bill futures contract is trading at 94.80. Jim views these rates as very attractive relative to recent history, and he would like to lock them in. His first impulse is to buy 100 June T-Bill futures contracts, but his recent experience leads him to believe that he should be buying something less.

(a) Why is a one-to-one hedge ratio inappropriate in Jim's situation?

A one-to-one hedge ratio is not appropriate because the daily settlement gains and losses can be invested or must be financed. If no interest was earned on settlement cash flows, then a one-to-one hedge would be appropriate because the settlement cash flows would exactly offset the change in value of the underlying instrument. Because interest can be earned on the settlement cash flows, the delivery date value of the settlement cash flows will not exactly equal the change in value of the underlying instrument. The terminal value of the settlement cash flows will exceed, in magnitude, the change in value of the underlying instrument. The difference will be the interest earned and/or assessed between the settlement date and the delivery date.

(b) Compute an appropriate hedge ratio, given the market conditions faced by Jim.

The hedge ratio is slightly reduced to account for the interest that can be earned between daily settlement and the delivery date. This is called tailing the hedge. The amount by which the hedge is reduced is called the tailing factor. The tailing factor is the present value factor between the settlement date and the delivery date. When Jim initiates the hedge the discount yield for the 60-day T-Bill (his hedging time horizon) is 6.1 percent. Computing the tailing factor, which is the price per $1 of face value we have:

$$\text{Tailing factor} = 1 - \frac{0.061 \times 60}{360} = 0.9898$$

The tailed hedge for the \$100 million of 90-day T-Bills to be purchased in 2 months is:

$$\text{Tailed hedge} = \$100(0.9898) = \$98.98 \text{ million}$$

Jim would buy 99 June T-Bill futures contracts.

(c) Under what conditions might Jim need to adjust his hedge ratio between now and June?

The tailing factor is a function of the time to delivery and the level of interest rates. If interest rates change significantly, then Jim might need to adjust the hedge. The longer the time to settlement, the more influential interest rate movements will be. Even if interest rates do not move, the passage of time will call for adjustment as the tailing factor will move toward the value of 1 on the delivery date.

17. Alex Brown has just returned from a seminar on using futures for hedging purposes. As a result of what he has learned, he reexamines his decision to hedge \$500 million of long-term debt that his firm plans to issue in May. His current hedge is a short position of 5,000 T-Bond futures contracts (\$100,000 each). If the debt could be issued today, it would be priced at 119–22 to yield 6.5 percent. With its 8 percent coupon and 30 years to maturity, the duration of the debt would be 13.09 years. On the futures side, the futures prices are based on the cheapest-to-deliver bonds, which are trading at 124–14 to yield 5.6 percent. These bonds have a duration of 9.64 years.

    (a) List and briefly describe possible strategies Alex Brown could use to hedge his impending debt issue.

There are a number of different methods Alex could employ to hedge his impending debt issue:

Face Value Naive Model. In this method Alex would trade \$1 of nominal futures contract per \$1 of debt face value. The major benefit of this method is the ease of implementation. Unfortunately, it ignores market values and the differential responses of the bond and futures contract prices to interest rates.

Market Value Naive Model. In this method Alex would hedge \$1 of debt market value using \$1 of futures price value. That is, the hedge ratio is determined by the market prices instead of nominal and face values. Unfortunately, it does not consider the price sensitivities of the two instruments.

Conversion Factor Model. This model can be used when the hedging instrument is a T-Note or T-bond futures contract. The conversion factor adjusts the prices of deliverable bonds and notes that do not have an 8 percent coupon to make them "equivalent" to the 8 percent coupon bond or note that is called for in the contract. The hedge ratio is determined by multiplying the face value naive hedge ratio by the conversion factor. The appropriate conversion factor to use is the conversion factor of the cheapest to deliver T-bond or T-note. This model still ignores price sensitivity differences between the hedging and hedged instruments.

Basis Point Model. This model uses the price changes of the futures and cash positions resulting from a 1 basis point change in yields to determine the hedge ratio. It is calculated as:

$$HR = -\frac{BPC_C}{BPC_F}$$

This model works well if the cash and futures instruments face the same rate volatility. If they face different volatilities and that relationship can be quantified, then the basis point model can be adjusted to account for the differing volatilities.

Regression Model. In the regression model the historic relationship between cash market price changes and futures market price changes is estimated. This estimation is accomplished by regressing price changes in the cash market on futures price changes. The slope coefficient from this regression is then used as the hedge ratio. Alex may not find this model useful, as he is trying to hedge a new debt issue. Even if Alex had a historic price stream on 30-year corporate debt issues, the historic relationship with the futures price might prove to be an unreliable indicator of the present or future relationship. This stems from the fact that the price response of the futures contract is determined by the cheapest-to-deliver bond. The cheapest-to-deliver bond can vary in maturity from 15 to 30 years. This means that the futures contract can have very different price responses to interest rates at different points in time.

Price Sensitivity Model. This may be a good model for Alex to use. It is designed for interest rate hedging, and it accounts for the differential price responses of the hedging and the hedged instruments. The model is duration-based so that it accounts for maturity and coupon rate differences of the cash and the futures positions. It is computed as:

$$N = -\left(\frac{P_i \mathrm{MD}_i}{\mathrm{FP}_F \mathrm{MD}_F}\right)\mathrm{RYC}$$

where:

FP_F and P_i are the respective futures contract and cash instrument prices; MD_i and MD_F are the modified durations for the cash and futures instruments, respectively, and RYC is the change in the cash market yield relative to the change in the futures yield.

(b) What strategy is Alex Brown currently using? What are the strengths and weaknesses of this strategy?

Currently, Alex has employed a face value naive hedge. For each dollar of debt principal he plans to issue, he is short \$1 of nominal T-bond futures. The benefit of the strategy is its ease of implementation. The drawback is that cash instrument and the T-bond futures may have differential price responses to interest rate changes.

(c) Based on the knowledge Alex gained at the hedging seminar, he feels that a price sensitivity hedge would be most appropriate for his situation. Design a hedge using the price sensitivity method. Assume that the relative volatility between the corporate interest rate and the T-bond interest rate, RYC, is equal to one.

The price sensitivity hedge ratio is:

$$N = -\left(\frac{P_i \mathrm{MD}_i}{\mathrm{FP}_F \mathrm{MD}_F}\right)\mathrm{RYC}$$

$$\mathrm{FP}_F = 124.4375\% \times 0.1 \text{ million} \qquad \mathrm{MD}_F = 9.128788$$

$$P_i = 119.6875\% \times 500 \text{ million} \qquad \mathrm{MD}_i = 12.29108$$

$$N = -\frac{\$598{,}437{,}500 \times 12.29108}{\$124{,}437.50 \times 9.128788} = -6{,}475 \text{ contracts}$$

To hedge the risk, 6,475 contracts should be sold.

(d) At the time of refinancing, the T-bond futures price is 121–27 and B.I.G.'s new debt issue is priced at 116–08. Compute the net wealth change resulting from the naive hedge.

In the cash market, Alex suffers an opportunity loss because he anticipated issuing debt at 119–22, but he is only to get 116–08 for the debt. The opportunity loss is:

$$\text{Opportunity loss} = (1.1625 - 1.196875)\,\$500{,}000{,}000 = -\$17{,}187{,}500$$

In the futures market Alex realizes a gain because he was short T-bond futures. His gain is:

$$\text{Futures gain} = (1.244375 - 1.2184375)\,\$500{,}000{,}000 = \$12{,}968{,}750$$
$$\text{Net wealth change} = -\$17{,}187{,}500 + \$12{,}968{,}750 = -\$4{,}218{,}750$$

(e) Compute the net wealth change resulting from the price sensitivity hedge.
Examining the price sensitivity hedge, the cash market loss will still be $17,187,500.The futures market gain will now be:

$$\text{Futures gain} = (1.244375 - 1.2184375)\,647{,}500{,}000 = \$16{,}794{,}53$$
$$\text{Net wealth change} = -\$17{,}187{,}500 + \$16{,}794{,}531 = -\$392{,}969$$

The price sensitivity hedge is much more effective than the face value naive hedge at reducing the risk.

SECURITY FUTURES PRODUCTS: INTRODUCTION

1. Assume that the DJIA stands at 8340.00 and the current divisor is 0.25. One of the stocks in the index is priced at $100.00 and it splits 2 : 1. Based on this information, answer the following questions:

(a) What is the sum of the prices of all the shares in the index before the stock split?
The equation for computing the index is:

$$\text{Index} = \frac{\sum_{i=1}^{N} P_{\mathrm{i}}}{\text{Divisor}}$$

If the index value is 8340.00 and the divisor is 0.25, the sum of the prices must be 8340.00(0.25) = $2,085.00.

(b) What is the value of the index after the split? Explain.

After the split, the index value is still 8340.00. The whole purpose of the divisor technique is to keep the index value unchanged for events such as stock splits.

(c) What is the sum of the prices of all the shares in the index after the split?

The stock that was $100 is now $50, so the sum of the share prices is now $2035.00.

(d) What is the divisor after the split?

With the new sum of share prices at $2035.00, the divisor must be 0.244005 to maintain the index value at 8340.00.

2. What is the main difference in the calculation of the DJIA and the S&P 500 index? Explain.

The S&P 500 index gives a weight to each represented share that is proportional to the market value of the outstanding shares. The DJIA simply adds the prices of all of the individual shares, so the DJIA effectively weights each stock by its price level.

3. For the S&P 500 index, assume that the company with the highest market value has a 1 percent increase in stock prices. Also, assume that the company with the smallest market value has a 1 percent decrease in the price of its shares. Does the index change? If so, in what direction?

The index value increases. The share with the higher market value has a greater weight in the index than the share with the smallest market value. Therefore, the 1 percent increase on the high market value share more than offsets the 1 percent decrease on the low market value share.

4. The S&P 500 futures is scheduled to expire in half a year, and the interest rate for carrying stocks over that period is 11 percent. The expected dividend rate on the underlying stocks for the same period is 2 percent of the value of the stocks. (The 2 percent is the half-year rate, not an annual rate.) Ignoring the interest that it might be possible to earn on the dividend payments, find the fair value for the futures if the current value of the index is 945.00.

Assuming that the half-year rate is 0.11/2, the fair value is:

$$\text{Fair value} = 945.00(1.055 - 0.02) = 978.075$$

Assuming semiannual compounding, the interest factor would be 1.0536 and the fair value would be:

$$\text{Fair value} = 945.00(1.0536 - 0.02) = 976.752$$

5. Consider a very simple index such as the DJIA, except assume that it has only two shares, A and B. The price of A is \$100.00, and B trades for \$75.00. The current index value is 175.00. The futures contract based on this index expires in 3 months, and the cost of carrying the stocks forward is 0.75 percent per month. This is also the interest rate that you can earn on invested funds. You expect Stock A to pay a \$3 dividend in 1 month and Stock B to pay a \$1 dividend in 2 months. Find the fair value of the futures. Assume monthly compounding.

$$\text{Fair value} = 175.00(1.0075)^3 - \$3(1.0075)^2 - \$1(1.0075) = 174.91$$

6. Using the same data as in Problem 5, now assume that the futures trades at 176.00. Explain how you would trade with this set of information. Show your transactions.

At 176.00, the futures is overpriced. Therefore, the trader should sell the futures, buy the stocks and carry them forward to expiration, investing the dividend payments as they are received. At expiration, the total cost incurred to carry the stocks forward is 174.91, and the trader receives 176.00 as cash settlement, for a profit of 1.09 index units. (This ignores interest on daily settlement flows.)

7. Using the same data as in Problem 5, now assume that the futures trades at 174.00. Explain how you would trade with this set of information. Show your transactions.

At 174.00 the futures is underpriced. Therefore, the trader should buy the futures and sell the stocks short, investing the proceeds. The trader will have to borrow to pay the dividends on the two shares.

At expiration, the total outlays, counting interest, have been:

$$\$175.00(1.0075)^3 - \$3(1.0075)^2 - \$1(1.0075) = 174.91$$

With the convergence at expiration, the trader can buy the stocks for 174.00 and return them against the short sale. This gives a profit of $0.91.

8. For a stock index and a stock index futures constructed, such as the DJIA, assume that the dividend rate expected to be earned on the stocks in the index is the same as the cost of carrying the stocks forward. What should be the relationship between the cash and futures market prices? Explain.

The cash and futures prices should be the same. In essence, an investment in the index costs the interest rate to carry forward. This cost is offset by the proceeds from the dividends. If these are equal, the effective cost of carrying the stocks in the index forward is zero, and the cash and futures prices should then be the same.

9. Your portfolio is worth $100 million and has a beta of 1.08 measured against the S&P index, which is priced at 350.00. Explain how you would hedge this portfolio, assuming that you wish to be fully hedged.

The hedge ratio is:

$$-\beta_P\left(\frac{V_P}{V_F}\right) = \text{number of contracts}$$

With our data we have:

$$-1.08\left(\frac{\$100{,}000{,}000}{(350.00)(250)}\right) = -1234.286$$

The cash value of the futures contract is 250 times the index value of 350.00, or $87,500. Therefore, the complete hedge is to sell 617 contracts.

10. You have inherited $50 million, but the estate will not settle for 6 months and you will not actually receive the cash until that time. You find current stock values attractive and you plan to invest in the S&P 500 cash portfolio. Explain how you would hedge this anticipated investment using S&P 500 futures.

Buy S&P 500 index futures as a temporary substitute for actually investing the cash in the stock market. Probably, the best strategy is to buy the contract that expires closest in time to the expected date for receiving the cash. If the S&P 500 index value is 300.00, then the dollar value of one contract will be $75,000 (300.00 × $250). Therefore, you should achieve a good hedge by purchasing about 666 ($50,000,000/$75,000) contracts.

11. William's new intern, Jessica, is just full of questions. She is particularly inquisitive about stock index futures. She notices that the futures price is consistently higher than the current index level and that the difference gets smaller as the contracts near their expiration dates.

    (a) Explain the relationship between the futures price, the spot price, interest rates, and dividends.

The futures contract effectively allows one to commit to the sale or the purchase of the Dow index stocks at a specific point in the future at a price agreed upon today. There are costs and benefits related to using the futures market as opposed to the cash market to buy the Dow index stocks. Take for example an individual who wishes to own the Dow stocks in 3 months. The alternatives are to (1) buy the stocks today and hold them, or (2) buy a futures contract with 3 months to delivery.

The benefit of using the futures contract is that there is no cash outlay today. The purchase price of the stocks can be invested for 3 months to earn interest. The downside of using the futures contract is that since the stocks are not held over the next 3 months, no dividends are received. This suggests the following relationship between the spot and the futures market prices:

$$F_{0,t} = S_0(1 + C) - \sum_{i=1}^{n} D_i(1 + r_i)$$

where:

$F_{0,t}$ is the futures price today for delivery at time t

S_0 is the spot price for the index today

C is the cost-of-carry

D_i is the ith dividend paid between now and time t

r_i is the rate of return received on the ith dividend between the payment date and time t.

In general, the dividend yield is smaller than the cost-of-carry so the index futures markets are generally normal (futures index above the spot index). As the delivery date approaches, the futures index converges to the spot index.

(b) Jessica asks William to explain the Dow index to her. What type of index is the Dow? How is it constructed? How could she build a portfolio of stocks to replicate it?

The Dow is a price-weighted index. The stocks are represented in the index in proportion to their price. The index is computed as:

$$\text{Dow index} = \frac{\sum_{i=1}^{30} P_i}{\text{Divisor}}$$

where the P_i's are the prices of the stocks comprising the Dow, and the Divisor is a number used to compute the "average".

When the Dow first appeared with 30 stocks in 1928 (the Dow was first published in 1884 with 11 stocks), the divisor was 30. As stocks split or the components of the Dow changed, the divisor was adjusted to maintain continuity in the index. To form a portfolio that would replicate the Dow, Jessica should buy an equal number of shares of each stock in the Dow.

(c) Jessica wants a numerical example of the relationship between a price-weighted index and the futures contract based on that index. She supposes the following example. A futures contract is based on a price-weighted index of three stocks A, B, and C. The futures contract expires in 3 months. Stock A pays a dividend at the end of the first month, and Stock C pays a dividend at the end of the second month. The term structure is flat over this time period with the monthly interest rate equal to 0.5 percent. The stock prices and dividends are summarized below.

Stock	Price	Dividend
A	\$30	\$0.11 in 1 month
B	\$50	\$0
C	\$40	\$0.15 in 2 months

Compute the index assuming a divisor of 3. How many shares of each stock should be bought to replicate the index?

The index is computed as:

$$\text{Index} = \frac{\sum_{i=1}^{3} P_i}{\text{Divisor}} = \frac{30 + 50 + 40}{3} = 40$$

To replicate the index, you would buy 1/Divisor shares of each stock. In our example one-third share of each stock would replicate the index.

(d) Suppose the divisor had been 0.5. Compute the index. How many shares of each stock must be bought to replicate the index?

With the divisor equal to 0.5 the index is:

$$\text{Index} = \frac{\sum_{i=1}^{3} P_i}{\text{Divisor}} = \frac{30 + 50 + 40}{0.5} = 240$$

The number of shares of each stock to buy to replicate the index is $1/0.5 = 2$. Two shares of each stock will replicate the index.

(e) Assuming the divisor is 0.5, compute the fair value for the 3-month futures contract.

The fair market value is computed using:

$$F_{0,t} = S_0(1 + C) - \sum_{i=1}^{n} D_i(1 + r_i)$$
$$F_{0,t} = 240(1 + 0.005)^3 - 2(0.11)(1.005)^2 - 2(0.15)(1.005)$$
$$F_{0,t} = 243.09$$

(f) Right now, the Dow Jones industrial average is at 8635. Its dividend yield is 1.76 percent. The 90-day T-bill rate is 5.6 percent bond equivalent yield. Compute a fair price today for the index futures contract expiring in 90 days.

For an index composed of a large number of securities it is sometimes helpful to express the relationship between the futures and spot indexes as:

$$F_{0,t} = S_0(1 + C - \text{DIVYLD})$$

where DIVYLD is the dividend yield on the index stocks.

The fair price for the futures contract expiring in 90 days is:

$$F_{0,t} = S_0(1 + C - \text{DIVYLD}) = 8{,}635\left(1 + \frac{0.056 \times 90}{365} - \frac{0.0176 \times 90}{365}\right)$$
$$F_{0,t} = 8{,}717$$

12. Casey Mathers manages the \$60 million equity portion of Zeta Corporation's pension assets. This past Friday, August 7th, Zeta announced that it was downsizing its workforce and would be offering early retirement to many of its older employees. The impact on the portfolio Casey manages would be an anticipated \$10 million withdrawal over the next 4 months. The stock market has been good for the past 5 years, but recently there have been signs of weakness. Casey is concerned about a drop in asset prices before the \$10 million is withdrawn from the portfolio. Casey runs a fairly aggressive portfolio with a β of 1.2, relative to the S&P 500 index. Casey sees the following S&P 500 index futures prices:

Expiration	**Contract Value \$250 × Index**
SEP	1088.50
DEC	1100.00
MAR × 1	1110.50

(a) How much of the portfolio should Casey hedge? Justify your answer.

Casey anticipates \$10 million being withdrawn from the portfolio over the next 4 months. Casey is concerned about the risk that falling stock prices will result in more shares having to be sold to raise the \$10 million. It is just the anticipated withdrawal that should be hedged.

(b) Design a hedge based on your answer to (a) above.

To hedge the risk, Casey should use the futures contract with expiration as soon after the anticipated withdrawal as possible. This would be the December futures contract. Since Casey's portfolio has a beta of 1.2 relative to the S&P 500, Casey should sell \$1.2 of future contract value per \$1 of portfolio hedged. Then the number of contracts to trade is computed by:

$$N = -\beta_P \frac{V_P}{V_F} = -1.2\frac{\$10{,}000{,}000}{(1{,}100)\ \$250} = -43.6 \text{ contracts}$$

To hedge the risk of the \$10 million withdrawal, Casey should sell 44 contracts. As the assets are withdrawn from the portfolio, the hedge should be gradually unwound. The transactions would be as follows:

Date	**Cash Market**	**Futures Market**
Today	Anticipate the withdrawal of \$10 million from the portfolio over the next 4 months	Sell 44 December S&P 500 futures contracts
Between now and December	Sell securities to meet portfolio withdrawal demands	Enter reverse trades to unwind the hedge as assets are withdrawn from portfolio

13. Byron Hendrickson manages the $30 million equity portion of Fredrick and Sons' pension plan assets. Byron has been trying to get the management of Fredrick and Sons to move more of their assets from their fixed income portfolio (market value of $60 million) to the equity portfolio in order to achieve the objectives that management had set forth for growth of the plan. Management has decided to invest the proceeds of several bond issues that will be maturing over the next 3 months. The total proceeds from the bond issues will be $10 million. It is now December 15th. Byron believes that the January price runup will be particularly strong this year. Since his portfolio is not particularly aggressive, $\beta = 0.85$, he would really like to have that $10 million working for him in January. Design a hedge that will prevent Byron from missing the January action, based on the following current market prices for the S&P 500 index futures:

Expiration	**Contract Value $250 × Index**
MAR	1157.00
JUN	1170.80

Byron wishes to commit $10 million to the stock market today, but the cash will not be available until March. Therefore, he is implicitly short in the cash market at present. Byron should make a long hedge using the March S&P 500 futures index. The number of contracts Byron should buy is computed as:

$$N = -0.85\frac{(-\$10{,}000{,}000)}{(1{,}157)\ \$250} = 29.4 \text{ contracts}$$

Byron's implicit short position is reflected in the preceding equation by the −$10,000,000 cash market position. Byron should buy 29 March S&P futures contracts. This will generate gains to replace the opportunity loss from not having the $10 million invested between now and March. As the bonds mature and the proceeds are transferred to the equity side, the hedge should be unwound. Byron's transactions are summarized below:

Date	**Cash Market**	**Futures Market**
Today	Anticipate the investment of $10 million into the equity portfolio over the next 3 months	Buy 29 March S&P 500 futures index contracts
March	Buy stocks using $10 million plus any gains/losses from the futures position	Unwind the futures position as the investments are made into the equity portfolio

SECURITY FUTURES PRODUCTS: REFINEMENTS

1. Explain the market conditions that cause deviations from a computed fair value price and that give rise to no-arbitrage bounds.

The villains are market imperfections, principally transaction costs. When trading is sufficiently costly, the futures price can deviate somewhat from fair value, and no market forces will arise to drive the futures price back to its fair value. The greater the costs of trading, the farther the futures price can stray from its theoretical fair value without arbitrage coming into play to restore the relationship. These trading costs include: the bid-asked spread and direct transaction costs, such as brokerage commissions and taxes. Also, restrictions on the use of the proceeds from short sales can be important.

2. The no-dividend index consists only of stocks that pay no dividends. Assume that the two stocks in the index are priced at \$100 and \$48, and assume that the corresponding cash index value is 74.00. The cost of carrying stocks is 1 percent per month. What is the fair value of a futures contract on the index that expires in 1 year?

$$\text{Fair value} = 74.00(1.01)^{12} = 83.3851$$

3. Using the same facts as in Problem 2, assume that the round-trip transaction cost on a futures is \$30. The contract size, we now assume, is for 1,000 shares of each stock. Trading stocks costs \$0.05 per share to buy and the same amount to sell. Based on this additional information, compute the no-arbitrage bounds for the futures price.

From the cash-and-carry transactions we would buy the stocks, carry them to expiration, and sell the futures. This strategy would cost:

Purchase and carry stock	$-\$148{,}000(1.01)^{12} = -\$166{,}770$
Stock transaction cost	$+1{,}000(2)(\$.05) = -\$100$
Futures transaction cost	$-\$30$
Total outlay	$-\$166{,}900$

For this strategy to generate a profit, the futures must exceed 83.450/contract. For the reverse cash-and-carry, we would sell the stocks, invest the proceeds, and buy the futures:

Sell stock; invest proceeds	$\$148{,}000(1.01)^{12} = \$166{,}770$
Stock transaction cost	$1{,}000(2)(\$.05) = -\$100$
Futures transaction cost	$-\$30$
Total inflow	$-\$166{,}640$

For this strategy to generate a profit, the futures must be less than 83.320/contract. The no-arbitrage bounds on the futures range from 83.320 to 83.450.

4. Using the facts in Problems 2 and 3, we now consider differential borrowing and lending costs. Assume that 1 percent/month is the lending rate and assume that the borrowing rate is 1.5 percent/month. What are the no-arbitrage bounds on the futures price now?

From the cash-and-carry transactions we would buy the stocks, carry them to expiration, and sell the futures. Now the financing cost is 1.5 percent/month. This strategy would cost:

Purchase and carry stock	$-\$148{,}000(1.015)^{12} = -\$176{,}951$
Stock transaction cost	$+1{,}000(2)(\$.05) = -\$100$
Futures transaction cost	$-\$30$
Total outlay	$-\$176{,}821$

For this strategy to generate a profit, the futures must exceed 88.411/contract. The reverse cash-and-carry strategy is unaffected because the lending rate is still 1 percent. Therefore, the no-arbitrage bounds on the futures range from 83.320 to 88.411.

5. Using the facts in Problems 2–4, assume now that the short seller receives the use of only half of the funds in the short sale. Find the no-arbitrage bounds.

The cash-and-carry transactions are the same as in Problem 4, so they give an upper no-arbitrage bound of 88.411. For the reverse cash-and-carry, we would sell the stocks, invest the proceeds, and buy the futures:

Sell stock; invest 50% of proceeds	$+\$74{,}000(1.01)^{12} = \$83{,}385$
Stock transaction cost	$-1{,}000(2)(\$.05) = -\$100$
Futures transaction cost	$-\$30$
Recoup 50% of unused funds	$+\$74{,}000$
Total inflow	$+\$157{,}255$

For this strategy to generate a profit, the futures must be less than 78.628/contract. The no-arbitrage bounds on the futures range from 78.628 to 88.411.

6. Consider the trading of stocks in an index and trading futures based on the index. Explain how different transaction costs in the two markets might cause one market to reflect information more rapidly than the other.

Let us assume that it is more costly to trade the individual stocks represented in the index than it is to trade the futures based on the index. (Once in a while we assume something consistent with reality.) Traders with information about the future direction of stock prices will want to exploit that information as cheaply as possible. Therefore, they will be likely to trade futures rather than the stocks in the index. Trading futures causes the futures price to adjust, and through arbitrage links, the stock price adjusts to the new futures price. In this scenario, the futures market reflects the new information before the stock market does.

7. For index arbitrage, explain how implementing the arbitrage through program trading helps to reduce execution risk.

Execution risk is the risk that the actual trade price will not equal the anticipated trade price. The discrepancy arises largely from the delay between order entry and order execution. By using program trading, orders are conveyed to the floor more quickly and receive more rapid execution. (At least this is true in the absence of exchange-imposed delays on program trades.) Therefore, the use of program trading techniques should help to reduce execution risk.

8. Index arbitrageurs must consider the dividends that will be paid between the present and the futures expiration. Explain how overestimating the dividends that will be received could affect a cash-and-carry arbitrage strategy.

Assume that a trader estimates a dividend rate that is higher than the actual dividend rate that will be achieved. Further assume that the market as a whole correctly forecasts the dividend rate. For this investor, a strategy of cash-and-carry arbitrage will appear to be more attractive than it really is. This trader will be expecting to receive more dividends than will actually be forthcoming, so the trader will underestimate the net cost of carrying stocks forward. This overestimate of the dividend rate could lead the trader to expect a profit from the trade that will evaporate when adjusted for the actual dividends that will be received.

9. Explain the difference between the β in the Capital Asset Pricing Model (CAPM) and the β one finds by regressing stock returns against returns on a stock index.

The beta of the CAPM is a theoretical entity. The CAPM β is a measure based on the relationship between a particular security and an unobserved and probably unobservable market portfolio. The β estimated by regressing stock returns against the returns on an index is an estimate of that ideal CAPM β. Because the index fails to capture the true market portfolio, the actually estimated β must fail to capture the true CAPM β. Nonetheless, the estimated β may be a useful approximation of the true CAPM β.

10. Explain the difference between an *ex ante* and an *ex post* minimum risk hedge ratio.

The *ex ante* minimum risk hedge ratio is estimated using historical data. In hedging practice, this estimated hedge ratio is applied to a future time period. Almost certainly, the hedge ratio that would have minimized risk in the future period (the *ex post* hedge ratio) will not equal the estimated *ex ante* hedge ratio. However, the *ex post* minimum risk hedge ratio can only be known after the fact. Therefore, we must expect some inaccuracy in estimating a hedge ratio *ex ante* and comparing it with the ideal *ex post* hedge ratio.

11. Assume you hold a well-diversified portfolio with a β of 0.85. How would you trade futures to raise the β of the portfolio?

Buy a stock index futures. In effect, this action levers up the initial investment in stocks, effectively raising the β of the stock investment. In principle, this levering up can continue to give any level of β a trader desires.

12. An index fund is a mutual fund that attempts to replicate the returns on a stock index, such as the S&P 500. Assume you are the manager of such a fund and that you are fully invested in stocks. Measured against the S&P 500 index, your portfolio has a β of 1.0. How could you transform this portfolio into one with a zero β without trading stocks?

Sell S&P 500 index futures in an amount equal to the value of your stock portfolio. After this transaction, you are effectively long the index (your stock holdings) and short the index by the same amount (your short position in the futures). As a result, you are effectively out of the stock market, and the β of such a position must be zero.

13. You hold a portfolio consisting of only T-bills. Explain how to trade futures to create a portfolio that behaves like the S&P 500 stock index.

Buy S&P 500 index futures. You should buy an amount of futures that equals the value of funds invested in T-bills. The resulting portfolio will replicate a portfolio that is fully invested in the S&P 500.

14. In portfolio insurance using stock index futures, we noted that a trader sells additional futures as the value of the stocks falls. Explain why traders follow this practice.

The goal of portfolio insurance is to keep the value of a portfolio from falling below a certain level or, alternatively expressed, to ensure that the return achieved on a portfolio over a given horizon achieves a certain minimum level. At the same time, portfolio insurance seeks to retain as much potential for beating that minimum return as is possible. The difference between the portfolio's current value and the value it must have to meet the minimum target is called the *cushion*. If the portfolio has no cushion, the only way to ensure that the portfolio will achieve the target return, or the target value, is for the portfolio to be fully hedged.

We now consider the trader's response if the portfolio value is above the minimum level, that is, if there is some cushion and stock prices fall. The drop in stock prices reduces the cushion, so the trader must move to a somewhat more conservative position. This requires hedging a greater portion of the portfolio, which the trader does by selling futures. Therefore, an initial drop in prices requires the selling of futures, and each subsequent drop in prices requires the sale of more futures.

15. Casey Mathers, manager of the Zeta Corporation's equity portfolio, hires a new assistant, Alec. Alec is pretty sharp and immediately questions Casey's decision to hedge an anticipated $10 million withdrawal. Casey had hedged the portfolio using the S&P 500 index futures contract. In calculating the hedge, Casey used the portfolio β of 1.2 that was computed using the S&P 500 Index.

(a) Explain to Casey why his hedge may not be a risk-minimization hedge.

Casey's hedge may not be a risk-minimization hedge because the β used in calculating the hedge ratio was computed using the S&P 500 index. It is the index futures contract, though, that is used for hedging. So theoretically the portfolio's β computed using the futures contract prices is what should be used in calculating the hedge. Additionally, the S&P 500 index futures contract is not the only possible hedging vehicle available. For example, there are the Dow Jones Industrial Average index futures and the NYSE Composite index futures. The portfolio's returns may be more highly correlated with one of these other contracts than it is with the S&P 500 index futures contract.

Alec calculates possible risk-minimizing βs for the Zeta portfolio using the S&P 500 index futures, the Dow Jones Industrial Average futures, and the NYSE Composite index futures, with the following results:

	S&P 500 Index Futures	**DJIA Futures**	**NYSE Composite Index Futures**
Contract size	$250 × index	$10 × index	$500 × index
Current quote (MAR) contract	1110.59	8715.00	559.30
β_{RM}	1.30	1.35	1.10
R^2	0.83	0.75	0.90

(b) Given Alec's results, is the S&P 500 futures index the most appropriate hedging vehicle? Be sure to justify your answer.

In risk-minimization hedging the best vehicle to use is the instrument with the highest R^2. The R^2 tells the percentage of portfolio returns that is explained by the hedging instrument's price returns. Examining the R^2 figures from Alec's results, it can be seen that the NYSE Composite index contract has the highest

R^2 of 90 percent. This is greater than the S&P 500 contract's R^2 of 83 percent. The NYSE Composite contract would be the better hedging instrument according to the risk-minimization technique.

(c) Design a risk-minimization hedge using Alec's results.

Using Alec's results the risk-minimizing hedge would be accomplished by selling NYSE Composite index futures. The number of contracts to sell is computed as:

$$N = -1.10 \frac{\$10{,}000{,}000}{559.30\,(\$500)} = -39.3 \text{ contracts}$$

Alec would recommend selling 39 contracts.

(d) Will Alec's hedging strategy turn out to be superior to Casey's hedging strategy? Justify your answer.

While Alec's hedging strategy will probably be superior to Casey's, this is not guaranteed, because the hedge calculation is based on historic relationships that are measured with error. This relationship will not hold exactly in the future.

16. Raymond J. Johnson, Jr. manages a $20 million equity portfolio. It has been designed to mimic the S&P 500 index. Ray has a hunch that the market is going south during the coming month. He has decided that he wants to eliminate his exposure for the next month and take off for Montana to go fishing. Ray has the following information at hand:

S&P 500 index futures with 1 month to delivery	1084.50
Dividend yield on Ray's portfolio	2.1%
S&P 500 index today	1081.40

(a) Design a hedge to eliminate Ray's market risk for the next month.

Ray, in effect, would like to sell his portfolio for a month and put the money into T-bills. The transaction costs make this strategy cost prohibitive, though. Alternatively, Ray could sell futures contracts. Then over the next month any losses in the cash market will be offset by gains in the futures market. To eliminate his exposure to the market, Ray would calculate the number of S&P 500 futures contracts as:

$$N = -1.0 \frac{\$20{,}000{,}000}{(1084.50)\,(\$250)} = -73.8 \text{ contracts}$$

Ray should sell 74 S&P 500 futures contracts.

(b) Compute the return he can expect to receive over the next month.

The futures index pricing relationship is:

$$F_{0,t} = S_0(1 + C - \text{DIVYLD})$$

where:

$F_{0,t}$ = index futures value
S_0 = spot price
C = cost-of-carry
DIVYLD = dividend yield.

The cost-of-carry can also be viewed as an implied repo rate. This is the return Ray will receive over the next month:

$$C = \left(\frac{F_{0,t}}{S_0} + \text{DIV} - 1\right)12$$
$$C = \left(\frac{74(1084.50)(250)}{20{,}000{,}000} + \frac{0.021}{12} - 1\right)12$$
$$C = 5.9 \text{ percent}$$

Ray can expect to earn a 5.9 percent annual return over the next month.

17. Remember Ray? He is the guy running the S&P 500 index fund who wanted to go fishing. Ray has changed his mind. The fishing reports from Montana were not favorable so he has decided not to go. Since he is not leaving, he has decided to devise a portfolio insurance strategy for his $20 million portfolio. His objective is to not let his portfolio value fall below $18 million.
    (a) Design a portfolio insurance strategy that applies no hedges for portfolio values at or above $20 million and is fully hedged at or below $18 million.

Portfolio insurance is a dynamic hedging strategy that applies hedges as the hedged asset falls in value and removes hedges as the hedged asset increases in value. When the asset value is falling, hedges are applied until the assets are fully hedged. Theoretically, a money manager could prevent the value of his positions from falling below some prespecified level because further decline in the assets' value is offset by futures gains. Ray wishes to apply hedges as the value of this portfolio falls from $20 million (no hedges) down to $18 million (fully hedged). Assume that Ray will apply the hedges in a linear fashion. That is, the percentage of assets hedged will be given by:

$$\%\ \text{hedged} = \max\left(\left[\frac{20 - \max(V_P, 18)}{20 - 18}\right], 0\right)$$

When $V_P < 18$, the percentage of the portfolio hedged is 100 percent. When $V_P > 20$, the percentage of the portfolio hedged is 0 percent.

    (b) On day 1, the stock portfolio value falls from $20 million to $19.4 million, and the S&P 500 futures price falls to 1052. What action should Ray take?

If the portfolio value falls from $20 million to 19.4 million on day 1, the percentage of the assets that should be hedged is:

$$\%\ \text{hedged} = \left(\left[\frac{20 - \max(19.4, 18)}{20 - 18}\right], 0\right)$$
$$\%\ \text{hedged} = 30\%$$

Thirty percent of the assets should be hedged. This can be accomplished by selling contracts calculated as follows:

$$N = -1.0\,(0.30)\,\frac{\$19{,}400{,}000}{(1{,}052)\,(\$250)} = -22.13$$

At the end of day 1 Ray should sell 22 S&P 500 futures contracts.

(c) On day 2, the value of Ray's portfolio increases by 2 percent. The S&P 500 futures contract increases to 1073. What would be the change in value of Ray's portfolio (including any hedges that may be in place)? What action should Ray take?

Cash market gain: assets increase by 2 percent to $19,788,000.
Future market loss: 22 (1052 − 1073) ($250) = −$115,500.
End of day 2 assets: $19,788,000 − $115,500 = $19,672,500.

The change in value of Ray's portfolio on day 2 is $272,500. Part of the portfolio must be liquidated to mark the futures contract to market. At the end of day 2 Ray needs to adjust his hedge. The new percentage to hedge is:

$$\%\text{ hedged} = \frac{20 - 19.67}{20 - 18} = 16.5\%$$

The number of contracts to achieve this hedge is:

$$N = -1.0\,(0.164)\frac{\$19{,}672{,}500}{(1{,}073)\,(\$250)} = -12.02$$

Ray is already short 22 contracts so he is over-hedged. He should buy 10 S&P 500 futures contracts to bring his futures position to 12 short.

(d) Is Ray really protected against his portfolio value falling below $18 million in value? Explain. The protection Ray has depends upon several factors. First, the amount of protection depends on how closely Ray monitors his position. If Ray does not closely monitor his position, the portfolio value could fall below $18 million before Ray places a hedge. The more closely Ray monitors, and the more frequently he adjusts the hedge, the better the protection. Second, if market frictions prevent Ray from applying hedges in a timely fashion, the portfolio value could fall below $18 million. For many investors, this occurred during the stock market crash in 1987. The order flow overwhelmed the order handling and reporting systems of the stock market. Also, the price information coming from the stock markets was stale. This kind of event could mislead Ray in his hedging decision-making process. In this kind of extreme situation, Ray's assets could fall below $18 million before Ray knew it.

18. What real-world complications can hinder the effectiveness of a stock index arbitrage strategy?

(1) Dividend payments may be suspended or altered; (2) the composition of underlying index continually changes through time; (3) transaction costs; (4) stock exchange restrictions on short sales of stock; (5) the prices observed on trading screens may not reflect the actual state of the market at the time arbitrageurs make their decisions; (6) trade execution risk, that is, the arbitrageur's order may move the price; and (7) stock exchange "collars" on index arbitrage on days of large price moves. This is not an exhaustive list.

19. It is 7:00 a.m and the NYSE does not open until 9:30 a.m. The current S&P 500 e-mini nearby futures contract is trading on the CME's Globex system at 1150.00 index points. There are 21 days remaining until contract expiration. You expect the underlying index to pay out dividend equivalent to 1.1 index points over the contracts remaining life. You expect your financing cost to be 1.9 percent per year. If the closing price for the index from the previous day was 1135.00, use the

cost-of-carry relationship to determine whether you expect the stock market to open higher or lower, based on current information.

The annualized financing cost of an arbitrage position is 1.90 percent, or 0.0190 in decimal form. The financing cost for 21 days is therefore $(21/365) \times 0.0190 = 0.001093$. The expected dividend return is 1.1 index points. Therefore, the fair value stock index price is: $(1150 + 1.1)/(1 + 0.001093) = 1149.843$ index points.

Since the index closed at 1135, based on information at 7:00 a.m. it appears that the market will open strongly higher.

FOREIGN EXCHANGE FUTURES

1. The current spot exchange rate for the dollar against the Japanese yen is 146 yen/dollar. What is the corresponding U.S. dollar value of 1 yen?

The dollar value per yen is simply the inverse of the yen per dollar rate:

$$1/146 = \$.0068/\text{yen}$$

2. You hold the current editions of *The Wall Street Journal* and *The Financial Times*, the British answer to the *WSJ*. In the *WSJ*, you see that the dollar/pound 90-day forward exchange rate is \$2.00/pound. In *The Financial Times*, the pound 90-day dollar/pound rate is £0.45 per U.S. dollar. Explain how you would trade to take advantage of these rates, assuming perfect markets.

These rates are inconsistent because a rate of \$2.00/pound implies that the cost of \$1 should be £0.50. Therefore, an arbitrage opportunity is available by trading as follows:

Geographical arbitrage transactions

$t = 0$	
In New York, using the *WSJ* rates, sell \$2.00 for £1.00 90-days forward	\$0
In London, using *The Financial Times* rates, sell £1.00 for \$2.22 90-days forward	\$0
Total cash flow	\$0
$t = 90$	
In New York, fulfill the forward contract by delivering \$2.00 and collecting £1.00	−\$2.00 +£1.00
In London, fulfill the forward contract by delivering £1.00	−£1.00
and collecting \$2.22	+\$2.22
Total cash flow	+\$.22

3. In Problem 2, we assumed that markets are perfect. What are some practical impediments that might frustrate your arbitrage transactions in Problem 2?

Transaction costs would be the major impediment. Every trade of foreign exchange faces a bid-asked spread. In addition, there is likely to be some commission to be paid, either in the form of an outright commission or in the form of an implicit commission for maintaining a trading function. In addition, forward contracts sometimes require margin, and this would be an additional cost that the potential arbitrageur must bear.

4. In the *WSJ*, you see that the spot value of the euro is \$.63 and the Swiss franc is worth \$.72. What rate of exchange do these values imply for the Swiss franc and euro? Express the value in terms of euros per franc.

The rate of \$.63/euro implies a value of the euro equal to €1.5873/dollar. The rate of \$.72/franc implies a value of the franc equal to SF1.3889/dollar. Therefore, €1.5873 and SF1.3889 are equivalent amounts, both equal to \$1. As a consequence, the value of the euro per Swiss franc must equal 1.5873/1.3889 = 1.1429.

5. Explain the difference between a pegged exchange rate system and a managed float.

In a pegged exchange rate system, the value of a pegged currency is fixed relative to another currency. For example, many Caribbean countries peg the value of their currency to the U.S. dollar. In a managed float, the value of the currency is allowed to fluctuate as market conditions require. This is the floating part of the policy. In a managed float, the central bank intervenes in the market to influence the value of the currency by buying or selling its own currency.

6. Explain why covered interest arbitrage is just like our familiar cash-and-carry transactions from Chapter 3.

In a cash-and-carry transaction, a trader sells the futures and buys the underlying good. The trader carries the underlying good to the expiration of the futures, paying the carrying cost along the way, and delivers the good against the futures. In covered interest arbitrage, the transaction has a similar structure. The trader sells the futures and buys the foreign currency. The trader carries the foreign currency to the expiration of the futures, paying the carrying costs along the way, and delivers the good against the futures. The carrying cost for the foreign currency consists of two components. First, there is the financing cost in the home currency for the funds borrowed to buy the foreign currency. Second, the foreign currency that is carried forward to delivery against the futures earns interest. This interest on the foreign currency offsets the first component of the carrying cost.

7. For covered interest arbitrage, what is the cost-of-carry? Explain carefully.

The cost-of-carry is the difference between the home currency interest rate and the foreign currency interest rate. For covered interest arbitrage, the trader borrows the home currency and pays the domestic interest rate for these funds. The trader uses these funds to buy the foreign currency in the spot market, and invests the foreign currency to earn the foreign interest rate. Therefore, the cost-of-carry is the domestic interest rate minus the foreign interest rate.

8. The spot value of the euro is \$0.65, and the 90-day forward rate is \$0.64. If the U.S. dollar interest factor to cover this period is 2 percent, what is the EMU rate? What is the cost of carrying a euro forward for this period?

From the Interest Rate Parity Theorem, we know that \$1 invested in the United States must earn the same rate as the \$1 converted into a foreign currency, investing at the foreign rate and converting the proceeds back into dollars via a forward contract initiated at the outset of the transactions. For our data:

$$\$1(1.02) = (\$1/\$0.65)(1 + r_{\text{euro}})\$0.64$$

where r_{euro} = the EMU interest rate for this 90-day period. Therefore, $r_{euro} = 0.0359$. This is also the cost-to-carry a euro forward for the 90 days.

9. The Swiss franc is worth \$0.21 in the spot market. The Swiss franc futures that expires in one year trades for \$0.22. The U.S. dollar interest rate for this period is 10 percent. What should the Swiss franc interest rate be?

$$1.10 = (1/0.21)(1 + r_{SF})0.22$$

where r_{SF} = the Swiss franc interest rate for this period. Thus, $r_{SF} = 0.05$.

10. Using the data in Problem 9, explain which country is expected to experience the higher inflation over the next year. If the expected inflation rate in the United States is 7 percent, what inflation rate for the Swiss franc does this imply?

The franc is expected to increase in value against the dollar from being worth \$0.21 now to \$0.22 in 1 year. Assuming PPP, this implies that the purchasing value of the dollar will decline relative to the franc.

If the expected inflation rate in the United States is 7 percent, the real rate of interest is given by the equation:

$$1.10 = (1.07)(1 + r^*)$$

where r^* is the real rate of interest in the United States, and $r^* = 0.028$. Assuming identical real rates in the United States and Switzerland, the expected Swiss inflation rate is given by:

$$1.05 = \left[1 + E(I)\right](1.028)$$

where $E(I)$ is the expected inflation rate in Switzerland, and it equals 0.0214.

11. Using the data of Problem 9, assume that the Swiss interest rate for the year is also 10 percent. Explain how you might transact faced with these values.

Faced with the exchange rates of Problem 9 and interest rates in both the United States and Switzerland of 10 percent, we could sell dollars for francs in the spot market, invest the franc proceeds at 10 percent, and arrange now to convert the Swiss funds in 1 year at the forward rate of \$0.22. Assuming an initial amount of \$100, we would proceed as shown in the table below.

Dollar versus. Swiss franc arbitrage	
$t = 0$	
Borrow \$100 for 1 year at 10%	+\$100.00
Sell \$100 for SF476.19 in the spot market	+SF476.19 −\$100.00
Invest SF 476.19 at 10% in Switzerland	−SF476.19
Sell SF 523.81 1 year forward for \$115.24	0
Total cash flow	\$0

$t = 1$ year	
Collect SF523.81 on investment	+SF 523.81
Deliver SF523.81 on forward contract, collect \$115.24	−SF 523.81 +\$115.24
Repay debt from borrowing \$100.00	−\$110.00
Total cash flow	+\$5.24

12. Many travelers say that shoes in Italy are a big bargain. How can this be, given the PPP Theorem?

Travelers are wrong as a matter of fact, but we still must answer the question. If PPP is held with perfection, shoes would have the same cost in any currency and there would be no bargain shoes anywhere. Bargains can arise, however, due to market imperfections. First, transportation is costly. As a consequence, shoes in Italy could be cheaper than the same shoes in New York. The New York shoes must include the transportation cost. Second, even ignoring transportation costs, there are barriers to the free flow of shoes around the world. Governments impose tariffs and quotas, which can affect the price. Thus, if the United States protects its shoe industry by imposing tariffs or quotas on the Italian shoes, they can cost more in the United States, thereby making shoes in Italy a bargain.

13. For the most part, the price of oil is denominated in dollars. Assume that you are a French firm that expects to import 420,000 barrels of crude oil in 6 months. What risks do you face in this transaction? Explain how you could transact to hedge the currency portion of those risks.

Here we assume that the price of oil is denominated in dollars. Further, contracts traded on the NYMEX in oil are also denominated in dollars. Therefore, hedging on the NYMEX will not deal with the currency risk the French firm faces. However, the French firm can hedge the currency risk it faces by trading forwards for the euro. To see how the French firm can control both its risk with respect to oil prices and foreign exchange consider the following data. We assume a futures delivery date in 6 months for the oil and for foreign exchange forward contracts. The futures price of oil is \$30 barrel, and the 6-month forward price of a euro is \$0.20. With these prices, the French firm must expect a total outlay of \$12.6 million for the oil, and a total euro outlay of €63 million. By trading oil futures and euro forwards, it can lock in this euro cost. Because the crude oil contract is for 1,000 barrels, the French firm should buy 420 contracts. This commits it to a total outlay of \$12.6 million. The French firm then sells €63 million in the forward market for \$12.6 million dollars. These two transactions lock in a price of €63 million for the oil.

14. A financial comptroller for a U.S. firm is reviewing the earnings from a German subsidiary. This sub earns €1 million every year with exactitude, and it reinvests those earnings in its own German operations. This plan will continue. The earnings, however, are translated into U.S. dollars to prepare the U.S. parent's financial statements. Explain the nature of the foreign exchange risk from the point of view of the U.S. parent. Explain what steps you think the parent should take to hedge the risk that you have identified.

This risk is entirely translation risk, because we assume that the funds stay strictly in Germany. If the firm enters the futures or forward market to hedge the dollar value of the €1 million, it undertakes a transaction risk to hedge a translation risk. In other words, the firm increases its economic risk to hedge a purely accounting risk. From an economic point of view, this hedge would not make sense.

15. Joel Myers works for a large international bank. He has been watching the trading screen on this hot August morning and was disappointed in the lack of trading activity. He was just about to take a break when a flurry of activity in the Swiss bond and currency markets caught his attention. He quickly pulled up the following quotes:

Spot exchange rate	\$0.1656/SF
1-month forward	\$0.1659/SF
3-month forward	\$0.1665/SF
6-month forward	\$0.1673/SF

T-bill yields (bond equivalent)

1-month	4.95%
3-month	5.01%
6-month	5.11%

(a) Compute the 1-month (30-day), 3-month (91-day), and 6-month (182-day) yields Joel should expect to see in the Swiss money market.

For interest rate parity to hold, the Swiss interest rates should be such that Joel would be indifferent between investing in the U.S. money market and the Swiss money market. The interest rate parity relationship is:

$$1 + r_{DC} = \frac{1}{FC}(1 + r_{FC})F_{0,t}$$

where:

r_{DC} and r_{FC} are, respectively, the domestic and foreign interest rates, FC is the spot exchange rate expressed as the cost of one unit of foreign currency in terms of domestic currency, and $F_{0,t}$ is the forward exchange rate today for a transaction at time t expressed as the domestic currency cost of one foreign currency unit.

Solving for the foreign interest rate, r_{FC}, we have:

$$r_{FC} = \frac{FC}{F_{0,t}}(1 + r_{DC}) - 1$$

1-month rate:

$$r_{FC} = \left[\frac{0.1656}{0.1659}\left(1 + \frac{0.0495 \times 30}{365}\right) - 1\right]\frac{365}{30} = 2.74\%$$

3-month rate:

$$r_{FC} = \left[\frac{0.1656}{0.1665}\left(1 + \frac{0.0501 \times 91}{365}\right) - 1\right]\frac{365}{91} = 2.81\%$$

6-month rate:

$$r_{FC} = \left[\frac{0.1656}{0.1673}\left(1 + \frac{0.0511 \times 182}{365}\right) - 1\right]\frac{365}{182} = 3.02\%$$

(b) Suppose Joel sees that the 6-month yield in the Swiss money market is 4%. Assuming there are no market frictions, is arbitrage possible? If so, show the arbitrage transactions and compute the profit for a $1 million arbitrage.

Joel has already determined that the no-arbitrage 6-month return in the Swiss money market would be 3.02%. If the 6-month yield in the Swiss market is 4%, then Joel could borrow domestically, exchange the dollars for Swiss francs and invest in the Swiss money market. At the same time he would lock in a 6-month forward exchange rate to convert the francs back to dollars so the borrowing can be repaid. The profit on a $1 million arbitrage would be computed as:

Date	Cash Market	Forward Market
Today	Borrow $1 million for 6 months at 5.11% Convert $1 million to Swiss fracs at spot exchange rate of $0.1656/SF Invest SF6.0386 million for 6 months at 4% Anticipated proceeds are SF6.1591 million Net investment = 0	Sell SF6.1591 million 6 months forward at $0.1673/SF.
6 months	Receive anticipated SF6.1591 million Repay borrowing; amount due is $1.0255 million Profit = ($1.0304 − $1.0255) million = $4,936	Deliver SF6.1591 million and receive $1.0304 million.

16. As the Fall semester starts, David McElroy is making arrangements for Oklahoma State University's (OSU's) Summer in London program for the next summer. This is a program in which OSU faculty teach courses to OSU students at Regents College in London, England. Room and board is £1,500 per participant to be paid by May 15th. The enrollment is capped at 42 people, and OSU always operates at the cap. In the past, the Summer in London program has been burned by adverse movements in exchange rates. This happens because OSU has borne the exchange rate risk between the dollar denominated room and board rate quoted to the students and the British pound rate paid to Regents College. David wonders if there is some way that OSU could pass this risk off to someone else.

(a) Does OSU face translation or transaction exposure?

A trader faces transaction exposure when one currency must be converted into another. This differs from translation exposure in which one currency is restated but not converted to another currency. OSU faces transaction exposure because it will be converting dollars to pounds in May.

(b) What could OSU do to reduce this exchange rate risk?

There are several ways OSU could reduce its exchange rate risk. First, OSU could negotiate a room and board contract denominated in dollars. This would transfer the risk to Regents College. This may be a viable alternative for future years, but it is too late for this year as the contract has already been made. The second alternative is to buy British pounds forward using the futures market. This transfers the risk to a third party.

(c) David asks a finance professor for advice. The professor pulls up the following dollar per pound quotes on the £62,500 futures contract:

Delivery	**$/£**
SEP (this year)	1.6152
DEC (this year)	1.6074
MAR (next year)	1.6002
JUN (next year)	1.5936

What strategy might the professor recommend to reduce OSU's exchange rate exposure? (Make a recommendation.)

The professor might suggest buying British pounds using the June futures contract. The amount of exposure OSU has is equal to the enrollment in the program multiplied by the pound denominated room and board rate. The exposure will be:

$$\text{Exposure} = 42 \times £1500 = £63{,}000$$

To hedge this exposure, OSU should buy one June British pound futures contract at $1.5936/pound.

(d) May 15th arrives and the following situation is realized:

No of participants:	42
Dollar room and board rate:	$2,400
$/£ exchange rate:	$1.65
June futures contract:	$1.6451/pound

Compute OSU's gains and losses in the cash market and the futures market. Was the hedging strategy successful?

When May arrives, exchange rates have risen. That is, the British pound has become more expensive in dollar terms. Luckily, the June futures price has also increased resulting in gains from OSU's futures position. The gains and losses are as follows:

Date	**Cash Market**	**Futures Market**
Today	Anticipate the need for $100,397 on May 15th to make £63,000 room and board payment	Buy one June £62,500 futures contract at $1.5936/pound
May 15th	Buy £63,000 in the spot market at $1.65/pound for $103,950	Sell one June futures contract at $1.6451/pound
	Opportunity loss = − $3553	Profit = $3,219
	Net loss = −$334	

While the hedge did not totally eliminate OSU's transaction exposure, it did reduce it. Therefore, the hedge was a success.

17. Viva Soda is an up-and-comer in the highly competitive sports drink market. Viva owns three regional bottling facilities in the United States and one Canadian subsidiary that meets the demand for Viva in the Canadian provinces. Great North Bottling, the Canadian subsidiary, accounts for 25 percent of Viva's total sales and net earnings at the present exchange rates. Dave Baker, CFO for Viva, is very concerned about Viva's translation exposure. Viva will be in the debt refinancing market in 1 year. Dave is acutely aware of the relationship between the cost of debt and earnings results. Dave's assistant has made the following forecasts of Great North's earnings before taxes for the next four quarters:

Quarter	Great North Earnings before Taxes (CAN $)
DEC	10 million
MAR06	7.5 million
JUN	8.5 million
SEP	12 million

(a) What risks does Viva face with regard to its Canadian operations? What could Dave Baker do to hedge the risk?

The risk Viva faces with regard to its Canadian subsidiary is primarily translation exposure. Since Great North Bottling meets the demand of the Canadian provinces, Viva has a natural transaction hedge. This occurs when sales and expenses are denominated in the same local currency. The only risk then is the restatement (translation) of results in the home currency. Adverse movements in exchange rates could hurt Viva's reported results which could, in turn, impact their cost of debt. Dave Baker could hedge the translation exposure by selling Canadian dollars forward. Any adverse impacts of exchange rates on Great North's contribution to Viva's bottomline will be offset by gains in the futures market.

(b) Dave's assistant notes the following futures exchange rates for the Canadian dollar:

Delivery	US$/CAN$
DEC	0.6603
MAR06	0.6609
JUN	0.6615
SEP	0.6621

Design a hedge that will solve Dave's problem. Assume one futures contract is for $100,000 Canadian dollars.

To hedge the translation exposure, Dave should sell each of Great North's anticipated pretax earnings in the futures market. To do this, Dave would sell 100 December contracts, 75 March contracts, 85 June contracts, and 120 September contracts.

(c) Assuming that the spot prices shown in the following table are realized, compute the translated earnings in each quarter and the net impact on Viva's results considering the hedging activities.

Month	US$/CAN$
DEC	0.6271
MAR06	0.6827
JUN	0.5961
SEP	0.7100

The anticipated contribution of Great North to pretax earnings in each quarter is:

Quarter	Anticipated US$/CAN$	Anticipated Pre-tax Earnings in US$ (Millions)
DEC	0.6603	6.603
MAR06	0.6609	4.957
JUN	0.6615	5.623
SEP	0.6621	7.945

The realized contribution of Great North to earning before taxes each quarter is:

Quarter	Realized US$/CAN$	Realized Earning before Taxes in US$ (Millions)
DEC	0.6271	6.271
MAR06	0.6827	5.120
JUN	0.5961	5.067
SEP	0.7100	8.520

The impact of the exchange rate changes on the translated quarterly earnings is the anticipated earnings before taxes minus the realized earnings before taxes. These are:

Quarter	Realized EBT (US$ Millions)	Anticipated EBT (US$ Millions)	Gain (Loss) (US$ Millions)
DEC	6.271	6.603	(0.332)
MAR06	5.120	4.957	0.163
JUN	5.067	5.623	(0.556)
SEP	8.520	7.945	0.575

The gains and losses in the futures market are calculated as:

$$100{,}000(\text{selling price} - \text{buying price}) \times \text{no. of contracts}$$

Quarter	Selling Price	Buying Price	No.of Contracts	Gain (Loss) US$ Millions
DEC	0.6603	0.6271	100	0.332
MAR06	0.6609	0.6827	75	(0.163)
JUN	0.6615	0.5961	85	0.556
SEP	0.6621	0.7100	120	(0.575)

The gains (losses) in the futures markets offset the translation losses (gains). One thing to keep in mind is that the translation gains and losses are accounting in nature. As long as Great North cash flows are not converted back into U.S. dollars, the gains and losses are only on paper. The futures trading gains and losses are cash gain and losses. Unless there is some cash benefit to reducing the translated earnings volatility, hedging the translation exposure might not be a good idea.

AN OPTIONS PRIMER

1. Respond to the following claim: "Buying a call option is very dangerous because it commits the owner to purchasing a stock at a later date. At that time the stock may be undesirable. Therefore, owning a call option is a risky position."

Owning a call option does not involve any commitment to purchase a stock. It gives the owner the option to buy a stock if he or she wishes, but it is completely discretionary. The line of reasoning is completely incorrect. However, owning a call option is a risky position, because the value of the option is uncertain.

2. "I bought a call option with an exercise price of $110 on IBM when IBM was at $108 and I paid $6 per share for the option. Now the option is about to expire and IBM is trading at $112. There's no point in exercising the option, because I will wind up paying a total of $116 for the shares – $6 I already spent for the option plus the $110 exercise price." Is this line of reasoning correct? Explain.

Bad reasoning. The $6 option price that is already paid is a sunk cost. The only question is what to do now. With the price of IBM at $112, the option to buy it at $110 is worth at least $2, so the option should be exercised or sold before expiration.

3. What is the value of a call option on a share of stock if the exercise price of the call is $0 and its expiration date is infinite? Explain.

The call price would equal the stock price, because the option could be transformed into the stock without cost by exercise at a zero price.

4. Why is the value of a call option at expiration equal to the maximum of zero or the stock price minus the exercise price?

The price cannot be less than zero because the option involves no obligations. If the stock price exceeds the exercise price and the call price is less than the stock price minus the exercise price, there will be an arbitrage opportunity. One could buy the call, exercise it, and sell the stock for a profit. Similarly, if the call price exceeded the stock price, one could sell the call, buy the stock, and profit by the difference between the call price and the stock price.

5. Two call options on the same stock have the following features. The first has an exercise price of \$60, a time to expiration of 3 months, and a premium of \$5. The second has an exercise price of \$60, a time to expiration of 6 months, and a premium of \$4. What should you do in this situation? Explain exactly, assuming that you transact for just one option. What is your profit or loss at the expiration of the nearby option if the stock is at \$55, \$60, or \$65?

Buy a 6-month option at \$4, and sell a 3-month option at \$5. This gives a net cash inflow of \$1. If the stock price in 3 months is \$55, then the 3-month option cannot be exercised, and you can merely keep the \$1, plus your long position in the call. If the stock price in 3 months is \$60, then the 3-month option cannot be exercised, and you can merely keep the \$1, plus your long position in the call. If the stock price in 3 months is \$65, then the 3-month option will be exercised against you. You must surrender the stock and receive \$60. You exercise your call, and pay \$60 to acquire stock. This generates a zero net cash flow, but leaves you with the \$1 from the original transaction. In summary, no matter what the stock price in 3 months, you will have at least a \$1 profit. If the option you sold cannot be exercised against you, your profit will be \$1 plus the value of your long option.

6. Two call options are identical except that they are written on two different stocks with different risk levels. Which will be worth more? Why?

The call on the riskier stock should be worth more. A call option has inherent in it an insurance policy against extremely adverse outcomes, because a call option is a leveraged instrument and the value of the call cannot fall below zero.Insurance against an adverse outcome from a greater risk must be worth more, so the option on the riskier stock must be worth more.

7. Assume the following: a stock is selling for \$100, a call option with an exercise price of \$90 is trading for \$6 and matures in 1 month, and the interest rate is 1 percent per month. What should you do? Explain your transactions.

Buy the call for \$6 and exercise it, paying \$90. Sell the stock and receive \$100. This gives a total profit of \$4.

8. Two call options on the same stock expire in 2 months. One has an exercise price of \$55 and a price of \$5. The other has an exercise price of \$50 and a price of \$4. What transactions would you make to exploit this situation?

Buy the call with an exercise price of \$4, and sell the call with an exercise price of \$5. This gives a net cash flow of \$1. If the stock price at expiration is above \$55, the option you sold will be exercised against you, and your total profit will be \$1. If the stock price at expiration is \$50 or below, neither option can be exercised and your total profit will be \$1. If the stock price is greater than \$50 or equal to or less than \$55, you will make more. For example, assume that the stock price is \$53 at expiration. The option you sold cannot be exercised against you, but your option will be worth \$3. In this case, your total profit will be \$4, the \$3 current value of your option plus the \$1 cash flow you received originally.

OPTIONS ON FUTURES

1. A holder of a call futures option exercises her option. What position does she hold after exercise? Does the owner of the futures option receive any funds when she exercises? Are there any exceptions to this rule?

She receives a long position in the underlying futures contract. She should exercise only when the option is in-the-money. (In fact, she should exercise only when the futures price is above the critical futures price, which means the option is well in-the-money.) Therefore, she should receive a cash flow upon exercise. The only exception to this rule occurs if she exercises a call that is out of the money. This happens only rarely, but it does happen.

2. Compare and contrast the position of a seller of a futures option and the purchaser of a futures put as a result of exercise. Be sure to state what positions the two traders hold and the cash flows that they incur.

Let E = the exercise price of the futures option, and let F_0 = the futures price at the time of exercise. Upon exercise, the seller of a call futures option receives a short position in the futures and pays $F_0 - E$. Upon exercise, the seller of a put futures option receives a long futures position and pays $E - F_0$. Upon exercise, the owner of a put futures option receives a short position in the futures and receives $E - F_0$.

3. Explain why the Black Model applies to forward options but does not apply exactly to futures options.

The Black Model for forward options is given by:

$$C = e^{-rt}\left[F_{0,t}N(d_1^*) - EN(d_2^*)\right]$$

With our data we have:

$$d_1^* = \frac{\ln(4.00/3.80) + 0.5\,(0.25)\,(0.25)\,(0.5)}{0.25\sqrt{0.5}} = 0.3785$$

$$d_2^* = 0.3785 - 0.25\sqrt{0.5} = 0.2017$$

4. Consider the following data. A forward contract on wheat is priced at \$4/bushel and it expires in 6 months. The risk-free interest rate is 10 percent, and the standard deviation of the forward contract is 25 percent. A forward call option has an exercise price of \$3.80/bushel. Find the Black Model price for this forward option, assuming that the underlying contract amount is 5,000 bushels.

The Black Model is:

$$C = e^{-rt}\left[F_{0,t}N(d_1^*) - EN(d_2^*)\right]$$

With the values of this problem, we have:

$$d_1^* = \frac{\ln(4.00/3.80) + 0.5\,(0.25)\,(0.25)\,(0.5)}{0.25\sqrt{0.5}} = 0.3785$$

$$d_2^* = 0.3785 - 0.25\sqrt{0.5} = 0.2017$$

The cumulative probabilities are $N(0.3785) = 0.647470$ and $N(0.2017) = 0.579924$. Therefore, the Black Model value is:

$$C = e^{-0.5(0.10)}\left[\$4(0.647470) - \$3.80(0.579924)\right] = \$.3673/\text{bushel}$$

For 5,000 bushels, the option would cost $1,836.68.

5. For the forward option in Problem 4, find the value of the option if there is no uncertainty about the price of the forward contract.

With certainty, the call price is given by:

$$C_f = e^{-0.5(0.10)}\left[\$4.00 - \$3.80\right] = \$.1902/\text{bushel}$$

For 5,000 bushels, the option would cost $951.23.

6. For an option on a no-dividend stock, explain why a trader would never exercise a call option before expiration. Contrast your reasoning for the stock option to explain the circumstances that can make the early exercise of a futures call option rational.

Exercise of any option yields only the intrinsic value. For a call option on a stock this is $S - E$, where S is the stock price and E is the exercise price. As long as the option has some time remaining until expiration, the call will be worth more than $S - E$. Therefore, it is better to dispose of the call by selling it rather than exercising it before expiration. This principle has a potential exception if the underlying stock pays a dividend. In this case, there could be an incentive to exercise. By exercising, the call owner obtains the stock and the rights to dividends from the stock. In this case, it is the cash flow on the underlying asset that motivates the exercise. Early exercise of a futures call option can be rational for reasons similar to those that justify the early exercise of a call option on a dividend paying stock. Upon exercising a futures call option, the trader receives a long position in the futures and a cash payment of $F_0 - E$. This payment is now available to earn interest for the trader, but it is not available unless the trader exercises. Therefore, the exercise decision on the futures call option must weigh the benefit of that potential interest against accepting only the intrinsic value of the futures call option by exercising. As we have seen, there is a critical futures price, F^*. If the futures price is above F^*, early exercise is desirable because the interest that can be earned on $F_0 - E$ will more than compensate for discarding the call option's value in excess of the intrinsic value.

7. Which kind of futures call option will have a higher price, a European or an American style option? Explain the conditions that create the maximum difference in value for two such options on the same underlying good with the same time to expiration and the same striking price.

Because the American option gives all of the rights associated with a European option, the American option must always be worth at least as much as the European option. The American option also carries with it the right to exercise it early, and this early exercise privilege can sometimes have value. Early exercise would be desirable when the option is well into the money. Exercising in this situation discards the option value above the intrinsic value, but exercising gives immediate access to that intrinsic value. The intrinsic value received upon exercise can be invested to earn interest. Therefore, we would expect a maximal difference between the value of American and European futures call options for options well into-the-money. When the futures price attains the critical futures price F^*, the American futures option attains its maximum value of $F^* - E$, and for futures prices above F^*, the holder should exercise the futures option.

8. For European call options, what is the price relationship between an option on the physical and an option on the futures? Explain.

The prices must be the same. In both cases, the option can be exercised only at expiration. At expiration, both will be worth the difference between the value of the underlying physical good and the exercise price. Therefore, at times prior to expiration, the two options must have the same value as well.

9. For American futures options, explain the relationship between call option prices on futures and physicals, when the underlying good pays a continuous dividend. In particular, explain how the pricing depends on the relationship between the dividend rate and the interest rate.

If the physical asset pays a continuous dividend equal to the risk-free rate of interest, then the value of the option on the physical and the option on the futures will be the same. We can generalize this result. For an underlying asset that pays no dividend, such as gold, the option on the futures is worth more than the option on the physical. Likewise, if the dividend rate on the underlying asset is less than the interest rate, the call on the futures will be worth more than the call on the physical. The excess value of the option on the futures over the option on the physical falls as the dividend rate on the underlying asset increases. When the dividend rate on the underlying asset equals the risk-free rate, the call option on the physical and the futures have the same value. If the dividend rate on the physical asset exceeds the risk-free rate of interest, the call on the physical will be worth more than the call on the futures.

10. A futures contract on soybeans is priced at \$6/bushel and it expires in 3 months. The futures call on this contract has an exercise price of \$5.90 and it is priced at \$.50. If the interest rate for this period is 8 percent, what is the price of the futures put option?

Put-Call Parity for futures options is given by:

$$C_f - P_f = (F_{0,t} - E)e^{-rt}$$

Applying this parity relationship to our data, we have:

$$\$0.50 - P_f = (\$6 - \$5.90)e^{-0.08(0.25)}$$

Therefore, $P_f = \$0.500 - 0.098 = \$0.4020$/bushel.

11. The futures market just closed and you were unable to close a long position in wheat. Somehow, futures options are still trading. Explain how you would completely hedge the risk of your futures contract by trading futures options.

To completely hedge the risk associated with the futures contract, buy a put futures option and sell a call futures option. Together, these replicate a short position in the futures. Combining this synthetic short futures with the long futures that you actually hold gives a net zero position in the futures.

12. You hold a well-diversified stock portfolio and your pesky broker calls you and says: "Now is the time for you as an individual trader to get involved with portfolio insurance. Portfolio insurance

is wonderful, because it lets you avoid risk by simply buying a call option on the index. Further, this is effectively costless, because the call option is fairly priced. (You get what you pay for in the options market.) With portfolio insurance, you get the upside potential, but you are protected from downside risk. With portfolio insurance, your insured portfolio can have exactly the same value at the end of the horizon as your uninsured portfolio. You can't lose!" Analyze the sales pitch for potential weaknesses.

First, to insure a long stock portfolio, you cannot buy a call option. If anything, you would buy a put option. Second, even if options are fairly priced and even if we ignore the time decay that will reduce the value of an option, the insurance is not costless. Holding the insurance reduces the potential for large gains on the portfolio. This is the case, because you must pay for the put option, thereby increasing the net investment in the portfolio. Yes, it is true that the insured portfolio can have exactly the same value at the end of the horizon as an uninsured portfolio. However, the insured portfolio will cost more. This means that the returns on the insured portfolio will be less than the returns on the uninsured portfolio when stock prices go up. Thus, virtually every point the salesman raises is either wrong or misleading.