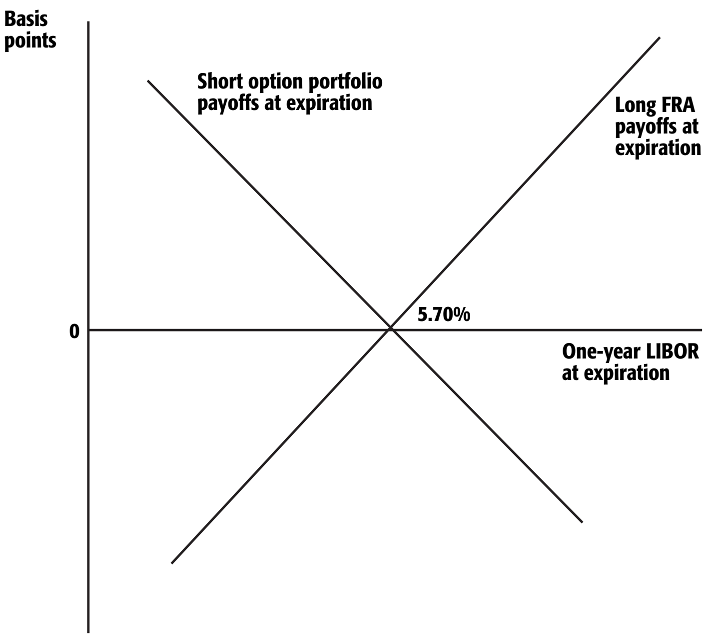
22 Swaps: Applications

Answers to Questions and Problems

1. At present, you observe the following rates: $FRA_{0,1}$ = 5.25 percent and $FRA_{1,2}$ = 5.70 percent, where the subscripts refer to years. You also observe prices on calls and puts on one-year LIBOR that expire in one year, with payment the following year. For a call on LIBOR with a strike rate of 5.70 percent, the price is 150 basis points. The corresponding put has a strike rate of 5.70 percent, and a cost of 143 basis points.

A. Explain how these prices represent an arbitrage opportunity. Draw a diagram illustrating the arbitrage opportunity.

From the forward put-call parity relationship, we know that a long FRA position is equivalent to a long call/short put portfolio with matching quantities and dates. Therefore, the cost of a long call/short put portfolio with a common strike rate equal to the FRA rate should be zero. In our situation, the long call/short put portfolio costs 7 basis points, the difference between the call cost and the put cost. Therefore, we can sell the option portfolio (sell the call and buy the put) and take a long position in the FRA. As the diagram shows, at expiration, the short call/long put portfolio and the long FRA will have exactly offsetting payoffs, for a net zero payoff.



B. Assuming a notional principal of $100 million, state the transactions that you would enter to secure the arbitrage profit.

> $t = 0$
> Sell 1 call for 150 basis points = 0.0150 × $100,000,000 = +$1,500,000.
> Buy 1 put for 143 basis points = 0.0143 × $100,000,000 = − $1,430,000.
> Enter a pay-fixed position in the FRA on one-year LIBOR with a one-year maturity at a contract rate of 5.70 percent.
> Net Cash Flow: + $70,000
>
> $t = 1$
> There will be a net zero obligation no matter what the one-year spot rate for LIBOR is. Consider two examples, with one-year LIBOR at 5.60 and 5.80 percent:
>
> *LIBOR = 5.60 percent*
> Call expires worthless.
> Long put is worth (0.0570 − 0.0560) × $100,000,000 = +$100,000.
> FRA obligation is (−0.057 + 0.056) × $100,000,000 = −$100,000.
> Net Obligation: 0
>
> *LIBOR = 5.80 percent*
> Short call is exercised against arbitrageur: (0.0580 − 0.0570) × $100,000,000 = −$100,000.
> Put expires worthless.
> Pay-fixed FRA pays (0.0580 − 0.0570) × $100,000,000 = +$100,000.
> Net Obligation: 0

C. Compute the present value of the arbitrage profit.

The present value of the arbitrage profit is $70,000, because it is received immediately at the time of contracting and no other cash flows are incurred.

2. An inverse floating rate note, or inverse floater, is a debt instrument with a floating rate that moves inversely with market rates. Generally, an inverse floater pays a fixed rate minus LIBOR. Consider an FRN with a principal amount of $50 million paying LIBOR with a five-year maturity. Consider also a plain vanilla interest rate swap with a fixed rate of 7 percent, a floating rate equal to LIBOR, a notional principal of $100 million, and a tenor of five years. For both instruments assume annual payments.

A. Explain how to construct the inverse floater from the instruments described above. Assume that there is no floor rate on the inverse floater. What is the net annual payment on the inverse floater?

To construct the inverse floater, one should issue the FRN and initiate a swap as the pay-fixed party. An annual payment on the whole position would be:

On the FRN:	Pay LIBOR × $50,000,000
On the swap:	Pay $7,000,000
	Receive LIBOR × $100,000,000
Net Payment:	$7,000,000 − LIBOR × $50,000,000

B. Compute the value of a net annual payment if LIBOR is 5.5 or 6.5 percent.

LIBOR = 5.5 percent:	$7,000,000 − 0.055 × $50,000,000 = $4,250,000
LIBOR = 6.5 percent:	$7,000,000 − 0.065 × $50,000,000 = $3,750,000

This shows that the construction is an inverse floater; as rates rise, the payment drops.

C. What happens to this instrument if LIBOR exceeds 14 percent?

As an example, let LIBOR equal 15 percent. Then the cash flow is:

$$\$7,000,000 - 0.15 \times \$50,000,000 = -\$500,000$$

The bond, instead of giving payments, requires payments. The breakeven point is 14 percent. We might say that the inverse floater sank. Typically, inverse floaters have a minimum 0 percent coupon payment so that they cannot really "sink" as in this situation. This also indicates why the fixed rate is set so high relative to LIBOR—to provide buoyancy.

D. Explain how to construct an inverse floater from a fixed rate bond and a plain vanilla swap.

One can create an inverse floater by buying a fixed rate bond and initiating a receive-fixed plain vanilla swap. The total periodic cash flow on the constructed inverse floater will consist of a fixed coupon inflow on the bond, C, a fixed inflow on the swap, $SFR \times NP$, and a floating outflow on the swap, $\text{LIBOR} \times NP$. This gives a total periodic cash flow of:

$$C + SFR \times NP - \text{LIBOR} \times NP$$

The size of this cash flow will vary inversely with LIBOR, showing that it is an inverse floater.

E. What modifications to the analysis would be required to replicate an inverse floater with a floor rate?

To replicate the floor that inverse floaters typically feature, one would need a swaption as well as the other instruments. Consider the inverse floater from the buyer's perspective. With a floor, the buyer is guaranteed to receive a rate no less than the floor rate. Therefore, the owner of the inverse floater needs a receiver swaption with a fixed rate equal to the floor rate on the inverse floater. When the rate on the inverse floater goes below the floor, the owner of the inverse floater continues to receive the low floating rate. With the receiver swaption, the floater owner can exercise the swaption to pay the floating rate and receive the fixed rate. The receipt of the floating rate on the inverse floater and the payment of the floating rate upon exercise of the swaption offset each other, leaving the owner of the inverse floater and swaption receiving a fixed rate equal to the floor rate.

3. A portfolio manager handles a Japanese equity portfolio for her clients, who are principally long-term dollar-based investors seeking exposure to the Japanese economy. The current value of the portfolio is \$1.5 billion, and the spot exchange rate is \$1 = ¥125. The portfolio pays an annual dividend of 3 percent. However, the quarterly distributions are weighted heavily toward the last quarter, with a mix of approximately 20-20-20-40 percent across the four quarters. The manager feels that her clients, many of whom rely on dollar income from the portfolio, would appreciate a dollar income stream less subject to foreign exchange risk. Therefore, the portfolio manager would like to reduce the exchange rate risk associated with the dividend payments while retaining the equity exposure to the Japanese market and while retaining the foreign exchange exposure on the principal. For the purpose of this analysis, we assume that the dividend payments are made on the last day of each quarter and that we are presently approaching year end. The manager has invited you to make proposals for avoiding this exchange rate risk over a horizon of three years. Note that the problem is not the uneven amounts of the dividends across quarters, but the foreign exchange risk of those payments.

A. Briefly compare and contrast the merits of using futures, FOREX forwards, or a swap to solve this problem.

The quarterly yen payments are ¥1.125 billion for the smaller quarters and ¥2.250 billion for the large quarter. Evaluated at the spot exchange rate, the dollar value of these payments is about \$9,000,000 and \$18,000,000, respectively. Hedging the exchange risk would essentially involve selling the yen receipts forward in favor of dollars.

The futures market would be a conceptually appropriate vehicle for this task, except the hedging horizon is three years and foreign exchange futures are really viable only for a horizon of one year. Therefore, futures can provide only part of the solution at best. The risk could also be hedged by entering twelve FOREX forward contracts, one for each dividend payment over the next three years. A plain vanilla foreign currency swap is available with the necessary tenor, but it has two drawbacks. First, it involves an exchange of principal, which really is not needed in this context. Second, the dividend payments have a seasonal character that is not amenable to a plain vanilla swap.

B. Explain how you would address this problem by using at least one swap among the instruments you would recommend.

One might approach this problem with a fixed-for-fixed seasonal foreign currency swap with no exchange of principal. The problem does not provide information on prevailing forward FOREX rates, but the swap could be established to provide ¥1.125 billion for each of the next three quarters, followed by ¥2.250 billion for the fourth quarter, each payment being in exchange for a fixed dollar amount that reflects current FOREX forward rates. The last two years of the swap would repeat the pattern of the first year. Alternatively, one might go to the FOREX forward market.

4. Assume you can borrow at a fixed rate for ten years for 11 percent or that you can borrow at a floating rate of LIBOR plus 40 basis points for ten years. Assume also that LIBOR stands at 10.60 percent. Under these circumstances, your financial advisor states: "The all-in cost is the same on both deals—11 percent. Therefore, the two are equivalent, and one should be indifferent between these two financing alternatives." How would you react? Explain.

   Your financial advisor is naive, because she is neglecting the shape of the term structure. Her advice would have some foundation if the yield curve were flat at 11 percent. In this situation, the expected cost on the two alternatives would be the same. However, the short-term strategy still involves risks (and opportunities) that the fixed rate strategy does not possess.

5. A firm needs to secure fixed rate financing of $40 million for a horizon of five years. It is considering two alternatives that seem equally attractive to management in terms of the instruments involved. Thus, the choice revolves solely around the relative cost. Alternative A is to issue a straight bond with annual coupon payments. The bond would sell at par, the firm would net 98 percent of the sale proceeds, and the coupon rate would be 7 percent. Administrative costs would be $30,000 for each coupon payment, and would be payable on the payment date. Under Alternative B, the firm would engage in a sequence of one-year financings, obtained at the prevailing rate each year, with each annual financing in the amount of $40 million. The firm would pair this financing with a pay-fixed interest rate swap. The firm's investment bank has guaranteed to make this sequence of loans at LIBOR, but would charge a fee of 15 basis points on each loan amount, payable at each loan date. The investment bank would also take the receive-fixed side of the swap that the firm needs with a fixed rate of 7.3 percent. The two alternatives differ slightly in the dollar amount of funds that they would provide, but this difference can be ignored. What is the all-in cost for each alternative?

   We first need to set out all of the cash flows from each alternative as shown in the following table:

	Cash Flows for the Two Financing Alternatives			
Year	**A**	**B-Bond**	**B-Swap**	**B-Net**
0	+0.98 × ($40,000,000)	$40,000,000 − 0.0015 × ($40,000,000)		$40,000,000 − 0.0015 × ($40,000,000)
1	− 0.07 × ($40,000,000) − $30,000	$40,000,000 − 0.0015 × ($40,000,000) − LIBOR × $40,000,000 − $40,000,000	(LIBOR − 0.073) × $40,000,000	− (0.0015 + 0.073) × $40,000,000
2	− 0.07 × ($40,000,000) − $30,000	$40,000,000 − 0.0015 × ($40,000,000) − LIBOR × $40,000,000 − $40,000,000	(LIBOR − 0.073) × $40,000,000	− (0.0015 + 0.073) × $40,000,000
3	− 0.07 × ($40,000,000) − $30,000	$40,000,000 − 0.0015 × ($40,000,000) − LIBOR × $40,000,000 − $40,000,000	(LIBOR − 0.073) × $40,000,000	− (0.0015 + 0.073) × $40,000,000
4	− 0.07 × ($40,000,000) − $30,000	$40,000,000 − 0.0015 × ($40,000,000) − LIBOR × $40,000,000 − $40,000,000	(LIBOR − 0.073) × $40,000,000	− (0.0015 + 0.073) × $40,000,000
5	− 0.07 × ($40,000,000) − $30,000 − $40,000,000	− LIBOR × $40,000,000 − $40,000,000	(LIBOR − 0.073) × $40,000,000	− (0.073) × $40,000,000 − $40,000,000

The following table summarizes and consolidates all of the cash flow information for the two alternatives and shows the IRRs for each alternative. The financing alternative using a sequence of FRNs and a swap saves about 11 basis points per year.

	Net Flows and IRRs for Each Alternative	
Year	**Net Flows–Alternative A**	**Net Flows–Alternative B**
0	39,200,000	39,940,000
1	−2,830,000	−2,980,000
2	−2,830,000	−2,980,000
3	−2,830,000	−2,980,000
4	−2,830,000	−2,980,000
5	−42,830,000	−42,920,000
	IRR 0.075703	IRR 0.074612

6. Today the following rates may be observed: $FRA_{0,3} = 0.0600$; $FRA_{3,6} = 0.0595$; $FRA_{6,9} = 0.0592$; $FRA_{9,12} = 0.0590$; $FRA_{12,15} = 0.0590$; $FRA_{15,18} = 0.0588$; $FRA_{18,21} = 0.0587$; $FRA_{21,24} = 0.0586$, where the subscripts pertain to months. Consider a two-year plain vanilla interest rate swap with quarterly payments, and also consider a forward interest rate swap to be initiated in twelve months, with quarterly payments, a notional principal of \$20 million, and a tenor of twelve months.

A. Without computing the *SFR* for the forward swap, compare and contrast the *SFR* for the plain vanilla swap of the previous question with the *SFR* for the forward swap. What should the relationship be between the two *SFR*s?

The FRA rates for the second year are lower than those for the first year, so the forward swap should have a lower *SFR* than the swap covering both years.

B. Find the *SFR* for the forward swap.

We first find the zero-coupon factors that pertain to the forward swap:

$$Z_{0,3} = 1 + FRA_{0,3}/4 = 1 + 0.0600/4 = 1.01500$$
$$Z_{0,6} = Z_{0,3}(1 + FRA_{3,6}/4) = 1.015(1 + 0.0595/4) = 1.030098$$
$$Z_{0,9} = Z_{0,6}(1 + FRA_{6,9}/4) = 1.030098(1 + 0.0592/4) = 1.045343$$
$$Z_{0,12} = Z_{0,9}(1 + FRA_{9,12}/4) = 1.045343(1 + 0.0590/4) = 1.060762$$
$$Z_{0,15} = Z_{0,12}(1 + FRA_{12,15}/4) = 1.060762(1 + 0.0590/4) = 1.076409$$
$$Z_{0,18} = Z_{0,15}(1 + FRA_{15,18}/4) = 1.076409(1 + 0.0588/4) = 1.092232$$
$$Z_{0,21} = Z_{0,18}(1 + FRA_{18,21}/4) = 1.092232(1 + 0.0587/4) = 1.108260$$
$$Z_{0,24} = Z_{0,21}(1 + FRA_{21,24}/4) = 1.108260(1 + 0.0586/4) = 1.124496$$

To find the *SFR* for the forward swap, we observe that both sides of the swap must have the same present value at the time the swap is initiated:

$$PART \times NP\left(\frac{FRA_{12,15}}{Z_{0,15}} + \frac{FRA_{15,18}}{Z_{0,18}} + \frac{FRA_{18,21}}{Z_{0,21}} + \frac{FRA_{21,24}}{Z_{0,24}}\right)$$
$$= PART \times NP \times SFR\left(\frac{1}{Z_{0,15}} + \frac{1}{Z_{0,18}} + \frac{1}{Z_{0,21}} + \frac{1}{Z_{0,24}}\right)$$

Therefore,

$$SFR = \frac{\dfrac{FRA_{12,15}}{Z_{0,15}} + \dfrac{FRA_{15,18}}{Z_{0,18}} + \dfrac{FRA_{18,21}}{Z_{0,21}} + \dfrac{FRA_{21,24}}{Z_{0,24}}}{\dfrac{1}{Z_{0,15}} + \dfrac{1}{Z_{0,18}} + \dfrac{1}{Z_{0,21}} + \dfrac{1}{Z_{0,24}}}$$

$$= \frac{\frac{0.0590}{1.076409} + \frac{0.0588}{1.092232} + \frac{0.0587}{1.108260} + \frac{0.0586}{1.124496}}{\frac{1}{1.076409} + \frac{1}{1.092232} + \frac{1}{1.108260} + \frac{1}{1.124496}}$$

$$SFR = \frac{0.054812 + 0.053835 + 0.052966 + 0.052122}{0.929015 + 0.915556 + 0.902315 + 0.889287} = \frac{0.213725}{3.636173} = 0.058777$$

C. Assume that today the following rates are observed: $FRA_{0,3} = 0.0590$; $FRA_{3,6} = 0.0588$; $FRA_{6,9} = 0.0587$; $FRA_{9,12} = 0.0586$, where the subscripts pertain to months. What would be the *SFR* for a plain vanilla swap based on these rates, assuming a tenor of one year and quarterly payments? Explain.

First, we find the zero-coupon factors:

$Z_{0,3} = 1 + 0.0590/4 = 1.014750$
$Z_{0,6} = 1.014750 \times (1 + 0.0588/4) = 1.029667$
$Z_{0,9} = 1.029667 \times (1 + 0.0587/4) = 1.044777$
$Z_{0,12} = 1.044777 \times (1 + 0.0586/4) = 1.060083$

Next, we apply Equation 22.15, treating the numerator and denominator separately for convenience:

$$NUMERATOR = \frac{0.0590}{1.014750} + \frac{0.0588}{1.029667} + \frac{0.0587}{1.044777} + \frac{0.0586}{1.060083} = 0.226711$$

$$DENOMINATOR = \frac{1}{1.014750} + \frac{1}{1.029667} + \frac{1}{1.044777} + \frac{1}{1.060083} = 3.857117$$

$$SFR = \frac{0.226711}{3.857117} = 0.058777$$

The forward swap that we considered earlier and this plain vanilla swap have exactly the same pattern of rates. In other words, the four rates in this plain vanilla swap are the same as the last four FRA rates on our original swap. Therefore, the plain vanilla swap and a forward swap will have the same *SFR*.

7. Consider an amortizing interest rate swap with an initial notional principal of $50 million, annual payments, and a tenor of five years that is negotiated today. The notional principal is $50 million for the first payment, and then drops $10 million per year. FRA rates covering the tenor of this swap are as follows, where the subscripts indicate years: $FRA_{0,1} = 0.0600$; $FRA_{1,2} = 0.0615$; $FRA_{2,3} = 0.0623$; $FRA_{3,4} = 0.0628$; $FRA_{4,5} = 0.0633$.

A. Assuming that Equation 22.15 does not pertain to an amortizing swap, which numbered equation from the chapter is best suited to computing the *SFR* for this amortizing swap? Explain.

Equation 22.16 works best, because it allows for the notional principal to be different for each payment on the swap.

B. Find the relevant zero-coupon factors for each of the payments in the swap.

$Z_{0,1} = 1 + FRA_{0,1} = 1.0600$
$Z_{0,2} = Z_{0,1}(1 + FRA_{1,2}) = 1.0600\,(1.0615) = 1.125190$
$Z_{0,3} = Z_{0,2}(1 + FRA_{2,3}) = 1.125190\,(1.0623) = 1.195289$
$Z_{0,4} = Z_{0,3}(1 + FRA_{3,4}) = 1.195289\,(1.0628) = 1.270354$
$Z_{0,5} = Z_{0,4}(1 + FRA_{4,5}) = 1.270354\,(1.0633) = 1.350767$

C. Compute the *SFR* for this swap.

Letting NP_t = the payment at year t, we have: $NP_1 = \$50,000,000$; $NP_2 = \$40,000,000$; $NP_3 = \$30,000,000$; $NP_4 = \$20,000,000$; $NP_5 = \$10,000,000$. Therefore, the SFR must satisfy this equation:

$$\frac{FRA_{0,1} \times NP_1}{Z_{0,1}} + \frac{FRA_{1,2} \times NP_2}{Z_{0,2}} + \frac{FRA_{2,3} \times NP_3}{Z_{0,3}} + \frac{FRA_{3,4 \times} NP_4}{Z_{0,4}} + \frac{FRA_{4,5} \times NP_5}{Z_{0,5}}$$
$$= SFR\left(\frac{NP_1}{Z_{0,1}} + \frac{NP_2}{Z_{0,2}} + \frac{NP_3}{Z_{0,3}} + \frac{NP_4}{Z_{0,4}} + \frac{NP_5}{Z_{0,5}}\right)$$

Treating the numerator and denominator separately, we have:

$$SFR = \frac{NUMERATOR}{DENOMINATOR}$$

where:

$$NUMERATOR = \frac{0.0600 \times \$50,000,000}{1.0600} + \frac{0.0615 \times \$40,000,000}{1.125190}$$
$$+ \frac{0.0623 \times \$30,000,000}{1.195289} + \frac{0.0628 \times \$20,000,000}{1.270354} + \frac{0.0633 \times \$10,000,000}{1.350767}$$
$$= \$2,830,189 + \$2,186,297 + \$1,563,639 + \$988,701 + \$468,623 = \$8,037,488$$

$$DENOMINATOR = \frac{\$50,000,000}{1.0600} + \frac{\$40,000,000}{1.125190} + \frac{\$30,000,000}{1,195289} + \frac{\$20,000,000}{1.270354} + \frac{\$10,000,000}{1.350767}$$
$$+ \$47,169,811 + \$35,549,552 + \$25,098,533 + \$15,743,643 + \$7,403,201 = \$130,964,740$$

$$SFR = \frac{\$8,037,448}{\$130,964,740} = 0.061371$$

Note: The following interest rate data for the United States and Britain are used in almost all of the remaining problems. These data cover the next 10 years by semiannual periods.

8. Complete the following table for US interest rates.

	US Interest Rates			
Semiannual Periods	**Annualized Par Yield**	**Semiannual Par Yield**	**Zero-Coupon Factor**	**Forward Rate Factor**
1	0.055329	0.027665	1.027665	1.027665
2	0.055682	0.027841	1.056462	1.028022
3	0.056260	0.028130	1.086817	1.028733
4	0.057375	0.028688	1.119922	1.030461
5	0.057944	0.028972	1.153741	1.030197
6	0.057961	0.028981	1.187231	1.029027
7	0.059389	0.029695	1.228150	1.034466
8	0.059813	0.029907	1.266943	1.031587
9	0.059910	0.029955	1.305460	1.030401
10	0.060649	0.030325	1.350134	1.034221
11	0.060982	0.030491	1.393981	1.032476
12	0.061214	0.030607	1.438803	1.032154
13	0.061532	0.030766	1.486458	1.033121
14	0.061702	0.030851	1.534379	1.032238
15	0.062721	0.031361	1.596588	1.040544
16	0.063835	0.031918	1.665192	1.042969
17	0.064431	0.032216	1.729578	1.038666
18	0.064785	0.032393	1.792790	1.036547
19	0.066198	0.033099	1.884719	1.051277
20	0.066626	0.033313	1.958708	1.039258

9. Complete the following table for British interest rates.

	British Interest Rates			
Semiannual Periods	Annualized Par Yield	Semiannual Par Yield	Zero-Coupon Factor	Forward Rate Factor
1	0.053832	0.026916	1.026916	1.026916
2	0.054334	0.027167	1.055079	1.027425
3	0.056197	0.028099	1.086778	1.030044
4	0.057242	0.028621	1.119689	1.030284
5	0.058987	0.029494	1.156934	1.033264
6	0.059890	0.029945	1.194448	1.032425
7	0.061010	0.030505	1.235377	1.034266
8	0.062106	0.031053	1.279148	1.035431
9	0.062947	0.031474	1.324413	1.035387
10	0.064515	0.032258	1.378273	1.040667
11	0.064558	0.032279	1.423121	1.032539
12	0.066683	0.033342	1.491341	1.047937
13	0.067310	0.033655	1.548687	1.038453
14	0.068090	0.034045	1.611660	1.040662
15	0.069823	0.034912	1.694301	1.051277
16	0.069891	0.034946	1.754696	1.035646
17	0.070146	0.035073	1.821253	1.037931
18	0.070253	0.035127	1.887592	1.036425
19	0.071105	0.035553	1.975971	1.046821
20	0.071393	0.035697	2.054628	1.039807

10. Consider a forward interest rate swap on British rates from the perspective of time zero. The forward swap has semiannual payments, and the first payment will be made in three years. The swap has a tenor of five years and a notional principal of £100,000,000. From the perspective of time zero, find the SFR for this forward swap. If we were to treat the forward swap as an immediate swap, at what time would the swap be valued? From that point in time, treat the forward swap as an immediate swap and determine the SFR. Show that the two SFRs computed are the same.

First, from time zero, the following table shows the cash flows on the forward swap, and various intermediate calculations for the analysis:

	Forward Swap Analysis Standpoint of Time Zero			
Period	Zero-Coupon Factor, $Z_{0,t}$	Forward Rate Factor, $FRF_{t-1,t}$	$1/Z_{0,t}$	$FR_{t-1,t}/Z_{0,t}$
6	1.194448	1.032425	0.837207	0.027146
7	1.235377	1.034266	0.809469	0.027737
8	1.279148	1.035431	0.781770	0.027699
9	1.324413	1.035387	0.755051	0.026719
10	1.378273	1.040667	0.725546	0.029506
11	1.423121	1.032539	0.702681	0.022865
12	1.491341	1.047937	0.670537	0.032144
13	1.548687	1.038453	0.645708	0.024829
14	1.611660	1.040662	0.620478	0.025230
15	1.694301	1.051277	0.590214	0.030264
SUMS			7.138663	0.274139

As we have noted, a forward swap can be priced using the plain vanilla swap equation, Equation 21.13:

$$SFR = \frac{\sum_{n=1}^{N} \frac{FRA_{(n-1)\times MON,\, n\times MON}}{Z_{0,\, n\times MON}}}{\sum_{n=1}^{N} \frac{1}{Z_{0,\, n\times MON}}} = \frac{0.274139}{7.138663} = 0.038402$$

This is expressed in semiannual terms, so the annualized rate is twice as large, or 0.076804.

We can also price this swap from the point of view of a swap that is immediately negotiated. If the first payment is to occur at year 3, we price the swap from the standpoint of time 2.5 years. Where the subscripts refer to semiannual periods, we have $Z_{0,5} = 1.156934$ from the table in question 9.

	Forward Swap Analysis Standpoint of Time 2.5 Years			
Period	**Zero-Coupon Factor, $Z_{0,t}/Z_{0,5}$**	**Forward Rate Factor, $FRF_{t-1,t}$**	$\frac{1/Z_{0,t}}{Z_{0,5}}$	$\frac{FR_{t-1,t}/Z_{0,t}}{Z_{0,5}}$
6	1.032425	1.032425	0.968593	0.031407
7	1.067802	1.034266	0.936503	0.032090
8	1.105636	1.035431	0.904457	0.032046
9	1.144761	1.035387	0.873545	0.030912
10	1.191315	1.040667	0.839408	0.034136
11	1.230080	1.032539	0.812955	0.026453
12	1.289046	1.047937	0.775768	0.037188
13	1.338613	1.038453	0.747042	0.028726
14	1.393044	1.040662	0.717852	0.029189
15	1.464475	1.051277	0.682839	0.035014
SUMS			8.258962	0.317161

$$SFR = \frac{0.317161}{8.258962} = 0.038402$$

This is the same answer we found from the standpoint of time zero.

11. In the United States, a retailer is faced with a seasonal cash flow pattern over the next five years. The retailer has determined to enter a pay-fixed seasonal swap with a tenor of five years. In the first part of each year, the notional principal will be $50,000,000. In the second half of each year, the notional principal will be $75,000,000. Determine the *SFR* for this seasonal swap, and complete the following table.

	Seasonal Swap Cash Flows		
Semiannual Periods	**Notional Principal**	**Fixed Payment**	**Implied Floating Payment**
1	$50,000,000	1,517,950	1,383,250
2	$75,000,000	2,276,925	2,101,650
3	$50,000,000	1,517,950	1,436,650
4	$75,000,000	2,276,925	2,284,575
5	$50,000,000	1,517,950	1,509,850
6	$75,000,000	2,276,925	2,177,025
7	$50,000,000	1,517,950	1,723,300
8	$75,000,000	2,276,925	2,369,025
9	$50,000,000	1,517,950	1,520,050
10	$75,000,000	2,276,925	2,566,575

Because the notional principal varies in a seasonal swap, we cannot use the plain vanilla pricing equation. Instead, we need to use Equation 22.16:

$$SFR = \frac{\sum_{n=1}^{N} \frac{NP_n \times FRA_{(n-1) \times MON, n \times MON}}{Z_{0, n \times MON}}}{\sum_{n=1}^{N} \frac{NP_n}{Z_{0, n \times MON}}}$$

The following table shows the intermediate calculations in applying Equation 22.16:

	Seasonal Swap Analysis		
Period	**NP_t**	**$(NP_t \times FR_{t-1,t}) / Z_{0,t}$**	**$NP_t / Z_{0,t}$**
1	\$50,000,000	1,346,013	48,653,987
2	\$75,000,000	1,989,329	70,991,668
3	\$50,000,000	1,321,888	46,005,905
4	\$75,000,000	2,039,941	66,968,950
5	\$50,000,000	1,308,656	43,337,283
6	\$75,000,000	1,833,700	63,172,205
7	\$50,000,000	1,403,167	40,711,639
8	\$75,000,000	1,869,875	59,197,612
9	\$50,000,000	1,164,379	38,300,676
10	\$75,000,000	1,900,978	55,550,042
SUMS		16,177,925	532,889,967

$$SFR = \frac{16{,}177{,}925}{532{,}889{,}967} = 0.030359$$

This computed SFR is for a semiannual period. In annual terms, $SFR = 0.060718$. To complete the table showing the cash flows, we note that the fixed payment is just the semiannual SFR times the notional principal for the period. Each floating payment is the implied forward rate times the notional principal for the period.

12. In Great Britain, the manager of a fixed rate mortgage portfolio at a building society is quite satisfied with the composition of her portfolio, but she would like to protect the value of the portfolio from rising interest rates. The current market and principal value of the portfolio is £200,000,000. The mortgages in the portfolio are somewhat unusual in that they do not allow prepayment. However, the principal on the mortgages is being repaid over the next five years consistent with the schedule in the following table. The manager is considering two possible swaps to hedge the interest rate risk of the portfolio. First, the manager might initiate a plain vanilla interest rate swap with a notional principal of £100,000,000 and a tenor of five years. Alternatively, the manager is wondering if she might prefer an amortizing swap with a notional principal schedule that matches the table below and a tenor of five years.

Semiannual Periods	Notional Principal
1	£200,000,000
2	175,000,000
3	133,000,000
4	127,000,000
5	107,000,000
6	85,000,000
7	77,000,000
8	65,000,000
9	45,000,000
10	22,000,000

A. By inspecting the British interest rates, but without computation, which swap would have the higher *SFR*? Explain.

It is a close call by visual inspection, but the yield curve is rising and the plain vanilla swap has a higher notional principal in the later periods subject to the higher rate. Therefore, the plain vanilla swap probably has the higher rate.

B. Find the *SFR* for the plain vanilla swap.

Using the data from the table below:

$$SFR = \frac{0.274455}{8.508239} = 0.032258$$

Therefore, in annualized terms, the *SFR* is 0.064516.

C. Find the *SFR* for the amortizing swap.

For this amortizing swap, the notional principal varies, so that we must use Equation 22.16 and the calculations in the table below:

$$SFR = \frac{28{,}074{,}241}{923{,}158{,}660} = 0.030411$$

In annualized terms, $SFR = 0.060822$.

The following table presents the intermediate calculations useful in the *SFR* for both swaps.

Semiannual Periods	British Building Society Amortizing Swap					
	NP	$FRA_{t-2,t}$	$1/Z_{0,t}$	$FRA_{t-1,t}/Z_{0,t}$	$NP/Z_{0,t}$	$(NP \times FRA_{t-1,t})/Z_{0,t}$
1	£200,000,000	0.026916	0.973789	0.026211	194,757,896	5,242,104
2	175,000,000	0.027425	0.947796	0.025993	165,864,357	4,548,830
3	133,000,000	0.030044	0.920151	0.027645	122,380,100	3,676,788
4	127,000,000	0.030284	0.893105	0.027047	113,424,353	3,434,943
5	107,000,000	0.033264	0.864354	0.028752	92,485,829	3,076,449
6	85,000,000	0.032425	0.837207	0.027146	71,162,579	2,307,447
7	77,000,000	0.034266	0.809469	0.027737	62,329,151	2,135,771
8	65,000,000	0.035431	0.781770	0.027699	50,815,074	1,800,429
9	45,000,000	0.035387	0.755051	0.026719	33,977,317	1,202,355
10	22,000,000	0.040667	0.725546	0.029506	15,962,005	649,127
SUMS			8.508239	0.274455	923,158,660	28,074,241

D. Which of the two swaps would you recommend to the manager? Which side of the swap should the manager take? Explain.

The portfolio manager receives a fixed rate on the mortgages, so she needs a pay-fixed swap. This leaves a position in which the mortgage portfolio essentially earns a floating rate. Of the two swaps, the amortizing swap more closely matches the cash flows on the portfolio. Also, the amortizing swap has a lower fixed rate. Since she will be the fixed-rate payer, both considerations point toward the amortizing swap.

The following table presents the intermediate calculations useful in finding the *SFR* for both swaps.

13. At time zero, the five-year forward FOREX rate is $1 = £0.580024. What is the current spot exchange rate to the nearest cent? Rounding the spot exchange rate to the nearest full cent, complete the following table.

10-period zero-coupon factor for the United States = 1.350134
10-period zero-coupon factor for Great Britain = 1.378273
10-period foreign exchange forward rate = 0.580024

The proceeds of investing £1 for 10 semi-annual periods in Britain and then converting to dollars is:

$$£1 \times 1.378273 \times 1/0.580024 = \$2.376234$$

This must have the same value as converting £1 to dollars at the (unknown) spot rate and investing the proceeds at the dollar rate for the same period:

$$£1 \times \text{SPOT} \times 1.350134 = \$2.376234$$

Therefore, the spot rate is £1 = $1.76.

	US and British Foreign Exchange Rates	
Semiannual Periods	**$ per £**	**£ per $**
0	1.760000	0.568182
1	1.761284	0.567768
2	1.762307	0.567438
3	1.760063	0.568161
4	1.760366	0.568064
5	1.755143	0.569754
6	1.749366	0.571636
7	1.749704	0.571525
8	1.743207	0.573655
9	1.734814	0.576431
10	1.724068	0.580024
11	1.723962	0.580059
12	1.697997	0.588929
13	1.689280	0.591968
14	1.675606	0.596799
15	1.658498	0.602955
16	1.670225	0.598722
17	1.671408	0.598298
18	1.671606	0.598227
19	1.678722	0.595691
20	1.677835	0.596006

14. At time zero, a British exporter has just entered a contract to supply Princess Diana memorabilia to a US customer. The contract calls for the purchase of $50,000,000 worth of material each six months over the next four years. The US customer will pay in dollars. The British exporter would like to avoid the foreign exchange risk inherent in this transaction, but is unsure whether to accept exposure to British interest rates. The exporter also needs additional financing to cope with this increase in business and is considering issuing a quanto note to provide the financing and to cope with the interest rate risk and possibly the foreign exchange risk inherent in the large new order. The quanto note the exporter is considering would be based on British LIBOR, would have payments in US dollars, and would have terms that match the Princess Diana order. Specifically, the issue would be for $400,000,000, with semiannual interest-only payments, and would have a maturity of four years. The spot exchange rate is £1 = $1.760000.

A. Explain the interest rate and currency exposure that the British borrower would face if he issued the quanto note in conjunction with the Princess Diana cash flows.

From the Princess Diana order, the exporter has foreign exchange risk, because he will incur costs in British pounds to produce the merchandise, but will receive US dollars in payment. By issuing a quanto note, the exporter will receive a large dollar inflow at time zero that involves some FOREX risk. However, the exporter could immediately convert those dollars to pounds at the spot rate. In addition, the periodic outflows on the quanto note will be in dollars. These outflows will largely offset the dollar inflows, helping to deal with the foreign exchange risk inherent in the Princess Diana order. However, there may be some residual foreign exchange risk due to a mismatch in the periodic flows. The interest rate risk position after issuing the quanto note is that the exporter is exposed to floating British interest rates, versus a fixed dollar receipt.

B. By visually comparing the British and US yield curves, what can you infer about the likely spread? Price the quanto note. What spread should the exporter expect on the issuance?

The British rates are initially lower than the US rates, but the British yield curve is more strongly upward sloping. Therefore, the spread should be close to zero. As the following calculations show, the spread is *very* narrow.

Fair pricing for the quanto note equates the \$400,000,000 received at issuance versus the present value of the repayment stream. In the following equation, the LIBOR values can be thought of as representing the British forward rates that prevail at issuance. Alternatively, the actual cash flows on the quanto note will depend on the rates of British LIBOR that prevail on each determination date. Given the term structure information we already possess, the only unknown in this equation is the spread over or under British LIBOR (*SPRD*) on the note.

$$\$400{,}000{,}000 = \sum_{t=1}^{8} \frac{(\pounds\ LIBOR_t + SPRD) \times \$400{,}000{,}000}{Z_{0,t}} + \frac{\$400{,}000{,}000}{Z_{0,8}}$$

Notice that the principal amount does not affect the spread, as the preceding equation simplifies to:

$$1 - \frac{1}{Z_{0,8}} = \sum_{t=1}^{8} \frac{(\pounds\ LIBOR_t + SPRD)}{Z_{0,t}}$$

The left-hand term is 0.21069851. *SPRD* is found by an iterative search to be –0.00115631. Therefore, the issuance should be at British LIBOR minus about 11 or 12 basis points. The following table illustrates the accuracy of the calculation:

	The Quanto Note Spread Calculation		
Semiannual Periods	$Z_{0,t}$	**£** FR_t	**(£** $FR_t + SPRD)/Z_{0,t}$
1	1.027665	0.026916	0.02506623
2	1.056462	0.027425	0.02486478
3	1.086817	0.030044	0.02658009
4	1.119922	0.030284	0.02600868
5	1.153741	0.033264	0.02782920
6	1.187231	0.032425	0.02633749
7	1.228150	0.034266	0.02695900
8	1.266943	0.035431	0.02705306
SUM			0.21069853

C. Complete the following table, identifying the implied dollar cash flows on the quanto note. How closely do these match the dollar inflows? What do you think of the proposed issuance size as a means of offsetting the foreign exchange risk?

The following table shows the quanto note outflows implied by the term structure and the difference. Considering the interest payments on the quanto note in isolation, the quanto note accounts for only about 20 percent of the dollar inflows, leaving roughly \$35–40,000,000 mismatch. In addition, the quanto note creates a large mismatch in the final period when the principal is returned. At best, issuing the quanto note is only a partial solution to the FOREX risk.

	Quanto Note Cash Flow Comparison		
Semiannual Periods	Dollar Inflow from Sales	Implied Quanto Note Outflow	Net Dollar Flow
1	$50,000,000	10,303,876	39,696,124
2	$50,000,000	10,507,476	39,492,524
3	$50,000,000	11,555,076	38,444,924
4	$50,000,000	11,651,076	38,348,924
5	$50,000,000	12,843,076	37,156,924
6	$50,000,000	12,507,476	37,492,524
7	$50,000,000	13,243,876	36,756,124
8	$50,000,000	413,709,876	−363,709,876

D. The exporter is uncertain that the quanto note approach is correct, but is convinced that the US provides the best capital pool for achieving his financing. An alternative approach is to issue a four-year semiannual payment, amortizing note in the US for $350,000,000 at the fixed US rate. Find the rate for this issuance that is compatible with the US term structure, and complete the following table for the issuance amount of $350,000,000. Discuss the suitability of this approach in terms of avoiding the foreign exchange risk inherent in the Princess Diana project.

	Amortizing Note Cash Flow Comparison–$350,000,000 Principal		
Semiannual Periods	Dollar Inflow from Sales	Implied Note Outflow	Net Dollar Flow
1	$50,000,000	$49,678,870	+$321,130
2	$50,000,000	$49,678,870	$321,130
3	$50,000,000	$49,678,870	$321,130
4	$50,000,000	$49,678,870	$321,130
5	$50,000,000	$49,678,870	$321,130
6	$50,000,000	$49,678,870	$321,130
7	$50,000,000	$49,678,870	$321,130
8	$50,000,000	$49,678,870	$321,130

The payment on the amortizing note, PAY, is found as:

$$PAY = \frac{\$350{,}000{,}000}{\sum_{t=1}^{8} \frac{1}{Z_{0,t}}} = \$49{,}678{,}870$$

The yield to maturity on this note is the IRR of the cash flows. The semiannual IRR is 0.029139 for an issuance rate of 0.058279. Note that this is slightly less than the par yield (0.059813) for a four-year coupon bond, because the yield curve is rising and the amortized note has an effectively shorter maturity (duration) than a coupon bond. The amortizing note with a principal of $350,000,000 creates a dollar obligation that almost exactly offsets the $50,000,000 semiannual inflow, leaving the British exporter with a net dollar inflow of $321,130 semiannually.

E. As another alternative, the British exporter may just finance in England and confront the financial risk of the dollar inflows with a swap. That is, the British exporter is interested in swapping the eight $50,000,000 inflows against British pounds. Propose two alternative currency swaps in which the exporter pays his dollar receipts in exchange for British pounds. One swap gives the exporter a fixed British pound payment, while the other gives a floating payment. Price the fixed rate swap, and complete the following table detailing the cash flows on the floating rate swap.

These two swaps are currency annuity swaps, as there is no principal to be exchanged, but just a sequence of even dollar payments. In both swaps, the exporter will be paying the present value of the dollar flows discounted at the US rate. In return, one swap will pay eight payments of a fixed amount in pounds that have a present value that equals that of the US dollar cash flow stream. In the second swap, the exporter will receive a sequence of pound payments that depend on changes in British LIBOR. However, the table illustrates the payment stream consistent with the British term structure at the inception of the swap.

First, the present value of the dollar cash flow stream is:

$$PV = \$50{,}000{,}000 \times \sum_{t=1}^{8} \frac{1}{Z_{0,t}} = \$50{,}000{,}000 \times 7.045249 = \$352{,}262{,}450$$

With a spot exchange rate of £1 = \$1.760000, the value of the dollar stream is £200,149,119. For the fixed British pound swap, the payment stream must have a present value of £200,149,119 when the payments are discounted according to the British term structure, with each payment, PAY, being given by:

$$PV = PAY \times \sum_{t=1}^{8} \frac{1}{Z_{0,t}}$$

$$£200{,}149{,}119 = PAY \times 7.027642$$

$$PAY = £28{,}480{,}267$$

For the swap that pays a floating rate in British pounds, the sequence of floating rate payments consistent with the British term structure must have a present value equal to the present value of the dollar inflows converted to British pounds at the spot rate, which is £200,149,119. Therefore, the British pound floating payments must be computed on a notional principal satisfying the following equation

$$£200{,}149{,}119 = NP \times \sum_{t=1}^{8} \frac{FR_{t-1,t}}{Z_{0,t}}$$

$$NP = \frac{£200{,}149{,}119}{0.218230}$$

$$= £917{,}147{,}592$$

For example, the fifth payment, consistent with the British term structure, and as shown in the table, would be:

$$NP \times FR_{4,5} = £917{,}147{,}592 \times 0.033264 = £30{,}507{,}998$$

	British Exporter's Currency Annuity Cash Flow Analysis	
Semiannual Periods	**$FR_{t-1,t}$**	**Implied British Pound Cash Inflow on Floating Rate Swap**
1	0.026916	£24,685,945
2	0.027425	25,152,773
3	0.030044	27,554,782
4	0.030284	27,774,898
5	0.033264	30,507,998
6	0.032425	29,738,511
7	0.034266	31,426,979
8	0.035431	32,495,456

15. A US oil field equipment manufacturer, DrillBit, is currently negotiating a contract with a Houston-based oil exploration firm, FindIt, for a medium-term contract to deliver equipment each six months for two years. The present is time zero. If the deal goes through, the first equipment will be delivered in six months, with payment for the equipment being made six months following delivery. The contract calls for four deliveries spaced six months apart, with equipment valued at \$60,000,000 for each delivery. Most of DrillBit's financing is at a floating rate, so DrillBit is considering initiating a pay-fixed interest rate swap to match the cash flows on the FindIt deal. By entering this pay-fixed swap, DrillBit would agree to pay a sequence of

four payments of \$60,000,000 in return for a sequence of four payments tied to LIBOR of equal present value. The floating rate inflows on the swap would approximately match DrillBit's current floating rate debt obligations (not described here). So the swap would effectively convert DrillBit's existing financing position for a floating rate obligation to a fixed rate obligation. However, it is presently uncertain whether the deal with FindIt will be completed.

A. Describe the swap being proposed in detail with regard to its timing and cash flows. What is the implicit notional principal on which the floating payments must be computed? Complete the following table showing the cash flows on the swap consistent with the existing US term structure.

	Annuity Swap Cash Flow Analysis			
Semiannual Periods	**Fixed Payment**	$FR_{t-1,t}$	$FR_{t-1,t}/Z_{0,t}$	**Implied Floating Payment**
2	\$60,000,000	0.028022	0.026524	57,338,540
3	60,000,000	0.028733	0.026438	58,793,386
4	60,000,000	0.030461	0.027199	62,329,215
5	60,000,000	0.030197	0.026173	61,789,019

In terms of semiannual payments, the swap would consist of four payments of \$60,000,000 at times 2, 3, 4, and 5 in return for a sequence of floating rate payments with a present value equal to the present value of the fixed rate payments, satisfying the following equation.

$$\$60{,}000{,}000 \times \sum_{t=2}^{5} \frac{1}{Z_{0,t}} = NP \times \sum_{t=2}^{5} \frac{FR_{t-1,t}}{Z_{0,t}}$$

$$\$60{,}000{,}000 \times 3.626339 = \$2{,}046{,}197{,}265 \times 0.106334$$

The present value of the fixed rate payments is \$217,580,340, and the floating rate payments should be computed on a notional principal of \$2,046,197,265. The implied floating rate payments are shown in the table.

B. What is the *SFR* on this swap?

This swap is a forward swap to cover periods 2–5. Therefore, it may be priced as a plain vanilla swap from the standpoint of time zero.

$$SFR = \frac{\sum_{t=2}^{T} \frac{FRA_{(t-1),t}}{Z_{0,t}}}{\sum_{t=2}^{T} \frac{1}{Z_{0,t}}} = \frac{0.106334}{3.626339} = 0.029323$$

In annualized terms, $SFR = 0.058645$.

C. Explain how DrillBit might structure a swaption now to match its needs described above if the deal goes through.

The swaption would be a payer swaption with an expiration of six months, semiannual payments, a tenor of two years, and a notional principal of \$2,046,197,265. If the deal goes through, DrillBit could exercise the swaption to enter the swap analyzed in section A.

D. Price the swaption, assuming that the standard deviation of all interest rates is 0.22 and the strike rate is 6 percent.

The price of a payer swaption is given by Equation 22.32:

$$Payer\ Swaption_t = NP \times FRAC \times [SFR\, N(d_1) - SR\, N(d_2)] \quad \sum_{n=1}^{N} \frac{1}{Z_{t,T_n}}$$

where:

$$d_1 = \frac{\ln\left(\frac{SFR_t}{SR}\right) + (.5\sigma^2)(T - t)}{\sigma\sqrt{T - t}}$$

$$d_2 = d_1 - \sigma\sqrt{T - t}$$

For our swap, the valuation date is time zero, $t = 0$, $SFR = 0.058645$, $FRAC = 0.5$, $SR = 0.06$, $\sigma = 0.22$, and $NP = \$2{,}046{,}197{,}265$. The payment dates are 2–5 semiannual periods from now, so:

$$\sum_{n=2}^{5} \frac{1}{Z_{0,T_n}} = 3.626339$$

$$d_1 = \frac{\ln\left(\frac{SFR_t}{SR}\right) + (.5\sigma^2)(T - t)}{\sigma\sqrt{T - t}}$$

$$= \frac{\ln\left(\frac{0.058645}{0.06}\right) + 0.5(0.22)^2(0.5)}{0.22(0.7071)}$$

$$= -0.069054$$

$$d_2 = d_1 - \sigma\sqrt{T - t}$$

$$= -0.069054 - 0.155563$$

$$= -0.224617$$

The cumulative normal values are: $N(d_1) = N(-0.069054) = 0.472473$; $N(d_2) = N(-0.224617) = 0.411139$.

$$Payer\ Swaption_t$$

$$= NP \times FRAC \times [SFR\ N(d_1) - SR\ N(d_2)] \sum_{n=1}^{N} \frac{1}{Z_{t,T_n}}$$

$$= \$2{,}046{,}197{,}265 \times 0.5 \times [0.058645 \times 0.472473 - 0.06 \times 0.411139] \times 3.62633$$

$$= \$11{,}278{,}115$$

Therefore, the cost of the payer swaption is \$11,278,115, which is 55.12 basis points.

16. A US industrial firm, Sterling Industries, with a less than sterling credit rating, seeks new financing to upgrade its manufacturing facilities. Sterling has determined that it wants fixed rate financing without any foreign exchange exposure. Sterling needs to arrange its financing in two steps. First, it needs \$175,000,000 at present in the form of a semiannual coupon bond with a maturity of six years. Second, as its modernization plans proceed, Sterling will need another \$50,000,000 in financing at year 4. This too will be a semiannual coupon bond, and it will have a maturity of four years. Sterling has been exploring the possibilities for meeting its financing needs with investment bankers in both the United States and Great Britain and is considering several alternatives. It is now time zero, which is when Sterling will issue first financing. Although Sterling is sure that it wants its ultimate interest rate exposure to be as just described, Sterling's management remains open to alternative ways of achieving that exposure. Assuming, for the moment, that Sterling could issue at the rates shown in the US yield curve, at what rates would it expect to issue its two financings? Unfortunately, given its credit rating, Sterling is unable to issue at the rates in the interest rate tables completed in problems 8 and 9. Instead, Sterling faces the following three alternatives for meeting its financing needs:

A. Issue a six-year semiannual coupon bond at prevailing rates plus 135 basis points with a principal of \$175,000,000. Wait until year 4 and issue the second semiannual coupon bond. Sterling's US investment banker indicates that Sterling can reasonably expect similar terms on its second financing, but that the spread will probably be only about 125 basis points, as the loan amount is substantially less. Flotation costs on both alternatives are estimated to be 120 basis points, payable at the time of flotation on each issue. This alternative involves some interest rate risk, but Sterling is willing to consider this alternative seriously if it is truly price-effective.

B. Issue an FRN with a principal amount of $175,000,000 and a six-year maturity at a spread of 110 basis points over the LIBOR rates in the table. Sterling's US investment banker is also willing to commit to financing Sterling's residual needs itself for a commitment fee of 90 basis points for a second issuance of an FRN at year 4 in the amount of $50,000,000 with a maturity of four years. (This commitment would not be structured as an option, but as a firm commitment by both parties.) The FRN from the investment banker would also have the same 110 basis point spread over LIBOR, and the commitment fee would be payable at time zero. Flotation costs for any FRN will be 85 basis points, which is in addition to the spread and any commitment fee. Sterling is determined to have a fixed interest rate position. The investment banker agrees to act as counterparty for any necessary swap or swaps at the rates implied by the US yield curve of the table for a fee of 45 basis points of the entire financing package ($175,000,000 + $50,000,000) payable immediately. Except for the fee, the swap will be fairly priced according to the term structure.

C. Through its British investment advisers, Sterling is also considering financing in England. Sterling can issue a British FRN with a six-year maturity at British LIBOR, from the table, plus 85 basis points. The flotation costs for the service of the British investment banking firm would be 80 basis points on an FRN. The British investment banker is willing to act as a counterparty to Sterling for any desired swap that it might need in conjunction with this financing. Sterling is interested in this financing alternative but concerned about how it will plan for and meet the second part of its financing need. The British advisor is unwilling to predict Sterling's access to British markets for the second part of its financing need. Sterling discusses the entire matter fully with its US investment banker; the US investment banker is willing to allow Sterling to have the commitment for the second tranche described in Alternative B above, without going through the US investment banker for the immediate financing. The terms would be the same as described in Alternative B, except the commitment fee would be 105 basis points and the swap initiation fee would be 55 basis points if Sterling only takes that part of the deal. The US investment banking firm is still willing to provide a swap for the entire deal, so the swap initiation fee would apply to the entire financing package that Sterling contemplates ($175,000,000 plus $50,000,000).

For each of the three financing plans, describe in detail the instruments that Sterling would issue and the swaps that it would enter. For each alternative, prepare a table showing the cash flows for each period and determine the all-in cost of each alternative. (For simplicity, ignore any swap cash flows in determining the all-in cost.) Which alternative would you adopt if you were Sterling's CEO? Explain.

If Sterling could issue at the rates implied by the U.S. yield curve in problem 8, it would expect to issue the six-year bond right now at a rate of 6.1214 percent, read directly from the table. If it were to wait until year 4 and issue a four-year bond, it would now expect to issue at the forward rate implied by the term structure from the standpoint of year 4. The dollar coupon, $COUP$, would have to satisfy this equation:

$$\$50{,}000{,}000 = COUP \times \sum_{t=9}^{16} \frac{1}{\dfrac{Z_{0,t}}{Z_{0,8}}} + \frac{\$50{,}000{,}000}{\dfrac{Z_{0,16}}{Z_{0,8}}}$$
$$= COUP \times 6.930697 + \$38{,}041{,}950$$

Therefore, the semiannual dollar coupon payment is: $COUP = \$1{,}725{,}375$. This implies a semiannual yield of 0.0345075 and an annualized yield of 0.069015.

Alternative A

To these rates from the yield curve, Sterling must add its risk premium. For its six-year bond, Sterling would issue at the yield curve rate of 6.1214 percent plus 135 basis points, or 0.074714. With that rate, the six-year coupon bond has the cash flows shown in the second column of the table for Alternative A. As computed above, the term structure implies a coupon rate for a four-year bond at time 4 to be 0.069015. Because Sterling would probably issue at a rate 125 basis points higher, the expected rate would be 0.081515, leading to the cash flows in the third column. The fourth column shows the flotation costs at 120 basis points of the principal amount. The final column aggregates the cash flows of the other columns. Finding the IRR of the final column of cash flows gives a semiannual yield to maturity of 0.039140, or 0.078280 in annual terms. Under Alternative A, Sterling already has fixed rate financing, so it will not need to enter the swap market.

Cash Flow Analysis
Sterling Industries, Financing Alternative A

Semiannual Periods	Issue Six-Year Coupon Bond	Implied Cash Flows for Second Bond	Flotation Costs	Total Cash Flow
0	175,000,000		−2,100,000	172,900,000
1	−6,537,475			−6,537,475
2	−6,537,475			−6,537,475
3	−6,537,475			−6,537,475
4	−6,537,475			−6,537,475
5	−6,537,475			−6,537,475
6	−6,537,475			−6,537,475
7	−6,537,475			−6,537,475
8	−6,537,475	50,000,000	−600,000	42,862,525
9	−6,537,475	−2,037,875		−8,575,350
10	−6,537,475	−2,037,875		−8,575,350
11	−6,537,475	−2,037,875		−8,575,350
12	−181,537,475	−2,037,875		−183,575,350
13		−2,037,875		−2,037,875
14		−2,037,875		−2,037,875
15		−2,037,875		−2,037,875
16		−52,037,875		−52,037,875

Alternative B

Under Alternative B, Sterling will meet its financing need by issuing at a floating rate and then using a swap to convert the floating rate financing to a fixed rate. The second column of the table for Alternative B shows the forward rates from the term structure, and Sterling will issue a six-year FRN at those semiannual rates plus an annual spread of 110 basis points. This leads to the cash flows for the six-year FRN. For example, the cash flow at time 10 is a payment of \$6,951,175, and the rate for this semiannual period is 0.034221. Adding half of Sterling's spread of 110 basis points implies a forward rate for period 10 of 0.039721. Applying this spread-adjusted forward rate to the principal of \$175,000,000 gives the payment for period 10 that is consistent with the term structure: $0.039721 \times \$175{,}000{,}000 = \$6{,}951{,}175$. For the second FRN, to be issued at year 4 with a maturity of four years, the spread is estimated to be 110 basis points also, and the cash flows on the second bond were derived in the same way as those for the first. Flotation costs are 85 basis points, so the flotation cost for the first bond would be:

$$0.0085 \times \$175{,}000{,}000 = \$1{,}487{,}500$$

The commitment fee charged at time zero for the promise to issue the FRN at year 4 is 90 basis points. As the second FRN will be for \$50,000,000, this fee is \$450,000. In addition, the investment banker will charge a swap initiation fee of 45 basis points applied to the entire financing package, or \$1,012,500. The final column shows the aggregate cash flows. The last column includes the swap initiation fees, but does not include the swap flows themselves because the swap is fairly priced. The IRR for these flows is 0.038292, implying an annualized yield of 0.076584.

Under Alternative B, the two different financings require different swap analyses. For the initial financing, Sterling would want a pay-fixed swap with cash flows to match the periodic cash flows for the first financing.

Under this swap, Sterling would receive a floating payment each period that would match its obligation on the FRN and would make a fixed rate payment. If this were a plain vanilla swap, the *SFR* on this swap would match the yield on a par instrument with a maturity of six years, or 0.061214. The actual *SFR* on this swap would differ slightly, as the swap would have varying periodic cash flows. Similarly, for the second issuance, Sterling would require a pay-fixed forward swap to cover from year 4 to year 8. We have seen under Alternative A that the implied issuance rate for a coupon bond covering years 4 to 8 is 0.069015. Sterling's second swap would be at this rate plus the impact of the fees. These swaps would not affect the present value of the cash flows shown in the last column, as a fairly priced swap requires the exchange of cash flow streams with equal value at the time the swap is negotiated.

Cash Flow Analysis
Sterling Industries, Financing Alternative B

Semiannual Periods	Forward Rates	Issue Six-Year FRN	Issue Four-Year FRN at Year 4	Flotation Costs	Swap Initiation Fee	Commitment Fee	Total Cash Flow (Without Swap Flows)
0		175,000,000		−1,487,500	−1,012,500	−450,000	172,050,000
1	0.027665	−5,803,875					−5,803,875
2	0.028022	−5,866,350					−5,866,350
3	0.028733	−5,990,775					−5,990,775
4	0.030461	−6,293,175					−6,293,175
5	0.030197	−6,246,975					−6,246,975
6	0.029027	−6,042,225					−6,042,225
7	0.034466	−6,994,050					−6,994,050
8	0.031587	−6,490,225	50,000,000	−425,000			43,084,775
9	0.030401	−6,282,675	−1,795,050				−8,077,725
10	0.034221	−6,951,175	−1,986,050				−8,937,225
11	0.032476	−6,645,800	−1,898,800				−8,544,600
12	0.032154	−181,589,450	−1,882,700				−183,472,150
13	0.033121		−1,931,050				−1,931,050
14	0.032238		−1,886,900				−1,886,900
15	0.040544		−2,302,200				−2,302,200
16	0.042969		−52,423,450				−52,423,450

Alternative C

The third alternative involves financing in both the United States and Great Britain. The first issuance would be for £99,431,818 = $175,000,000 at the spot exchange rate of £1 = $1.760000. The following table shows the implied British pound cash flows at British floating rates for this issuance. The British issuance would be at a spread of 85 basis points over the British forward rates in the table. Thus, for the tenth period, as an example, the cash flow would be the forward rate, plus the semiannual spread times the British pound issuance amount:

$$(0.040667 + 0.00425) \times £99{,}431{,}818 = £4{,}466{,}179$$

At the tenth-period forward rate of exchange of £1 = $1.724068, the dollar equivalent of this flow would be $7,699,996, as shown in the final column of the table. We can then use the dollar equivalent flows for the subsequent portion of the analysis.

Cash Flow Analysis
Sterling Industries, Financing Alternative C, British FRN

Semiannual Periods	British Forward Rates	Exchange Rates	Cash Flows in British Pounds	Dollar Equivalent Cash Flows
0		1.760000	99,431,818	175,000,000
1	0.026916	1.761284	−3,098,892	−5,458,029

2	0.027425	1.762307	−3,149,503	−5,550,391
3	0.030044	1.760063	−3,409,915	−6,001,665
4	0.030284	1.760366	−3,433,778	−6,044,706
5	0.033264	1.755143	−3,730,085	−6,546,833
6	0.032425	1.749366	−3,646,662	−6,379,347
7	0.034266	1.749704	−3,829,716	−6,700,869
8	0.035431	1.743207	−3,945,554	−6,877,917
9	0.035387	1.734814	−3,941,179	−6,837,213
10	0.040667	1.724068	−4,466,179	−7,699,996
11	0.032539	1.723962	−3,657,997	−6,306,248
12	0.047937	1.697997	−104,620,866	−177,645,917

The following table shows the cash flow analysis for Alternative C. The third column of the table shows the dollar equivalent cash flows from the British issuance derived in the previous table. Under Alternative C, Sterling would make the second issuance through the investment banking firm, as in Alternative B. These fourth column cash flows are the same as those explored in Alternative B. The flotation cost on the British issuance is 80 basis points, or $1,400,000. For the second issuance through the US investment banking firm, the flotation cost is now 85 basis points, or $425,000. The swap initiation fee is now 55 basis points (instead of the 45 basis points in Alternative B), for a total of:

$$(\$175{,}000{,}000 + \$50{,}000{,}000) \times 0.0055 = \$1{,}237{,}500$$

Finally, the commitment fee charged by the US investment bank for the second issuance is 105 basis points on $50,000,000, or $525,000. The final column shows the total cash flow for the entire financing, without the swap flows. (Again, we can ignore the swap flows because the swap is fairly priced. We do need, however, to take into account the various fees associated with the swap.) The IRR for these flows is 0.037309, implying an annualized issuance cost of 0.074617. If Sterling were to adopt Alternative C, it would need to enter swaps. The financing in Alternative C is all at a floating rate. Therefore, Sterling would need to convert these floating obligations to fixed obligations by pay-fixed interest rate swaps. It would require a swap (or swaps) with varying cash flows.

Comparing the Alternatives

Alternative A is the simplest, but fails to give complete fixed rate financing. At 7.8280 percent it is also the most expensive. Because Sterling strongly desires fixed rate financing, Alternative A can be dropped from consideration. Alternative B costs about 17 basis points less than Alternative A at 7.6584 percent, and Alternative C is the cheapest at 7.4617 percent. Unfortunately, Alternative C is the most complicated, involving a foreign issuance and dealing through two investment banking firms. Both Alternatives B and C provide routes to fixed rate financing, so Sterling must decide if the added complications of Alternative C are worth bearing to save almost 20 basis points. Alternative C saves about $450,000 per year for six years while costing an extra $212,500 up-front. This substantial saving suggests that Alternative C is the preferable strategy.

Note: We ignored the cash flows on the swaps in computing the all-in costs on the grounds that a fairly priced swap has a zero *NPV*. However, with a shaped yield curve and varying periodic swap payments, the swap cash flows can still affect the all-in cost very slightly.

Use the following data on Mid-Continent National Bank for the remaining problems. Use the US interest rate data from problem 8 above.

The asset/liability committee at Mid-Continent National Bank (MCNB) has recently been reconstituted and is beginning its analysis of the bank's financial position. The ALCO (asset/liability committee) is interested in using swaps and a duration-based approach to manage the bank's interest rate exposure. The following balance sheet shows MCNB's current position in market value terms.

Mid Continent National Bank
Stylized Balance Sheet–Market Values

Assets		Liabilities and Net Worth	
A: Cash	$73,000,000	E: Demand Deposits	$225,000,000
B: Marketable Securities (6-month maturity; yield 7%)	135,000,000	F: Three-Month Money Market Obligations (average yield 6.2%)	175,000,000
C: Amortizing Loans (10-year average maturity; semiannual payments; 8% average yield)	475,000,000	G: Six-Month Money Market Obligations (average yield 6.8%)	85,000,000
D: Commercial Loans (5-year average maturity; semiannual payments; 7.4% average yield; par value $235,000,000)	$248,415,824	H: FRN (4-year maturity; semiannual payments)	130,000,000
		I: Coupon Bond (7-year maturity; semi-annual payments; 8.5% coupon; par value $180,000,000)	203,670,556
		Total Liabilities	$818,670,556
		Net Worth	$112,745,268
Total Assets	$931,415,824	Total Liabilities and Net Worth	$931,415,824

17. Using the zero-coupon factors for the United States from problem 8, find the payment on the amortizing loan. For balance sheet items C, D, and I, find the semiannual and annualized yield-to-maturity, and compute the Macaulay duration based on the yield-to-maturity (not the zero-coupon rates). For each of these items, prepare a table of the form shown below showing the computations.

Dollar Equivalent Cash Flow Analysis
Sterling Industries, Financing Alternative C

Semiannual Periods	US Forward Rates	Dollar Equivalent of British Six-Year FRN	Issue Four-Year FRN at Year 4	Flotation Costs	Swap Initiation Fee	Commitment Fee	Total Cash Flow
0		175,000,000		−1,400,000	−1,237,500	−525,000	171,837,500
1	0.027665	−5,458,029					−5,458,029
2	0.028022	−5,550,391					−5,550,391
3	0.028733	−6,001,665					−6,001,665
4	0.030461	−6,044,706					−6,044,706
5	0.030197	−6,546,833					−6,546,833
6	0.029027	−6,379,347					−6,379,347
7	0.034466	−6,700,869					−6,700,869
8	0.031587	−6,877,917	50,000,000	−425,000			42,697,083
9	0.030401	−6,837,213	−1,795,050				−8,632,263
10	0.034221	−7,699,996	−1,986,050				−9,686,046
11	0.032476	−6,306,248	−1,898,800				−8,205,048
12	0.032154	−177,645,917	−1,882,700				−179,528,617
13	0.033121		−1,931,050				−1,931,050
14	0.032238		−1,886,900				−1,886,900
15	0.040544		−2,302,200				−2,302,200
16	0.042969		−52,423,450				−52,423,450

	Duration Computation Worksheet		
Semiannual Periods	Cash Flow	Present Value of Cash Flow	Weighted Present Value of Cash Flow
1			
2			
Maturity			
SUMS			

The following table gives the yield-to-maturities for the various items, found by an iterative search using a spreadsheet:

	Yield-to-Maturities	
Balance Sheet Item	Semiannual Yield-to-Maturity	Annualized Yield-to-Maturity
Item C: Amortizing Loans	0.03135534	0.06271068
Item D: Commercial Loans	0.03029731	0.06059462
Item I: Coupon Bond	0.03079531	0.06159062

	Duration Computation Worksheet C: Amortizing Loans		
Semiannual Periods	Cash Flow	Present Value of Cash Flow	Weighted Present Value of Cash Flow
1	32,328,875	31,346,010	31,346,010
2	32,328,875	30,393,027	60,786,053
3	32,328,875	29,469,016	88,407,047
4	32,328,875	28,573,096	114,292,385
5	32,328,875	27,704,415	138,522,075
6	32,328,875	26,862,143	161,172,860
7	32,328,875	26,045,479	182,318,350
8	32,328,875	25,253,642	202,029,136
9	32,328,875	24,485,879	220,372,910
10	32,328,875	23,741,457	237,414,575
11	32,328,875	23,019,668	253,216,348
12	32,328,875	22,319,822	267,837,868
13	32,328,875	21,641,253	281,336,295
14	32,328,875	20,983,315	293,766,403
15	32,328,875	20,345,378	305,180,674
16	32,328,875	19,726,837	315,629,386
17	32,328,875	19,127,100	325,160,698
18	32,328,875	18,545,596	333,820,735
19	32,328,875	17,981,772	341,653,665
20	32,328,875	17,435,089	348,701,774
SUMS		474,999,994	4,502,965,247

$$D = \frac{4{,}502{,}965{,}247}{475{,}000{,}000} = 9.48 \text{ semiannual periods} = 4.74 \text{ years}$$

	Duration Computation Worksheet Item D: Commercial Loans		
Semiannual periods	Cash Flow	Present Value of Cash Flow Cash Flow	Weighted Present Value of Cash Flow
1	8,695,000	8,439,312	8,439,312
2	8,695,000	8,191,142	16,382,284
3	8,695,000	7,950,270	23,850,811
4	8,695,000	7,716,482	30,865,926
5	8,695,000	7,489,568	37,447,839
6	8,695,000	7,269,327	43,615,961
7	8,695,000	7,055,562	49,388,935
8	8,695,000	6,848,084	54,784,669
9	8,695,000	6,646,706	59,820,357
10	243,695,000	180,809,381	1,808,093,810
SUMS		248,415,833	2,132,689,904

$$D = \frac{2{,}132{,}689{,}904}{248{,}415{,}824} = 8.59 \text{ semiannual periods} = 4.29 \text{ years}$$

	Duration Computation Worksheet Item I: Coupon Bond		
Seminannual periods	Cash Flow	Present Value of Cash Flow	Weighted Present Value of Cash Flow
1	7,650,000	7,421,454	7,421,454
2	7,650,000	7,199,736	14,399,472
3	7,650,000	6,984,642	20,953,925
4	7,650,000	6,775,974	27,103,894
5	7,650,000	6,573,539	32,867,697
6	7,650,000	6,377,153	38,262,918
7	7,650,000	6,186,634	43,306,435
8	7,650,000	6,001,806	48,014,449
9	7,650,000	5,822,500	52,402,504
10	7,650,000	5,648,552	56,485,515
11	7,650,000	5,479,799	60,277,794
12	7,650,000	5,316,089	63,793,066
13	7,650,000	5,157,269	67,044,499
14	187,650,000	122,725,411	1,718,155,750
SUMS		203,670,557	2,250,489,372

$$D = \frac{2{,}250{,}489{,}372}{203{,}670{,}556} = 11.05 \text{ semiannual periods} = 5.52 \text{ years}$$

18. Complete the following table summarizing the durations for all of the balance sheet items for MCNB.

	Summary of Durations for MCNB Balance Sheet Items				
Item	Assets Market Value	Duration (Years)	Item	Liabilities Market Value	Duration (Years)
A: Cash	$73,000,000	0.00	E: Demand Deposits	$225,000,000	0.00
B: Marketable Securities	135,000,000	0.50	F: Three-Month Money Market	175,000,000	0.25

C: Amortizing Loans	475,000,000	4.74	G: Six-Month Money Market	85,000,000	0.50
D: Commercial Loans	248,415,824	4.29	H: FRN	130,000,000	0.50
			I: Coupon Bond	203,670,556	5.52

19. Compute the duration of the asset portfolio and the liability portfolio for MCNB individually.

$$D_A = \frac{\$73,000,000}{\$931,415,824}(0.00) + \frac{\$135,000,000}{\$931,415,824}(0.50)$$
$$+ \frac{\$475,000,000}{\$931,415,824}(4.74) + \frac{\$248,415,824}{\$931,415,824}(4.29)$$
$$= 0.000 + 0.0725 + 2.4173 + 1.1442$$
$$= 3.6340$$

$$D_L = \frac{\$225,000,000}{\$818,670,556}(0.00) + \frac{\$175,000,000}{\$818,670,556}(0.25)$$
$$+ \frac{\$85,000,000}{\$818,670,556}(0.50) + \frac{\$130,000,000}{\$818,670,556}(0.50)$$
$$+ \frac{\$203,670,556}{\$818,670,556}(5.52)$$
$$= 0.0000 + 0.0534 + 0.0519 + 0.0794 + 1.3733$$
$$= 1.5580$$

20. Find the duration gap for MCNB.

The duration gap, D_G, is given by Equation 22.3:

$$D_G = D_A - \frac{Total\ Liabilities}{Total\ Assets} D_L$$

$$D_G = 3.6340 - \frac{\$818,670,556}{\$931,415,824} \times 1.5580 = 2.2646$$

21. Find the durations for the fixed rate side and floating rate side of an eight-year plain vanilla interest rate swap in terms of semi annual periods and years. What are the durations of the receive-fixed and pay-fixed plain vanilla swap?

The fixed rate side of the swap is just an eight-year semiannual par coupon bond. From the table of problem 8, the par yield-to-maturity for an eight-year coupon bond is 0.063835, or 0.031918 in semiannual terms. The following table shows the duration computation.

	Duration Computation Worksheet Fixed Rate Side of Swap		
Semiannual Periods	**Cash Flow**	**Present Value of Cash Flow**	**Weighted Present Value of Cash Flow**
1	31.9180	30.9308	30.9308
2	31.9180	29.9740	59.9481
3	31.9180	29.0469	87.1408

4	31.9180	28.1485	112.5939
5	31.9180	27.2778	136.3891
6	31.9180	26.4341	158.6046
7	31.9180	25.6165	179.3153
8	31.9180	24.8241	198.5931
9	31.9180	24.0563	216.5068
10	31.9180	23.3122	233.1223
11	31.9180	22.5912	248.5028
12	31.9180	21.8924	262.7088
13	31.9180	21.2153	275.7983
14	31.9180	20.5590	287.8267
15	31.9180	19.9231	298.8471
16	1031.9180	624.1977	9,987.1636
SUMS		1000.00	12,773.9920

$$D = \frac{12{,}773.9920}{1{,}000} = 12.7740 \text{ semiannual periods} = 6.3870 \text{ years}$$

The floating rate on the swap resets semiannually, so the duration of the floating rate side is 0.50 years.

Duration of a Receive-Fixed Swap

= Duration of Underlying Coupon Bond − Duration of Underlying Floating Rate Bond
= 6.3870 − 0.50
= 5.8870

Duration of a Pay-Fixed Swap

= Duration of Underlying Floating Rate Bond − Duration of Underlying Coupon Bond
= 0.50 − 6.3870
= –5.8870

22. Explain how to hedge the asset portfolio alone using this swap to give the combined asset portfolio/interest rate swap a duration of zero. Show your calculations.

Equation 22.2 gives a general rule for finding a hedge for a zero duration:

$$D_X \times MV_X + D_H \times \mathrm{MV}_H^* = 0$$

where:

D_X = duration of position to be hedged
D_H = duration of hedging instrument
MV_X = market value of position to be hedged
MV_H^* = market value of hedging vehicle

Because the asset portfolio has a positive duration, we will use a pay-fixed swap with a negative duration as the hedging vehicle:

$$3.6340 \times \$931{,}415{,}824 + -5.8870 \times MV_H^* = 0$$

Solving for MV_H^*, the notional principal for the swap, gives: $MV_H^* = \$574{,}955{,}853$. Therefore, to hedge the asset portfolio alone, MCNB should enter a pay-fixed swap with a notional principal of about $575,000,000.

23. Explain how to hedge the liability portfolio alone using this swap to give the combined liability portfolio/interest rate swap a duration of zero. Show your calculations.

To hedge the liability portfolio, we use the same approach:

$$D_X \times MV_X + D_H \times \mathrm{MV}_H^* = 0$$

From the point of view of MCNB, the liability portfolio is, of course, a liability. Therefore, its duration is negative and the appropriate side of the swap is one that lengthens the duration of the liability portfolio from –1.5580 to zero. Therefore, a receive-fixed swap will be required:

$$-1.5580 \times \$818{,}670{,}556 + 5.8870 \times MV_H^* = 0 \; MV_H^* = \$216{,}661{,}921$$

To hedge the liability portfolio considered in isolation, MCNB should enter a receive-fixed swap with a notional principal of \$216,661,921.

24. Using the duration gap already computed, explain how to hedge the interest rate risk of the entire bank so that the bank plus swap position has a duration of zero.

The hedging equation for the duration gap approach is given by Equation 22.4:

$$D_G^* = D_G + D_S\left(\frac{MV_H^*}{Total\ Assets}\right)$$

where:

D_G^* = the desired duration gap
D_S = the duration of the swap
MV_H^* = the required market value (notional principal) for the swap

Hedging the entire bank to a zero duration position means $D_G^* = 0$. Since the duration gap is positive, a pay-fixed swap with its negative duration will be the hedging vehicle:

$$D_G + D_S\left(\frac{MV_H^*}{Total\ Assets}\right) = 0$$

$$2.2646 - 5.8870\left(\frac{MV_H^*}{\$931{,}415{,}824}\right) = 0$$

$$MV_H^* = \$358{,}295{,}274$$

Hedging the entire bank to a duration of zero requires a pay-fixed swap with a notional principal of \$358,296,274. Notice that this is the same net result as hedging the asset portfolio separately with a pay-fixed swap with a notional principal of \$574,955,853 and hedging the liability portfolio separately with a receive-fixed swap with a notional principal of \$216,661,921. (Note: \$574,955,853 − \$216,661,921 = \$358,293,932. The difference between this result and \$358,296,274 is due to rounding.)

25. Using the duration gap approach, show how to set the duration of the entire bank plus swap position so that it has a duration of 2.0 years.

As the duration gap exceeds 2.0 years, we will need a pay-fixed swap with a notional principal given as:

$$D_G + D_S\left(\frac{MV_H^*}{Total\ Assets}\right) = 2$$

$$2.2646 - 5.8870\left(\frac{MV_H^*}{\$931{,}415{,}824}\right) = 2$$

$$MV_H^* = \$41{,}863{,}874$$

26. Using the duration gap approach, show how to set the duration of the entire bank plus swap position so that it has a duration of 8.0 years. Compute the effect on MCNB's net worth if the yield curve shifted up or down by 50, 100, and 500 basis points, assuming that the average yield-to-maturity for the entire bank is 6.50 percent.

To set the duration gap to 8.0 years, we will need to lengthen the duration gap with a receive-fixed swap. The needed notional principal is:

$$D_G + D_S\left(\frac{MV_H^*}{Total\ Assets}\right) = 8$$

$$2.2646 + 5.8870\left(\frac{MV_H^*}{\$931{,}415{,}824}\right) = 8$$

$$MV_H^* = \$907{,}430{,}324$$

For any interest-sensitive item, we can use the following duration price change formula to find the approximate price change for a given change in yields. The net worth of MCNB is $112,745,268. If the bank has an overall duration of 8.0 years and an average yield-to-maturity of 6.5 percent, the effect of a jump in interest rates of 50 basis points would be:

$$\Delta P = -D\frac{\Delta r}{1+r}P = -8\,\frac{+0.0050}{1.065}\,\$112{,}745{,}268 = -\$4{,}234{,}564$$

Under this policy, a rise in rates of just 50 basis points would cause a loss of $4,234,564. The effects of a 100 and 500 basis point rise are:

$$\Delta P = -D\frac{\Delta r}{1+r}P = -8\,\frac{+0.01}{1.065}\$112{,}745{,}268 = -\$8{,}469{,}128$$

$$\Delta P = -D\frac{\Delta r}{1+r}P = -8\,\frac{+0.05}{1.065}\,\$112{,}745{,}268 = -\$42{,}345{,}641$$

As these computations show, the effect of a shift in rates is directly proportional. The loss from a 100 basis point rise was twice as large as the loss from a 50 basis point rise. Also, a very large jump in rates can be quite destructive of firm value, as the result of the 500 basis point rise indicates.