14 Option Sensitivities and Option Hedging

Answers to Questions and Problems

1. Consider Call A, with: $X = \$70$; $r = 0.06$; $T - t = 90$ days; $\sigma = 0.4$; and $S = \$60$. Compute the price, DELTA, GAMMA, THETA, VEGA, and RHO for this call.

$$c = \$1.82$$
$$\text{DELTA} = .2735$$
$$\text{GAMMA} = .0279$$
$$\text{THETA} = -8.9173$$
$$\text{VEGA} = 9.9144$$
$$\text{RHO} = 3.5985$$

2. Consider Put A, with: $X = \$70$; $r = 0.06$; $T - t = 90$ days; $\sigma = 0.4$; and $S = \$60$. Compute the price, DELTA, GAMMA, THETA, VEGA, and RHO for this put.

$$p = \$10.79$$
$$\text{DELTA} = -.7265$$
$$\text{GAMMA} = .0279$$
$$\text{THETA} = -4.7790$$
$$\text{VEGA} = 9.9144$$
$$\text{RHO} = -13.4083$$

3. Consider a straddle comprised of Call A and Put A. Compute the price, DELTA, GAMMA, THETA, VEGA, and RHO for this straddle.

$$\text{price} = c + p = \$12.61$$
$$\text{DELTA} = 0.2735 - 0.7265 = -0.4530$$
$$\text{GAMMA} = 0.0279 + 0.0279 = 0.0558$$
$$\text{THETA} = -8.9173 - 4.47790 = -13.6963$$
$$\text{VEGA} = 9.9144 + 9.9144 = 19.8288$$
$$\text{RHO} = 3.5985 - 13.4083 = -9.8098$$

4. Consider Call A. Assuming the current stock price is $60, create a DELTA-neutral portfolio consisting of a short position of one call and the necessary number of shares. What is the value of this portfolio for a sudden change in the stock price to $55 or $65?

   As we saw for this call, DELTA = 0.2735. The DELTA-neutral portfolio, given a short call component, is 0.2735 shares -1 call, costs:

$$.2735 \,(\$60) - \$1.82 = \$14.59$$

   If the stock price goes to $55, the call price is $.77, and the portfolio will be worth:

$$.2735 \,(\$55) - \$.77 = \$14.27$$

   With a stock price of $65, the call is worth $3.55, and the portfolio value is:

$$.2735 \,(\$65) - \$3.55 = \$14.23$$

   Notice that the portfolio values are lower for both stock prices of $55 and $65, reflecting the negative GAMMA of the portfolio.

5. Consider Call A and Put A from above. Assume that you create a portfolio that is short one call and long one put. What is the DELTA of this portfolio? Can you find the DELTA without computing? Explain. Assume that a share of stock is added to the short call/long put portfolio. What is the DELTA of the entire position?

   The DELTA of the portfolio is $-1.0 = -0.2735 - 0.7265$. This is necessarily true, because the DELTA of the call is $N(d_1)$, the DELTA of the put is $N(-d_2)$, and $N(d_1) + N(d_2) = 1.0$. If a long share of stock is added to the portfolio, the DELTA will be zero, because the DELTA of a share is always 1.0.

6. What is the GAMMA of a share of stock if the stock price is $55 and a call on the stock with $X = \$50$ has a price $c = \$7$ while a put with $X = \$50$ has a price $p = \$4$? Explain.

   The GAMMA of a share of stock is always zero. All other information in the question is irrelevant. The GAMMA of a share is always zero because the DELTA of a share is always 1.0. As GAMMA measures how DELTA changes, there is nothing to measure for a stock, since the DELTA is always 1.0.

7. Consider Call B written on the same stock as Call A with: $X = \$50$; $r = 0.06$; $T - t = 90$ days; $\sigma = 0.4$; and $S = \$60$. Form a bull spread with calls from these two instruments. What is the price of the spread? What is its DELTA? What will the price of the spread be at expiration if the terminal stock price is $60? From this information, can you tell whether THETA is positive or negative for the spread? Explain.

   As observed in problem 1, for Call A, $c = \$1.82$, DELTA = 0.2735, and THETA = -8.9173. For Call B, $c = \$11.64$, DELTA = 0.8625, and THETA = -7.7191. The long bull spread with calls consists of buying the call with the lower exercise price (Call B) and selling the call with the higher exercise price (Call A). The spread costs $\$11.64 - \$1.82 = \$9.82$. The DELTA of the spread equals $\text{DELTA}_B - \text{DELTA}_A = 0.8625 - 0.2735 = 0.5890$. If the stock price is $60 at expiration, Call B will be worth $10, and Call A will expire worthless. If the stock price remains at $60, the value of the spread will have to move from $9.82 now to $10.00 at expiration, so the THETA for the spread must be positive. This can be confirmed by computing the two THETAs and noting: $\text{THETA}_A = -8.9173$ and $\text{THETA}_B = -7.7191$. For the spread, we buy Call B and sell Call A, giving a THETA for the spread of $-7.7191 - (-8.9173) = 1.1982$.

8. Consider again the sample options, $C2$ and $P2$, of the chapter discussion as given in Table 14.7. Assume now that the stock pays a continuous dividend of 3 percent per annum. See if you can tell how the sensitivities will differ for the call and a put without computing. Now compute the DELTA, GAMMA, VEGA, THETA, and RHO of the two options if the stock has a dividend.

   The presence of a continuous dividend makes d_1 smaller than it otherwise would be, because the continuous dividend rate, σ, is subtracted in the numerator of d_1. With a smaller d_1, $N(d_1)$ is also smaller. But,

$N(d_1)$ = DELTA for a call, so the DELTA of a call will be smaller with a dividend present. By the same reasoning, the DELTA of the put must increase.

Sensitivity	C2	P2
DELTA	0.5794	−0.4060
GAMMA	0.0182	0.0182
THETA	−10.3343	−5.5997
VEGA	26.93	26.93
RHO	23.9250	−23.4823

9. Consider three calls, Call C, Call D, and Call E, all written on the same underlying stock; $S = \$80$; $r = 0.07$; $\sigma = 0.2$. For Call C, $X = \$70$, and $T − t = 90$ days. For Call D, $X = \$75$, and $T − t = 90$ days. For Call E, $X = \$80$, and $T − t = 120$ days. Compute the price, DELTA, and GAMMA for each of these calls. Using Calls C and D, create a DELTA-neutral portfolio assuming that the position is long one Call C. Now use calls C, D, and E to form a portfolio that is DELTA-neutral and GAMMA-neutral, again assuming that the portfolio is long one Call C.

Measure	Call C	Call D	Call E
Price	$11.40	$7.16	$4.60
DELTA	.9416	.8088	.6018
GAMMA	.0147	.0343	.0421

For a DELTA-neutral portfolio comprised of Calls C and D that is long one Call C, we must choose a position of Z shares of Call D to satisfy the following equation:

$$0.9416 + 0.8088\ Z = 0$$

Therefore, $Z = −1.1642$, and the portfolio consists of purchasing one Call C and selling 1.1642 units of Call D.

To form a portfolio of Calls C, D, and E that is long one Call C and that is also DELTA-neutral and GAMMA-neutral, the portfolio must meet both of the following conditions, where Y and Z are the number of Call Cs and Call Ds, respectively.

DELTA-neutrality: $0.9416 + 0.8088\ Y + 0.6018\ Z = 0$
GAMMA-neutrality: $0.0147 + 0.0343\ Y + 0.0421\ Z = 0$

Multiplying the second equation by $(0.8088/.0343)$ gives:

$$0.3466 + 0.8088\ Y + 0.9927\ Z = 0$$

Subtracting this equation from the DELTA-neutrality equation gives:

$$0.5950 − 0.3909\ Z = 0$$

Therefore, $Z = 1.5221$. Substituting this value of Z into the DELTA-neutrality equation gives:

$$0.8088\ Y + 0.9416 + 0.6018\ (1.5221) = 0$$

$Y = −2.2968$. Therefore, the DELTA-neutral and GAMMA-neutral portfolio consists of buying one unit of Call C, selling 2.2968 units of Call D, and buying 1.5221 units of Call E.

10. Your largest and most important client's portfolio includes option positions. After several conversations it becomes clear that your client is willing to accept the risk associated with exposure to changes in volatility and stock price. However, your client is not willing to accept a change in the value of her portfolio resulting from the passage of time. Explain how the investor can protect her portfolio against changes in value due to the passage of time.

Your client wants to avoid changes in the value of her portfolio due to the passage of time. THETA measures the impact of changes in the time until expiration on the value of an option. Your client should create a THETA-neutral portfolio to protect the value of her option positions against changes in the time until expiration. To protect her portfolio against the wasting away effect associated with option contracts, she must first determine the THETA for her current portfolio. Given the THETA value of her portfolio, she should construct a position in option contracts that has a THETA value that is of an equal magnitude and opposite sign of the THETA of her portfolio. Thus, the THETA for the hedge portfolio, the original portfolio plus the additional options contracts used to create the hedge, is zero. The value of this portfolio should not change with the passage of time. However, the portfolio will have exposure to changes in other market variables, that is, interest rates, volatility, and stock price changes.

11. Your newest client believes that the Asian currency crisis is going to increase the volatility of earnings for firms involved in exporting, and that this earnings volatility will be translated into large stock price changes for the affected firms. Your client wants to create speculative positions using options to increase his exposure to the expected changes in the riskiness of exporting firms. That is, your client wants to prosper from changes in the volatility of the firm's stock returns. Discuss which "Greek" your client should focus on when developing his options positions.

    Your client wants to create exposure to changes in the volatility of stock returns. VEGA measures the change in the value of an option contract resulting from changes in the volatility of the underlying stock. Once you have identified stocks with traded options that have significant Asian exposures, you want to construct positions based on the VEGA of the option. Because your client wants exposures to volatility risk, you would construct a portfolio with a large VEGA.

12. A long-time client, an insurance salesperson, has noticed the increased acquisition activity involving commercial banks. Your client wishes to capitalize on the potential gains associated with this increased acquisition activity in the banking industry by creating speculative positions using options. Your client realizes that bank cash flows are sensitive to changes in interest rates, and she believes that the Federal Reserve is about to increase short-term interest rates. Realizing that an increase in the short-term interest rates will lead to a decrease in the stock prices of commercial banks, your client wants the value of her portfolio of options to be unaffected by changes in short-term interest rates. Explain how the investor can use option contracts to protect her portfolio against changes in value due to changes in the risk-free rate, and to capitalize on the expected price changes in bank stocks.

    Your client wants to create exposure to changes in bank stock prices. DELTA measures the change in the value of an option contract resulting from changes in the underlying stock price. Additionally, your client has a preference to construct a portfolio such that the value of the portfolio will not change as interest rates change. RHO measures the change in the value of an option contract resulting from changes in the risk-free rate of interest. Because your client wants exposure to stock price changes, you would construct a portfolio with a large DELTA. However, the RHO for the portfolio should be constrained to equal zero. Thus, the resulting portfolio would be RHO-neutral hedge with a large DELTA.

13. Your brother-in-law has invested heavily in stocks with a strong Asian exposure, and he tells you that his portfolio has a positive DELTA. Give an intuitive explanation of what this means. Suppose the value of the stocks that your brother-in-law holds increases significantly. Explain what will happen to the value of your brother-in-law's portfolio.

    DELTA measures the change in the value of an option due to a change in the price of the underlying asset, which is usually a stock. If an investor holds a portfolio consisting of a single stock, the DELTA of the portfolio is one, because a one dollar increase in the stock price will produce a one dollar per share increase in the value of the portfolio. If the asset in question is an option, then the DELTA of the option measures the change in the value of the option contract because of a change in the underlying stock price. If your brother-in-law's portfolio has a positive DELTA, the value of his portfolio will move in the same direction as the value of the underlying asset. If the value of the stocks he holds increases, then the value of his portfolio will increase at a rate of DELTA times the dollar change in the asset price.

14. Your mother-in-law has invested heavily in the stocks of financial firms, and she tells you that her portfolio has a negative RHO. Give an intuitive explanation of what this means. Suppose the Federal Reserve increases short-term interest rates. Explain what will happen to the value of your mother-in-law's portfolio.

    RHO measures the change in the value of an asset due to changes in interest rates. If the investor holds an option, then the RHO of the option measures the change in the value of the option contract because of a change in the risk-free interest rate. If your mother-in-law's portfolio has a negative RHO, that implies that the value of the portfolio moves in the opposite direction as changes in the interest rate. If the short-term interest rate is increased by the Federal Reserve, then the value of her portfolio will decrease.

15. Your brother, Daryl, has retired. With the free time necessary to follow the market closely, Daryl has established large option positions as a stock investor. He tells you that his portfolio has a positive THETA. Give an intuitive explanation of what this means. Daryl is also a big soccer fan, and is heading to France to watch the World Cup for a month. He believes that there is not sufficient liquidity in the market to close out his open option positions, and he is going to leave the positions open while he is in France. Explain what will happen to the value of your brother's portfolio while he is in France.

    THETA measures the change in the value of an option because of changes in the time until expiration for the option contract. That is, with the passage of time, the value of an option contract will change. In most cases, the option will experience a decrease in value with the passage of time. This is known as time decay. Formally, THETA is the negative of the first derivative of the option pricing model with respect to changes in the time until expiration. Since your brother has constructed a portfolio with a positive THETA, the passage of time should increase the value of his portfolio. Thus, he should, all things being equal, return from his vacation to find that the value of his portfolio has increased.

16. Consider the following information for a call option written on Microsoft's stock.

$S = \$.96$	DELTA $= 0.2063$
$X = \$100$	GAMMA $= 0.0635$
$T - t = 5$ days	THETA $= -48.7155$
$\sigma = 0.4$	VEGA $= 3.2045$
$r = 0.1$	RHO $= 0.2643$
	Price $= \$.5$

    If in two days Microsoft's stock price has increased by $1 to $97, explain what you would expect to happen to the price of the call option.

    Two variables are changing in this problem, the underlying stock price, S, and the time until expiration, $T - t$. Thus, one needs to assess the impact of both DELTA and THETA on the value of the Microsoft option. DELTA is 0.2063, and THETA is -48.7155. A one dollar increase in the price of Microsoft would be expected to increase the price of the call option by $\$.2063 = \$1 \times 0.2063$. However, as an option contract approaches expiration, the passage of time has a significant adverse effect on the value of the option. Here two days represent 40 percent of the life of the option. The THETA effect is equal to $-\$.2669 = (2/365) \times -48.7155$, which is a larger negative effect than the positive impact of a stock price increase on the value of the option. The combined DELTA and THETA effects are $-\$.0606 = \$.2063 - \$.2669$. Thus, the expected price of the call option is $\$.4394$. The price of the call option according to the Black–Scholes model is $\$.4162$.

17. Consider a stock, CVN, with a price of $50 and a standard deviation of 0.3. The current risk-free rate of interest is 10 percent. A European call and put on this stock have an exercise price of $55 and expire in three months (0.25 years).

A. If $c = \$1.61057$ and $N(d_1) = 0.3469$, then calculate the put option price.
$$p = c - S + Xe^{-r(T - t)} = \$1.61057 - \$50 + \$55e^{-0.1 \times 0.25} = \$5.25262$$

B. Suppose that you own 3,000 shares of CBC, a subsidiary of CVN Corporation, and that you plan to go Christmas shopping in New York City the day after Thanksgiving. To finance your shopping trip, you wish

to sell your 3,000 shares of CBC in one week. However, you do not want the value of your investment in CBC to fall below its current level. Construct a DELTA-neutral hedge using the put option written on CVN. Be sure to describe the composition of your hedged portfolio.

Construction of a DELTA-neutral hedge using the put option requires the investor to hold $-1/\text{DELTA}_{put}$ put options per 100 shares of stock held by the investor. The put option extends the right to sell 100 shares of stock. The DELTA for the put option is $-0.6531 = 0.3469 - 1$. Thus, the hedge ratio is $1.5312 = -1/-0.6531$, put options per 100 shares of stock. Because you hold 3,000 shares of CBC, you must purchase $45.9348 = (3,000/100) \times 1.5312$, put options written on CVN that expire in three months with a strike price of $55. Since purchasing a fraction of an option's contract is not possible, you would round this up to 46 options contracts purchased.

18. An investor holds a portfolio consisting of three options, two call options and a put option, written on the stock of QDS Corporation with the following characteristics.

	C_1	C_2	P_1
DELTA	0.8922	0.2678	−0.6187
GAMMA	0.0169	0.0299	0.0245
THETA	−5.55	−3.89	−3.72

The investor is long 100 contracts of option C_1, short 200 contracts of option C_2, and long 100 contracts of option P_1. The investor's options portfolio has the following characteristics.

DELTA = 0.8922 (100) − 0.2678 (200) − 0.6187 (100) = −26.21
GAMMA = 0.0169 (100) − 0.0299 (200) + 0.0245 (100) = −1.84
THETA = −5.55 (100) + 3.89 (200) − 3.72 (100) = −149

The investor wishes to hedge this portfolio of options with two call options written on the stock of QDS Corporation with the following characteristics.

	C_a	C_b
DELTA	0.5761	0.6070
GAMMA	0.0356	0.0247
THETA	−9.72	−7.04

A. How many contracts of the two options, C_a and C_b, must the investor hold to create a portfolio that is DELTA-neutral and has a THETA of 100?

The investor must determine how many of the call options a and b to hold to create a portfolio that has a DELTA of 26.21 and a THETA of 249. That is, the DELTA of the combined portfolio of options should be zero, and the THETA of the combined portfolio should be 100. We must solve this for N_a and N_b subject to the constraint that the DELTA of the portfolio is 26.21 and the THETA is 249.

$$0.5671\, N_a + 0.607\, N_b = 26.21$$
$$-9.72\, N_a - 7.04\, N_b = 249$$
$$N_a = -175.955$$
$$N_b = 207.568$$

To create a portfolio with a DELTA of 26.21 and a THETA of 249, the investor must sell 176 contracts of option C_a and purchase 208 contracts of option C_b.

B. If QDS's stock price remains relatively constant over the next month, explain what will happen to the value of the portfolio created in part A of this question.

If QDS's stock price remains relatively constant, then the passage of time should increase the value of the portfolio. The DELTA of the portfolio is zero and the THETA is positive. Thus, small changes in the stock

price will have little impact on the value of the portfolio of options, and the passage of time should increase the value of the option portfolio.

C. How many of the two options, C_a and C_b, must the investor hold to create a portfolio that is both DELTA- and GAMMA-neutral?

The investor must determine how many of the call options C_a and C_b to hold to create a portfolio that has a DELTA of 26.21 and a GAMMA of 1.84. That is, the DELTA and GAMMA of the combined portfolio of options are zero. We must solve the equation for N_a and N_b subject to the constraint that the DELTA of the portfolio is 26.21 and the GAMMA is 1.84.

$$0.5671\, N_a + 0.607\, N_b = 26.21$$
$$0.0356\, N_a + 0.0247\, N_b = 1.84$$
$$N_a = 61.7573$$
$$N_b = -14.51667$$

To create a portfolio with a DELTA of 26.21 and a GAMMA of 1.84, the investor must purchase 62 contracts of option C_a and sell 15 contracts of option C_b.

D. Suppose the investor wants to create a portfolio that is DELTA-, GAMMA-, and THETA-neutral. Could the investor accomplish this objective using the two options, C_a and C_b, which have been used in the previous problems? Explain.

No. Creating neutrality in three dimensions requires at least three different option contracts. We need at least as many option contracts as parameters we are trying to hedge. To create a DELTA-, GAMMA-, and THETA-neutral portfolio would require an additional new option contract.

19. Both put and call options trade on HWP. Put and call options with an exercise price of $100 expire in 90 days. HWP is trading at $95, and has an annualized standard deviation of return of 0.3. The three-month risk-free interest rate is 5.25 percent per annum.

A. Use a four-period binomial model to compute the value of the put and call options using the recursive procedure (single-period binomial option pricing model).

$$U = e^{\sigma\sqrt{\Delta t}} = e^{0.3\sqrt{22.5/365}} = 1.0773$$
$$D = 1/U = 1/1.0773 = 0.9282$$
$$\pi_U = \frac{e^{r\Delta t} - D}{U - D} = \frac{e^{.0525 \times 22.5/365} - 0.9282}{1.0773 - 0.9282} = 0.5031 \quad \pi_D = 1 - 0.5031 = 0.4969$$
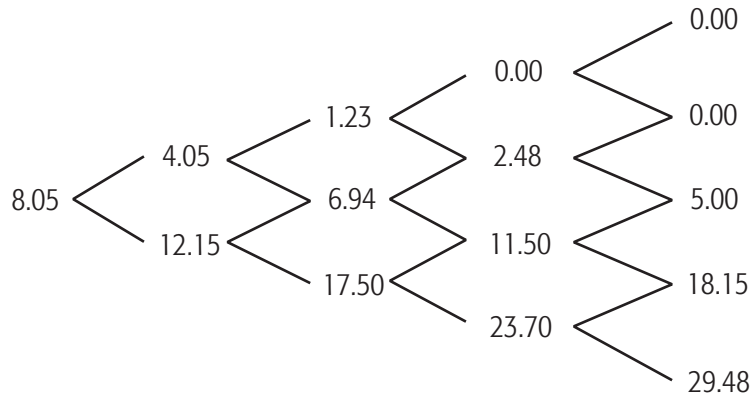$$e^{-r\Delta t} = 0.9968$$
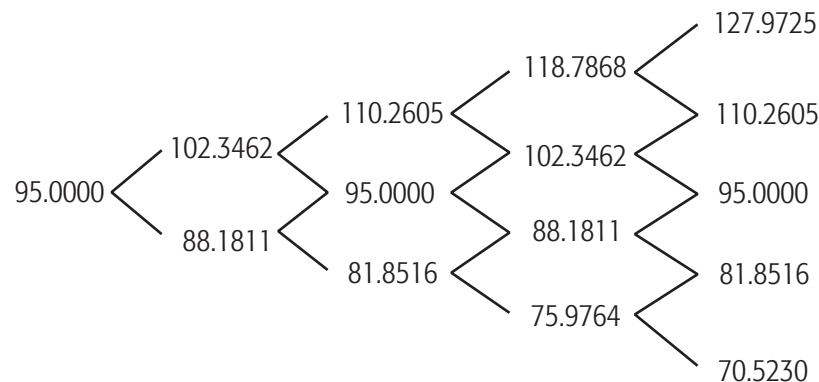
Stock price tree

Call price tree
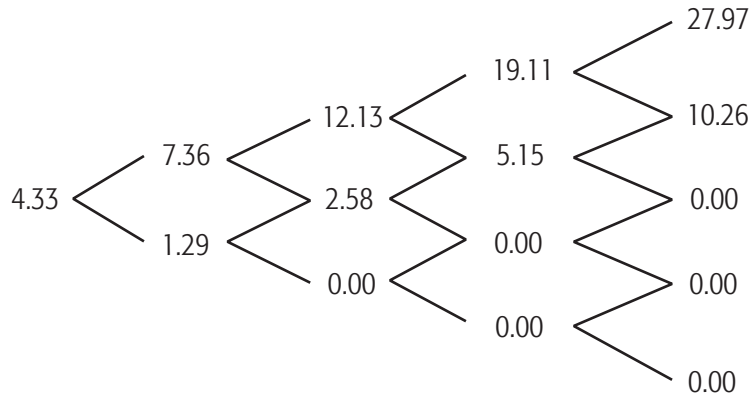
Put price tree

The call price is $4.33, and the put price is $8.05.

B. Increase the stock price by $.25 to $95.25 and recalculate the value of the put and call options using the recursive procedure (single-period binomial option pricing model).

If the stock price is increased by $.25, then the call price is $4.43 and the put price is $7.89.

C. For both the call and put options, calculate DELTA as the change in the value of the option divided by the change in the value of the stock.

Call DELTA	Put DELTA
$(4.43 - 4.33)/0.25 = 0.40$	$(7.89 - 8.05)/0.25 = -0.64$

D. Decrease the stock price by $.25 to $94.75 and recalculate the value of the put and call options using the recursive procedure (single-period binomial option pricing model).

When the stock price is decreased by $.25, then the call price is $4.24 and the put price is $8.20.

E. For both the call and put options, calculate DELTA as the change in the value of the option divided by the change in the value of the stock.

Call DELTA	Put DELTA
$(4.33 - 4.24)/0.25 = 0.36$	$(8.05 - 8.20)/0.25 = -0.60$

F. For both the call and put options, calculate GAMMA as the difference between the DELTA associated with a stock price increase and the DELTA associated with a stock price decrease, divided by the change in the value of the stock.

Call GAMMA	Put GAMMA
$(0.40 - 0.36)/0.5 = 0.08$	$(-0.60 - (-0.64))/0.50 = 0.08$

G. Increase the risk-free interest rate by 20 basis points to 5.45 percent per annum and recalculate the value of the put and call options using the recursive procedure (single-period binomial option pricing model).

   If the interest rate is increased by 20 basis points to 5.45 percent, then the call price is $4.35 and the put price is $8.02.

H. For both the call and put options, calculate RHO as the change in the value of the option divided by the change in the risk-free interest rate.

Call RHO	Put RHO
$(4.35 - 4.33)/0.002 = 10$	$(8.02 - 8.05)/0.002 = -15$

I. Increase the volatility of the underlying stock to 33 percent and recalculate the value of the put and call options using the recursive procedure (single-period binomial option pricing model).

   If the volatility is increased to 33%, then the call price is $4.89 and the put price is $8.60.

J. For both the call and put options, calculate VEGA as the change in the value of the option divided by the change in the volatility of the stock.

Call VEGA	Put VEGA
$(4.89 - 4.33)/0.03 = 18.6667$	$(8.60 - 8.05)/0.03 = 18.3333$

K. Notice that each branch in the binomial tree represents the passage of time. That is, as one moves forward through the branches in a binomial tree, the life of the option wastes away. Also notice that the initial stock price of $95 reappears in the middle of the second branch of the tree. Using the parameter values for U, D, π_U, π_D, Δt, and $e^{-r\Delta t}$ calculated using the initial information given, recalculate the value of the call and put options using a **two-period** model. That is, calculate the option prices after 45 days, assuming the stock price is unchanged.
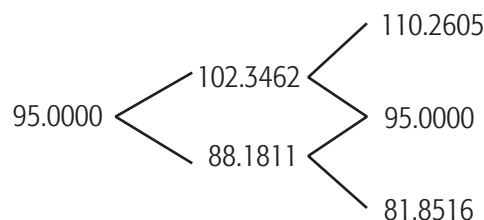
$$U = e^{\sigma\sqrt{\Delta t}} = e^{0.3\sqrt{22.5/365}} = 1.0773$$

$$D = 1/U = 1/1.0773 = 0.9282$$

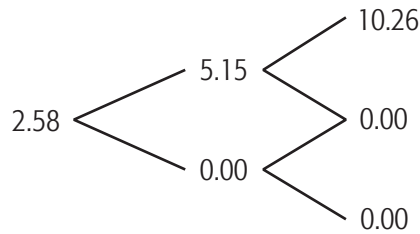$$\pi_U = \frac{e^{r\Delta t} - D}{U - D} = \frac{e^{.0525 \times 22.5/365} - 0.9282}{1.0773 - 0.9282} = 0.5031 \quad \pi_D = 1 - 0.5031 = 0.4969$$

$$e^{-r\Delta t} = 0.9968$$
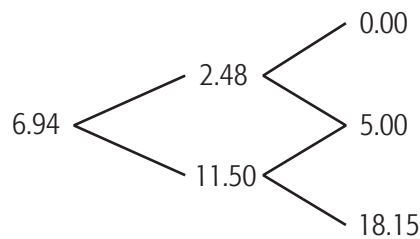
Stock price tree

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                                       110.2605
                          102.3462
       95.0000                         95.0000
                          88.1811
                                       81.8516
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Call price tree

Put price tree

The call price is $2.58, and the put price is $6.94.

L. For both the call and put options, calculate THETA as the difference in the value of the option with two periods to expiration less the value of the option with four periods to expiration, divided by twice the passage of time associated with each branch in the tree, that is, $2 \times \Delta t$.

Call THETA	Put THETA
$(2.58 - 4.33)/[2 \times (22.5/365)] = -14.1944$	$(6.94 - 8.05)/[2 \times (22.5/365)] = -9.0033$

20. Consider a stock, ABM, trading at a price of $70. Analysis of ABM's recent returns reveals that ABM has an annualized standard deviation of return of 0.4. The current risk-free rate of interest is 10 percent per annum.

A. What is the price of a European call and put written on ABM with a $75 strike price that expires in 180 days? Price the options using the Black–Scholes model.

$$d_1 = \frac{\ln(70/75) + ([0.10 + 0.5(0.4^2)](0.4932))}{0.40\sqrt{(0.4932)}} = 0.0704$$

$$d_2 = 0.0704 - 0.40\sqrt{(0.4932)} = -0.2105$$

$$N(d_1) = 0.528061 \quad N(d_2) = 0.416638$$

$$c = 70 \times 0.528061 - 75e^{-0.1 \times 0.4932} \times 0.416638 = \$7.2201$$

$$p = c - S + Xe^{-r(T-t)} = \$7.22 - \$70 + \$75e^{-0.1 \times 0.4932} = \$8.6111$$

The call price is $7.2201, and the put price is $8.6111.

B. Increase the stock price by $.25 to $70.25 and recalculate the value of the put and call options.

If the stock price is increased by $.25, then the call price is $7.3527 and the put price is $8.4938.

C. For both the call and put options, calculate DELTA as the change in the value of the option divided by the change in the value of the stock.

Call DELTA	Put DELTA
$(7.3527 - 7.2201)/0.25 = 0.5304$	$(8.4938 - 8.6111)/0.25 = -0.4692$

D. Decrease the stock price by \$.25 to \$69.75 and recalculate the value of the put and call options.

If the stock price is decreased by \$.25, then the call price is \$7.0887 and the put price is \$8.7298.

E. For both the call and put options, calculate DELTA as the change in the value of the option divided by the change in the value of the stock.

Call DELTA	Put DELTA
(7.2201 − 7.0887)/0.25 = 0.5256	(8.6111 − 8.7298)/0.25 = −0.4748

F. For both the call and put options, calculate GAMMA as the difference between the DELTA associated with a stock price increase and the DELTA associated with a stock price decrease divided by the change in the value of the stock.

Call GAMMA	Put GAMMA
(0.5304 − 0.5256)/0.5 = 0.0096	(−0.4692 − (−0.4748))/0.5 = .0112

G. Increase the risk-free interest rate by 25 basis points to 10.25 percent per annum and recalculate the value of the put and call options.

If the interest rate is increased by 25 basis points to 10.25 percent, then the call price is \$7.2568 and the put price is \$8.5599.

H. For both the call and put options, calculate RHO as the change in the value of the option divided by the change in the risk-free interest rate.

Call RHO	Put RHO
(7.2568 − 7.2201)/0.0025 = 14.68	(8.5599 − 8.6111)/0.0025 = −20.48

I. Increase the volatility of the underlying stock to 44 percent and recalculate the value of the put and call options.

If the volatility is increased to 44 percent, then the call price is \$8.0019 and the put price is \$9.3930.

J. For both the call and put options, calculate VEGA as the change in the value of the option divided by the change in the volatility of the stock.

Call VEGA	Put VEGA
(8.0019 − 7.2201)/0.04 = 19.5450	(9.3930 − 8.6111)/0.04 = 19.5475

K. Decrease the life of the option by 10 percent to 162 days and recalculate the value of the put and call options.

If the life of the option is decreased by 10 percent to 162 days, then the call price is \$6.6721 and the put price is \$8.4161.

L. For both the call and put options, calculate THETA as the difference in the value of the option with 162 days to expiration less the value of the option with 180 days to expiration divided by the passage of time.

Call THETA	Put THETA
(6.6721 − 7.2201)/(18/365) = −11.1122	(8.4161 − 8.6111)/(18/365) = −3.9542

21. Consider a stock that trades for \$75. A put and a call on this stock both have an exercise price of \$70, and they expire in 145 days. The risk-free rate is 9 percent per annum and the standard deviation of return for the stock is 0.35. Assume that the stock pays a continuous dividend of 4 percent.

A. What are the prices of a European call and put option written on this stock according to Merton's model?

$$d_1^M = \frac{\ln(75/70) + ([0.09 - 0.04 + 0.5(0.35^2)](0.3973))}{0.35\sqrt{0.3973}} = 0.5131$$

$$d_2^M = 0.5131 - 0.35\sqrt{0.3973} = 0.2925$$

$$N(d_1^M) = 0.696056 \quad N(d_2^M) = 0.615045$$

$$c^M = e^{-\delta(T-t)} S_t N(d_1^M) - Xe^{-r(T-t)} N_2(d^M)$$

$$c^M = e^{-0.04(0.3973)} 75 \times 0.696056 - 70e^{-0.09(0.3973)} \times 0.615045 = \$9.84$$

$$N(-d_1^M) = 0.303944 \quad N(-d_2^M) = 0.384955$$

$$p^M = Xe^{-r(T-t)} N(-d_2^M) - e^{-\delta(T-t)} S_t N(-d_1^M)$$

$$p^M = 70e^{-0.09(0.3973)} \times 0.384955 - e^{-0.04(0.3973)} 75 \times 0.303944 = \$3.56$$

The call price is $9.8402, and the put price is $3.5641.

B. Increase the stock price by $.25 to $75.25 and recalculate the value of the put and call options.

When the stock price is increased by $.25, then the call price is $10.0121 and the put price is $3.4899.

C. For both the call and put options, calculate DELTA as the change in the value of the option divided by the change in the value of the stock.

Call DELTA	Put DELTA
(10.0121 − 9.8402)/0.25 = 0.6876	(3.4899 − 3.5641)/0.25 = −0.2968

D. Decrease the stock price by $.25 to $74.75 and recalculate the value of the put and call options.

If the stock price is decreased by $.25, then the call price is $9.6696 and the put price is $3.6395.

E. Calculate DELTA for both the call and put options as the change in the value of the option divided by the change in the value of the stock.

Call DELTA	Put DELTA
(9.8402 − 9.6696)/0.25 = 0.6824	(3.5641 − 3.6395)/0.25 = −0.3016

F. Calculate GAMMA for both the call and put options as the difference between the DELTA associated with a stock price increase and the DELTA associated with a stock price decrease divided by the change in the value of the stock.

Call GAMMA	Put GAMMA
(0.6876 − 0.6824)/0.5 = 0.0104	(−0.2968 − (−0.3016))/0.5 = 0.0096

G. Increase the risk-free interest rate by 25 basis points to 9.25 percent per annum and recalculate the value of the put and call options.

When the interest rate is increased by 25 basis points to 9.25 percent, then the call price is $9.8815 and the put price is $3.5383.

H. Calculate RHO for both the call and put options as the change in the value of the option divided by the change in the risk-free interest rate.

Call RHO	Put RHO
$(9.8815 - 9.8402)/0.0025 = 16.5200$	$(3.5383 - 3.5641)/0.0025 = -10.32$

I. Increase the volatility of the underlying stock to 38.5 percent and recalculate the value of the put and call options.

When the volatility is increased to 38.5 percent, the call price is \$10.4136 and the put price is \$4.1374.

J. Calculate VEGA for both the call and put options as the change in the value of the option divided by the change in the volatility of the stock.

Call VEGA	Put VEGA
$(10.4136 - 9.8402)/0.035 = 16.3829$	$(4.1374 - 3.5641)/0.035 = 16.3800$

K. Decrease the life of the option by 20 percent to 116 days and recalculate the value of the put and call options.

When the life of the option is decreased by 20 percent to 116 days, then the call price is \$9.1028 and the put price is \$3.0764.

L. Calculate THETA for both the call and put options as the difference in the value of the option with 116 days to expiration less the value of the option with 145 days to expiration divided by the passage of time.

Call THETA	Put THETA
$(9.1028 - 9.8402)/(29/365) = -9.2811$	$(3.0764 - 3.5641)/(29/365) = -6.1383$

22. CSM is trading at \$78 and has an annualized standard deviation of return of 30 percent. CSM is expected to pay a dividend equal to 3 percent of the value of its stock price in 70 days. The current risk-free rate of interest is 7 percent per annum. Options written on this stock have an exercise price of \$80 and expire in 120 days.

A. Using a four-period binomial model, calculate the values of European put and call options written on CSM.
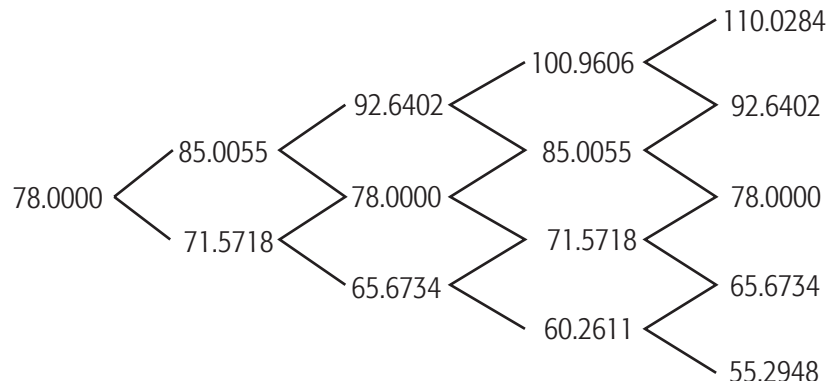
$$U = e^{\sigma\sqrt{\Delta t}} = e^{0.3\sqrt{30/365}} = 1.0898$$

$$D = 1/U = 1/1.0898 = 0.9176$$

$$\pi_U = \frac{e^{r\Delta t} - D}{U - D} = \frac{e^{0.07 \times 30/365} - 0.9176}{1.0898 - 0.9176} = 0.5120 \quad \pi_D = 1 - 0.5120 = 0.4880$$
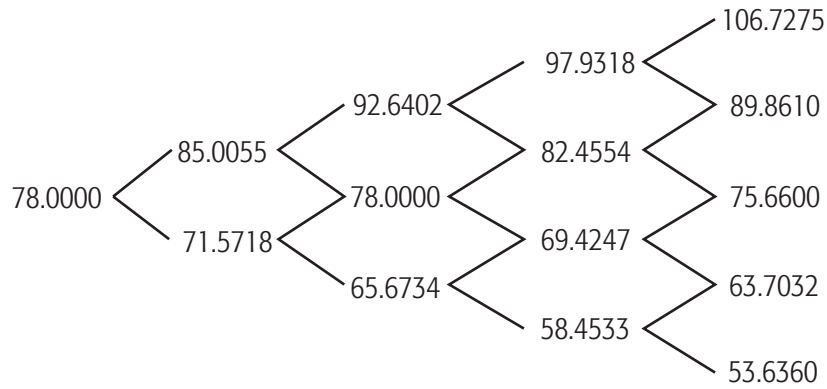
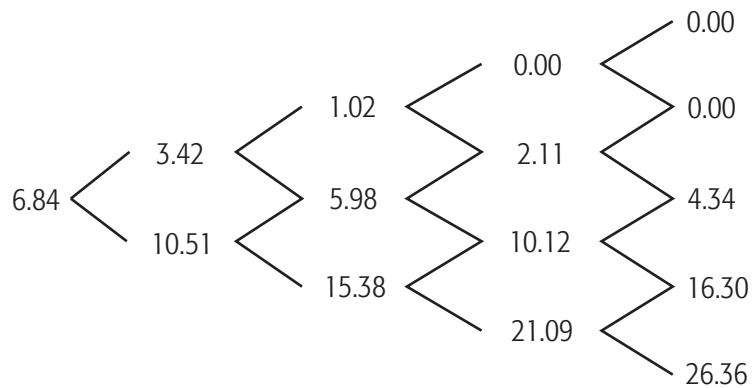$$e^{-r\Delta t} = 0.9943$$

Stock price tree

The stock prices in the tree must be adjusted for the dividend to be paid in 70 days before we can calculate the value of the options. Therefore, the stock prices in the tree in periods three and four must be adjusted downward by one minus the dividend yield paid by the firm (1 − 3%).
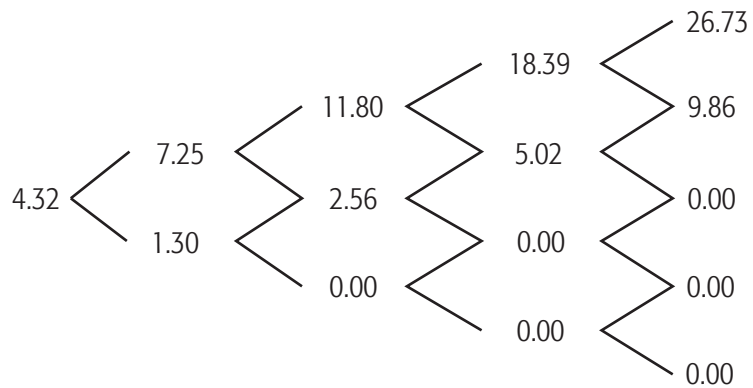
Dividend-adjusted stock price tree

```
                                                        106.7275
                                            97.9318
                                92.6402                 89.8610
                    85.0055                 82.4554
        78.0000                 78.0000                 75.6600
                    71.5718                 69.4247
                                65.6734                 63.7032
                                            58.4533
                                                        53.6360
```

Put price tree

```
                                                    0.00
                                        0.00
                            1.02                    0.00
                3.42                    2.11
        6.84                5.98                    4.34
                10.51                   10.12
                            15.38                   16.30
                                        21.09
                                                    26.36
```

The price of the put option is $6.84.

Call price tree

```
                                                    26.73
                                        18.39
                            11.80                   9.86
                7.25                    5.02
        4.32                2.56                    0.00
                1.30                    0.00
                            0.00                    0.00
                                        0.00
                                                    0.00
```

The price of the call option is $4.32.

B. Increase the stock price by \$.25 to \$78.25 and recalculate the value of the put and call options using the recursive procedure (single-period binomial option pricing model).

When the stock price is increased by \$.25, then the call price is \$4.42 and the put price is \$6.69.

C. Calculate DELTA for both the call and put options as the change in the value of the option divided by the change in the value of the stock.

Call DELTA	Put DELTA
(4.42 − 4.32)/0.25 = 0.40	(6.69 − 6.84)/0.25 = −0.60

D. Decrease the stock price by \$.25 to \$77.75 and recalculate the value of the put and call options using the recursive procedure (single-period binomial option pricing model).

When the stock price is decreased by \$.25, then the call price is \$4.22 and the put price is \$6.99.

E. Calculate DELTA for both the call and put options as the change in the value of the option divided by the change in the value of the stock.

Call DELTA	Put DELTA
(4.32 − 4.22)/0.25 = 0.40	(6.69 − 6.84)/0.25 = −0.60

F. Calculate GAMMA for both the call and put options as the difference between the DELTA associated with a stock price increase and the DELTA associated with a stock price decrease divided by the change in the value of the stock.

Call GAMMA	Put GAMMA
(0.3869 − 0.3869)/0.5 = 0	(−0.5831 − (−0.5831))/0.5 = 0

Calculating the price of each option to two decimal places using a four-period binomial process to model the price movements in a stock over a 90-day period produces symmetric price changes in the value of the options. There is not sufficient information in our modeling process to capture the nonlinearity of the pricing function for small symmetric changes in the stock price. This generates a value of zero for GAMMA.

G. Increase the risk-free interest rate by 20 basis points to 7.20 percent per annum and recalculate the value of the put and call options using the recursive procedure (single-period binomial option pricing model).

When the interest rate is increased by 20 basis points to 7.20 percent, then the call price is \$4.34 and the put price is \$6.81.

H. Calculate RHO for both the call and put options as the change in the value of the option divided by the change in the risk-free interest rate.

Call RHO	Put RHO
(4.34 − 4.32)/0.002 = 10	(6.81 − 6.84)/0.002 = −15

I. Increase the volatility of the underlying stock to 33 percent and recalculate the value of the put and call options using the recursive procedure (single-period binomial option pricing model).

When the volatility is increased to 33 percent, then the call price is \$4.83 and the put price is \$7.35.

J. Calculate VEGA for both the call and put options as the change in the value of the option divided by the change in the volatility of the stock.

Call VEGA	Put VEGA
$(4.83 - 4.32)/0.03 = 17$	$(7.35 - 6.84)/0.03 = 17$

K. Notice that each branch in the binomial tree represents the passage of time. That is, as one moves forward through the branches in a binomial tree, the life of the option wastes away. Also notice that the initial stock price of $78 reappears in the middle of the second branch of the tree. Using the parameter values for U, D, π_U, π_D, Δt, and $e^{-r\Delta t}$ calculated using the initial information given, recalculate the value of the call and put options using a **two-period** model. Assume that the dividend will be paid before period one and make the appropriate adjustments to the stock price tree. That is, calculate the option prices after 60 days, assuming that the stock price is unchanged.
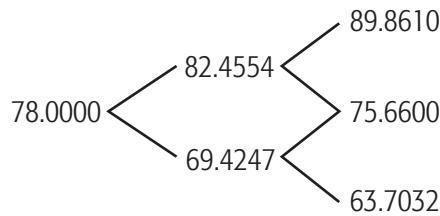
$$U = e^{\sigma\sqrt{\Delta t}} = e^{0.3\sqrt{30/365}} = 1.0898$$

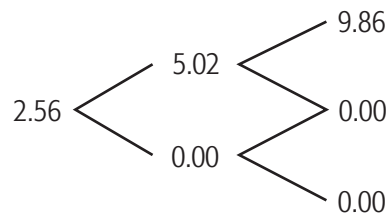$$D = 1/U = 1/1.0898 = 0.9176$$

$$\pi_U = \frac{e^{r\Delta t} - D}{U - D} = \frac{e^{0.07 \times 30/365} - 0.9176}{1.0898 - 0.9176} = 0.5120 \quad \pi_D = 1 - 0.5120 = 0.4880$$
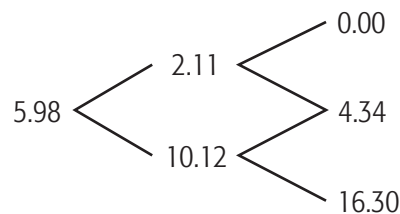
$$e^{-r\Delta t} = 0.9943$$

Dividend-adjusted stock price tree

```
                                    89.8610
                         82.4554
              78.0000              75.6600
                         69.4247
                                    63.7032
```

Call price tree

```
                                 9.86
                      5.02
            2.56              0.00
                      0.00
                                 0.00
```

Put price tree

```
                                 0.00
                      2.11
            5.98              4.34
                      10.12
                                 16.30
```

The call price is $2.56, and the put price is $5.98.

L. Calculate THETA for both the call and put options as the difference in the value of the option with two periods to expiration less the value of the option with four periods to expiration divided by twice the passage of time associated with each branch in the tree, that is, $2 \times \Delta t$.

Call THETA	Put THETA
$(2.56 - 4.32)/[2 \times (30/365)] = -10.7067$	$(5.98 - 6.84)/[2 \times (30/365)] = -5.2317$

23. DCC exports high-speed digital switching networks, and their largest and most important clients are in Asia. A recent financial crisis in Asia has diminished the prospects of new sales to the Asian market in the near term. However, you believe that DCC is a good investment for the long term. A quick check of Quote.com reveals that DCC is trading at $78.625 per share. Your previous calculation of the historical volatility for DCC indicated an annual standard deviation of return of 27 percent, but examining the implied volatility of several DCC options reveals an increase in annual volatility to 32 percent. There are two traded options series that expire in 245 days. The options have $75 and $80 strike prices respectively. The current 245-day risk-free interest rate is 4.75 percent per annum, and you hold 2,000 shares of DCC.

	X = 75		X = 80	
	Call	**Put**	**Call**	**Put**
DELTA	0.6674	−0.3326	0.574	−0.426
GAMMA	0.0176	0.0176	0.019	0.019
THETA	−7.5372	−4.0865	−7.7495	−4.0687
VEGA	23.4015	23.4015	25.2551	25.2551
RHO	27.6835	−21.0792	24.4395	−27.574

A. Construct a portfolio that is DELTA- and GAMMA-neutral using the call options written on DCC.

The investor owns 2,000 shares of DCC. An option contract extends the right to buy or sell 100 shares of the underlying stock at the strike price. Thus, by scaling the stock position by 100, we can convert the stock position to a scale that matches a single option contract. On a scaled basis, the shareholder has a long position in $20 = 2000/100$, contract units of stock. The investor must determine how many $75 call options and $80 call options to hold to create a portfolio that is DELTA- and GAMMA-neutral. The DELTA of the stock, DELTA_s, is one, and the GAMMA of the stock, GAMMA_s, is zero.

$$N_s \, \text{DELTA}_s + N_{75} \, \text{DELTA}_{75} + N_{80} \, \text{DELTA}_{80} = 0$$
$$N_s \, \text{GAMMA}_s + N_{75} \, \text{GAMMA}_{75} + N_{80} \, \text{GAMMA}_{80} = 0$$

$$20 \times 1 + N_{75}(0.6674) + N_{80}(0.5740) = 0$$
$$20 \times 0 + N_{75}(0.0176) + N_{80}(0.0190) = 0$$

We must solve the equation for N_{75} and N_{80} subject to the constraint that the DELTA of the portfolio is -20 and the GAMMA is 0.

$$N_{75}(0.6674) + N_{80}(0.5740) = -20$$
$$N_{75}(0.0176) + N_{80}(0.0190) = 0$$

$$N_{75} = -147.39$$
$$N_{80} = 136.53$$

To create a portfolio with a DELTA of -20 and a GAMMA of 0, the investor must sell 147.39 of the $75 call option contracts and purchase 136.53 of the $80 call option contracts.

B. Construct a portfolio that is DELTA- and GAMMA-neutral using the put options written on DCC.

The investor owns 2,000 shares of DCC. An option contract extends the right to buy or sell 100 shares of the underlying stock at the strike price. Thus, by scaling the stock position by 100, we can convert the stock position to a scale that matches a single option contract. On a scaled basis, the shareholder has a long position in $20 = 2000/100$, contract units of stock. The investor must determine how many \$75 put options and \$80 put options to hold to create a portfolio that is DELTA- and GAMMA-neutral. The DELTA of the stock, DELTA_s, is one, and the GAMMA of the stock, GAMMA_s, is zero.

$$N_s \, \text{DELTA}_s + N_{75} \, \text{DELTA}_{75} + N_{80} \, \text{DELTA}_{80} = 0$$
$$N_s \, \text{GAMMA}_s + N_{75} \, \text{GAMMA}_{75} + N_{80} \, \text{GAMMA}_{80} = 0$$

$$20 \times 1 + N_{75} \, (-0.3326) + N_{80} \, (-0.4260) = 0$$
$$20 \times 0 + N_{75} \, (0.0176) + N_{80} \, (0.0190) = 0$$

We must solve the equation for N_{75} and N_{80} subject to the constraint that the DELTA of the portfolio is -20 and the GAMMA is 0.

$$N_{75} \, (-0.3326) + N_{80} \, (-0.426) = -20$$
$$N_{75} \, (0.0176) + N_{80} \, (0.0190) = 0$$

$$N_{75} = -322.53$$
$$N_{80} = 298.76$$

To create a portfolio with a DELTA of -20 and a GAMMA of 0, the investor must sell 322.53 of the \$75 put option contracts and purchase 298.76 of the \$80 put option contracts.

C. Construct a portfolio that is DELTA- and THETA-neutral using the call options written on DCC.

The investor owns 2,000 shares of DCC. An option contract extends the right to buy or sell 100 shares of the underlying stock at the strike price. Thus, by scaling the stock position by 100, we can convert the stock position to a scale that matches a single option contract. On a scaled basis, the shareholder has a long position in $20 = 2000/100$, contract units of stock. The investor must determine how many \$75 call options and \$80 call options to hold to create a portfolio that is DELTA- and THETA-neutral. The DELTA of the stock, DELTA_s, is one, and the THETA of the stock, THETA_s, is zero.

$$N_s \, \text{DELTA}_s + N_{75} \, \text{DELTA}_{75} + N_{80} \, \text{DELTA}_{80} = 0$$
$$N_s \, \text{THETA}_s + N_{75} \, \text{THETA}_{75} + N_{80} \, \text{THETA}_{80} = 0$$

$$20 \times 1 + N_{75} \, (0.6674) + N_{80} \, (0.5740) = 0$$
$$20 \times 0 + N_{75} \, (-7.5372) + N_{80} \, (-7.7495) = 0$$

We must solve the equation for N_{75} and N_{80} subject to the constraint that the DELTA of the portfolio is -20 and the THETA is 0.

$$N_{75} \, (0.6674) + N_{80} \, (0.5740) = -20$$
$$N_{75} \, (-7.5372) + N_{80} \, (-7.7495) = 0$$

$$N_{75} = -183.28$$
$$N_{80} = 178.26$$

To create a portfolio with a DELTA of -20 and a THETA of 0, the investor must sell 183.28 of the \$75 call option contracts and purchase 178.26 of the \$80 call option contracts.

D. Construct a portfolio that is DELTA- and THETA-neutral using the put options written on DCC.

The investor owns 2,000 shares of DCC. An option contract extends the right to buy or sell 100 shares of the underlying stock at the strike price. Thus, by scaling the stock position by 100, we can convert the stock position to a scale that matches a single option contract. On a scaled basis, the shareholder has a long position in $20 = 2000/100$, contract units of stock. The investor must determine how many $75 put options and $80 put options to hold to create a portfolio that is DELTA- and THETA-neutral. The DELTA of the stock, $DELTA_s$, is one, and the THETA of the stock, $THETA_s$, is zero.

$$N_s \, DELTA_s + N_{75} \, DELTA_{75} + N_{80} \, DELTA_{80} = 0$$
$$N_s \, THETA_s + N_{75} \, THETA_{75} + N_{80} \, THETA_{80} = 0$$

$$20 \times 1 + N_{75}\,(-0.3326) + N_{80}\,(-0.426) = 0$$
$$20 \times 0 + N_{75}\,(-4.0865) + N_{80}\,(-4.0687) = 0$$

We must solve the equation for N_{75} and N_{80} subject to the constraint that the DELTA of the portfolio is -20 and the THETA is 0.

$$N_{75}\,(-0.3326) + N_{80}\,(-0.426) = -20$$
$$N_{75}\,(-4.0865) + N_{80}\,(-4.0687) = 0$$

$$N_{75} = -209.94$$
$$N_{80} = 210.86$$

To create a portfolio with a DELTA of -20 and a THETA of 0, the investor must sell 209.94 of the $75 put option contracts and purchase 210.86 of the $80 put option contracts.

E. How effective do you expect the DELTA- and GAMMA-neutral hedges to be? Explain.

The effectiveness of the DELTA-, GAMMA-neutral hedges will be conditional on movements in DCC's stock price over the hedge period. If DCC's stock remains relatively constant during the hedge period, then the hedge should be effective. However, if DCC's stock price is volatile during the hedge period, which is very likely to happen, then it will be necessary to rebalance the portfolio periodically to maintain the effectiveness of the hedge.

24. Consider a stock, PRN, that trades for $24. A put and a call on this stock both have an exercise price of $22.50, and they expire in 45 days. The risk-free rate is 5.5 percent per annum, and the standard deviation of return for the stock is .28.

A. Calculate the price of the put and call option using the Black–Scholes model.

$$d_1 = \frac{\ln(24/22.5) + ([0.055 + 0.5(0.28^2)](0.1233))}{0.28\sqrt{(0.1233)}} = 0.7746$$

$$d_2 = 0.7746 - 0.28\sqrt{(0.1233)} = -0.6763$$

$$N(d_1) = 0.780705 \quad N(d_2) = 0.750563$$

$$c = 24 \times 0.780705 - 22.5e^{-0.055 \times 0.1233} \times 0.750563 = \$1.96$$

$$p = c - S + Xe^{-r(T-t)} = \$1.96 - \$24 + \$22.5e^{-0.055 \times 0.1233} = \$.31$$

The call price is $1.96, and the put price is $.31.

Note: Use the following information for the remaining parts of this problem.
Suppose that you own 1,500 shares of PRN and you wish to hedge your investment in PRN using the traded PRN options. You are going on vacation in 45 days and want to use your shares to finance your vacation, so you do not want the value of your PRN shares to fall below $22.50.

B. Construct a hedge using a covered call strategy. In a covered call strategy, the investor sells call options to hedge against the risk of a stock price decline.

The investor owns 1,500 shares of PRN. An option contract extends the right to buy or sell 100 shares of the underlying stock at the strike price. Thus, by scaling the stock position by 100, we can convert the stock position to a scale that matches a single option contract. On a scaled basis, the shareholder has a long position in 15 = 1,500/100, contract units of stock. The investor wants to create a hedge position that expires in 45 days and the PRN options expire in 45 days. In this hedge, the investor will hold the option contracts to their expiration date and use the moneyness of the options to hedge the investment in PRN. The investor's objective is to create a price floor of $22.50 for the 1,500 shares of PRN. The minimum value desired for the PRN shares in 45 days is $33,750 = 1,500 × $22.50.

In this covered call hedge, the investor will sell one call option of every share of stock that he owns. On a scaled basis, the shareholder has a long position in 15 PRN contract units of stock. Thus, the investor should sell 15 PRN call options at $1.96. The cash inflow from the sale of the 15 call options is $2,940 = $1.96 × 15 × 100.

C. Construct a hedge using a protective put strategy. In a protective put strategy, the investor purchases put options to hedge against the risk of a stock price decline.

In this protective put hedge, the investor will buy one put option of every share of stock that he owns. On a scaled basis, the shareholder has a long position in 15 PRN shares. Thus, the investor should buy 15 PRN put options at $.31. The cash outflow from the purchase of the 15 put options is $465 = $.31 × 15 × 100.

D. If PRN is trading at $19 in 45 days, analyze and compare the effectiveness of the two alternative hedging strategies.

Covered call:
When PRN is trading at $19, the call options that the investor sold expire worthless. The investor keeps the proceeds of $2,940 from the sale of the call options. The investor's stock position is worth $28,500 = $19 × 1,500. The total value of the portfolio consisting of the stock position and the proceeds from the sale of the options is $31,440 = $28,500 + $2,940. Thus, the value of the portfolio is $2,310 less than the $33,750 minimum value for the portfolio established by the investor at the start of the hedge.

Protective put:
When PRN is trading at $19, the put options owned by the investor are in-the-money. The investor will exercise the 15 put options, selling 1,500, 15 × 100, shares of stock at $22.50 per share. The value of the investor's portfolio is $33,750, the minimum value for the portfolio established by the investor at the start of the hedge.

The covered call hedge strategy resulted in a loss of value to the investor's portfolio that was greater than the loss the investor was willing to accept at the start of the hedge period. The protective put strategy produced a portfolio that had a value that was equal to the minimum set by the investor at the start of the hedge.

E. If PRN is trading at $26 in 45 days, analyze and compare the effectiveness of the two alternative hedging strategies.

Covered call:
When PRN is trading at $26, the call options sold by the investor are in-the-money. The owner of the options will exercise the 15 call options against the investor. The investor will have to sell his 1,500 shares of PRN stock at $22.50 per share to the owner of the option. The investor will receive $33,750 from the sale of the stock. The proceeds of $2,940 from the sale of the call options plus the $33,750 from the sale of the stock result in a position that is worth $36,690. The value of 1,500 shares of PRN at $26 per share is $39,000. Thus, the investor incurs an opportunity cost of $2,310 from the sale of 15 PRN call options.

Protective put:
When PRN is trading at $26, the put options that the investor purchased expire worthless. The investor's stock position is worth $39,000, $26 × 1,500. The total value of the portfolio consisting of the stock position and the cost of the put options is $38,535, $39,000 − $465.

When the stock price exceeds the exercise price of an option at expiration, the investor incurs an opportunity cost when implementing a covered call hedge strategy. The protective put hedging strategy is not subject to this opportunity cost.

25. A friend, Audrey, holds a portfolio of 10,000 shares of Microsoft stock. In 60 days she needs at least $855,000 to pay for her new home. You suggest to Audrey that she can construct an insured portfolio using Microsoft stock options. You explain that an insured portfolio can be constructed several different ways, but the basic notion is to create a portfolio that consists of a long position in Microsoft's stock and a long position in put options written on Microsoft. If at the end of the hedge period Microsoft's stock is trading at a price below the strike price on the put option, Audrey has the right to sell her Microsoft stock to the owner of the put option for the strike price. Thus, at the end of the hedge period, Audrey has sufficient assets to cover her needs or obligations. This hedging strategy can be implemented using traded options. However, Audrey may not be able to find an option with the desired strike price or expiration date. Dynamic hedging permits Audrey to overcome these limitations associated with traded option contracts. In dynamic hedging, Audrey constructs a portfolio that consists of a long position in stock and a long position in Treasury bills. As the underlying stock price changes, Audrey dynamically alters the allocation of assets in the portfolio between stock and Treasury bills. Therefore, we can view dynamic hedging as an asset allocation problem where Audrey determines how much of her resources are allocated to the stock, and how much of her resources are allocated to Treasury bills. The amount of resources available for investment is simply the current cash value of Audrey's stock position. The proportion of resources committed to the stock, w, are calculated as

$$\frac{SN(d_1)}{S + P} = w$$

where S is the current stock price, P is the price of the relevant put option, and $N(d_1)$ comes from the Black–Scholes model. The proportion of resources committed to the Treasury bill is $1 - w$.

A. Microsoft is currently trading at $90. The annualized risk-free interest rate on a 60-day Treasury bill is 5 percent. The current volatility of Microsoft's stock is 0.32. Audrey wishes to create an insured portfolio. Since she needs $855,000 in 60 days, she decides to establish a position in a 60-day Microsoft put option with an $85.50 strike price. Audrey calls her broker, who informs her there is no 60-day Microsoft put option with a strike price of $85.50. Thus, Audrey must construct a dynamic hedge to protect the value of her investment in Microsoft stock. Using the Black–Scholes model, calculate the value of a put option with an $85.50 strike price with 60 days to expiration. Determine the allocation of assets in Audrey's insured portfolio. That is, find the proportion of resources committed to Microsoft stock, w, and the proportion of resources committed to Treasury bills, $1 - w$. Determine the dollar amount of her resources committed to Microsoft stock, and the dollar amount of her resources committed to Treasury bills.

$$d_1 = \frac{\ln(90/85.5) + ([0.05 + .5(0.32^2)](0.1644))}{0.32 \sqrt{(0.1644)}} = 0.5236$$

$$d_2 = 0.5236 - 0.32 \sqrt{(0.1644)} = 0.3938$$

$$N(d_1) = 0.699711 \quad N(d_2) = 0.653146$$

$$c = 90 \times 0.699711 - 85.5e^{-0.05 \times 0.1644} \times 0.653146 = \$7.59$$

$$p = c - S + Xe^{-r(T-t)} = \$7.59 - \$90 + \$85.5e^{-0.05 \times 0.1644} = \$2.39$$

The call price is $7.59, and the put price is $2.39. The proportion of resources committed to the stock is

$$w = \frac{SN(d_1)}{S + P} = \frac{\$90 \times 0.699711}{\$90 + \$2.39} = 0.6816$$

The proportion of resources committed to Treasury bills is

$$1 - w = 1 - 0.6816 = 0.3184$$

Audrey holds 10,000 shares of Microsoft with a market value of $90 per share, giving her a portfolio with a cash value of $900,000. Audrey will commit 68.16 percent of her resources to Microsoft stock and 31.84 percent of her resources to Treasury bills. Audrey holds $613,468 worth of Microsoft stock, $900,000 × 0.6816, and $286,532 worth of Treasury bills, $900,000 × 0.3184.

B. Twenty days later Microsoft is trading at $92. The annualized risk-free interest rate on a 40-day Treasury bill is 5 percent, and the volatility of Microsoft's stock is 0.32. Using the Black–Scholes model, calculate the value of a put option with an $85.50 strike price with 40 days to expiration. Determine the allocation of assets in Audrey's insured portfolio. That is, find the proportion of resources committed to Microsoft stock, w, and the proportion of resources committed to Treasury bills, $1 - w$.

$$d_1 = \frac{\ln(92/85.5) + ([0.05 + 0.5(0.32^2)](0.1096))}{0.32\sqrt{(0.1096)}} = 0.7964$$

$$d_2 = 0.7964 - 0.32\sqrt{(0.1096)} = -0.6904$$

$$N(d_1) = 0.787092 \quad N(d_2) = 0.755041$$

$$c = 92 \times 0.787092 - 85.5e^{-0.05 \times 0.1096} \times 0.755041 = \$8.21$$

$$p = c - S + Xe^{-r(T-t)} = \$8.21 - \$92 + \$85.5e^{-0.05 \times 0.1096} = \$1.24$$

The call price is $8.21, and the put price is $1.24. The proportion of resources committed to the stock is

$$w = \frac{SN(d_1)}{S + P} = \frac{\$92 \times 0.787092}{\$92 + \$1.24} = 0.7766$$

The proportion of resources committed to Treasury bills is

$$1 - w = 1 - 0.7766 = 0.2234$$

C. Twenty days later Microsoft is trading at $86. The annualized risk-free interest rate on a 20-day Treasury bill is 5 percent, and the volatility of Microsoft's stock is 0.32. Using the Black–Scholes model, calculate the value of a put option with an $85.50 strike price with 20 days to expiration. Determine the allocation of assets in Audrey's insured portfolio. That is, find the proportion of resources committed to Microsoft stock, w, and the proportion of resources committed to Treasury bills, $1 - w$.

$$d_1 = \frac{\ln(86/85.5) + ([0.05 + 0.5(0.32^2)](0.0548))}{0.32\sqrt{(0.0548)}} = 0.1519$$

$$d_2 = 0.1519 - 0.32\sqrt{(0.0548)} = 0.0770$$

$$N(d_1) = 0.560356 \quad N(d_2) = 0.530674$$

$$c = 86 \times 0.560356 - 85.5e^{-0.05 \times 0.0548} \times 0.530674 = \$2.94$$

$$p = c - S + Xe^{-r(T-t)} = \$2.94 - \$86 + \$85.5e^{-0.05 \times 0.0548} = \$2.21$$

The call price is $2.94, and the put price is $2.21. The proportion of resources committed to the stock is

$$w = \frac{S\,N(d_1)}{S + P} = \frac{\$86 \times 0.560356}{\$86 + \$2.21} = 0.5463$$

The proportion of resources committed to Treasury bills is

$$1 - w = 1 - 0.5463 = 0.4537$$

D. Discuss the adjustments Audrey has made in the allocation of resources between Microsoft stock and Treasury bills as Microsoft's stock price has changed.

In dynamic hedging, the investor's commitment to Microsoft stock increases as Microsoft's stock price increases and decreases as Microsoft's stock price decreases. With 60 days to the end of the hedge period, Audrey holds 68.16 percent of her resources in Microsoft stock. With 40 days to the end of the hedge period and Microsoft trading at $92, Audrey holds 77.66 percent of her resources in Microsoft stock. At 20 days to the end of the hedge period and Microsoft trading at $86, Audrey holds 54.63 percent of her resources in Microsoft stock. The trading strategy associated with dynamic hedging requires the investor to purchase additional Microsoft shares as the market price of the stock goes up and to sell additional Microsoft shares as the market price of the stock goes down. This buying of stock at successively higher prices and selling at successively lower prices is a direct cost of the insurance created.