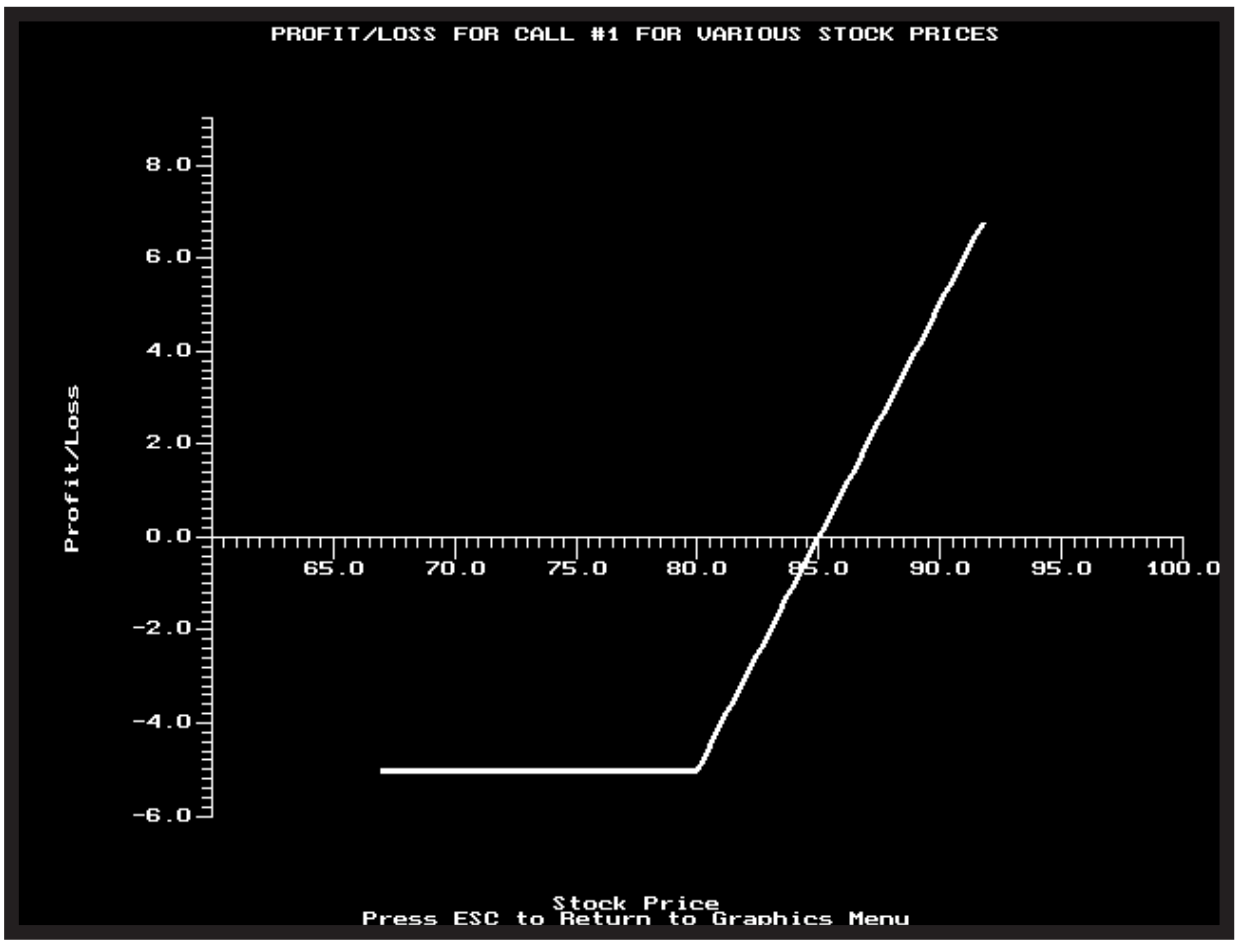
11 Option Payoffs and Option Strategies

Answers to Questions and Problems

1. Consider a call option with an exercise price of $80 and a cost of $5. Graph the profits and losses at expiration for various stock prices.



2. Consider a put option with an exercise price of \$80 and a cost of \$4. Graph the profits and losses at expiration for various stock prices.

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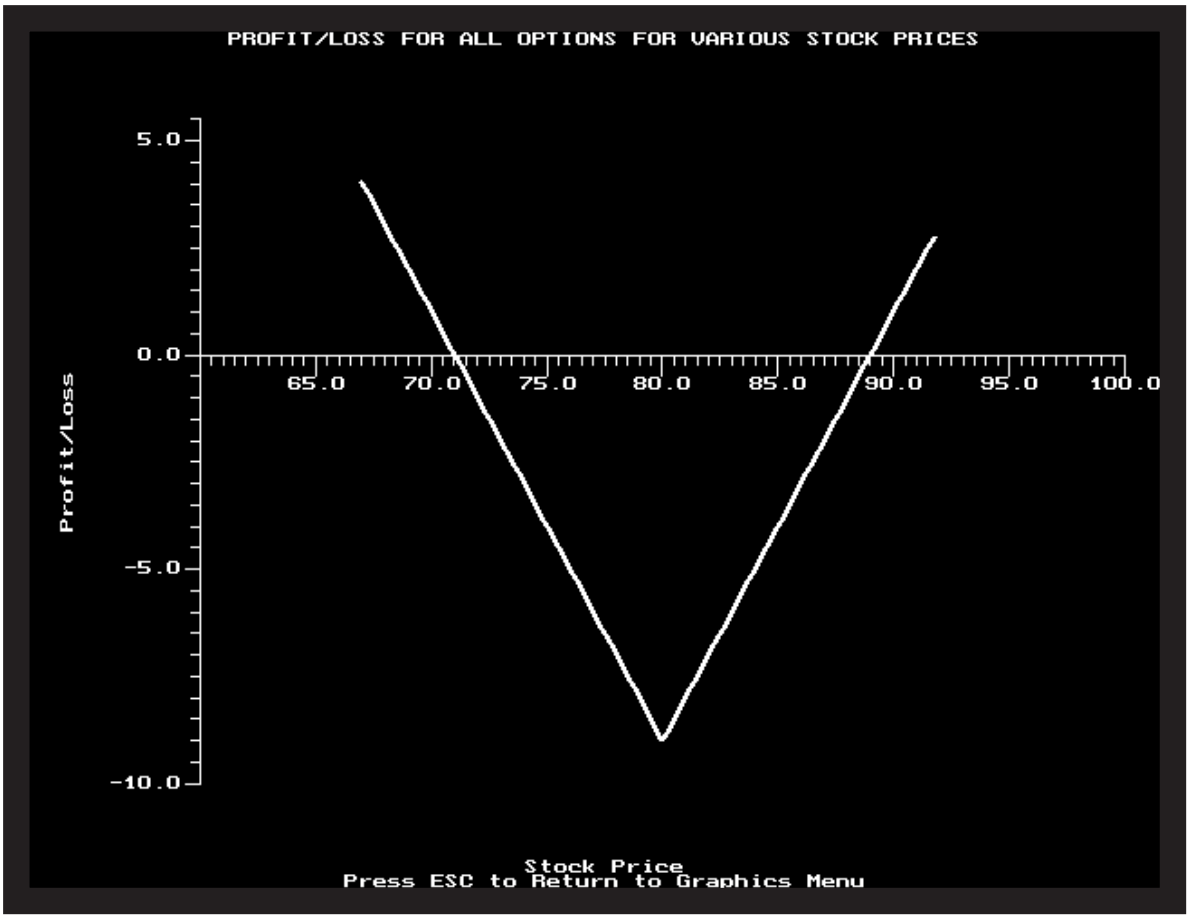
3. For the call and put in questions 1 and 2, graph the profits and losses at expiration for a straddle comprising these two options. If the stock price is $80 at expiration, what will be the profit or loss? At what stock price (or prices) will the straddle have a zero profit?



With a stock price at $80 at expiration, neither the call nor the put can be exercised. Both expire worthless, giving a total loss of $9. The straddle breaks even (has a zero profit) if the stock price is either $71 or $89.

4. A call option has an exercise price of $70 and is at expiration. The option costs $4, and the underlying stock trades for $75. Assuming a perfect market, how would you respond if the call is an American option? State exactly how you might transact. How does your answer differ if the option is European?

With these prices, an arbitrage opportunity exists because the call price does not equal the maximum of zero or the stock price minus the exercise price. To exploit this mispricing, a trader should buy the call and exercise it for a total out-of-pocket cost of $74. At the same time, the trader should sell the stock and deliver the stock just acquired through exercise for a $75 cash inflow. This produces a riskless profit without investment of $1. Because the option is at expiration, both the American and European options have the same right to exercise. Therefore, the American or European character of the option has no effect on the trading strategy.

5. A stock trades for $120. A put on this stock has an exercise price of $140 and is about to expire. The put trades for $22. How would you respond to this set of prices? Explain.

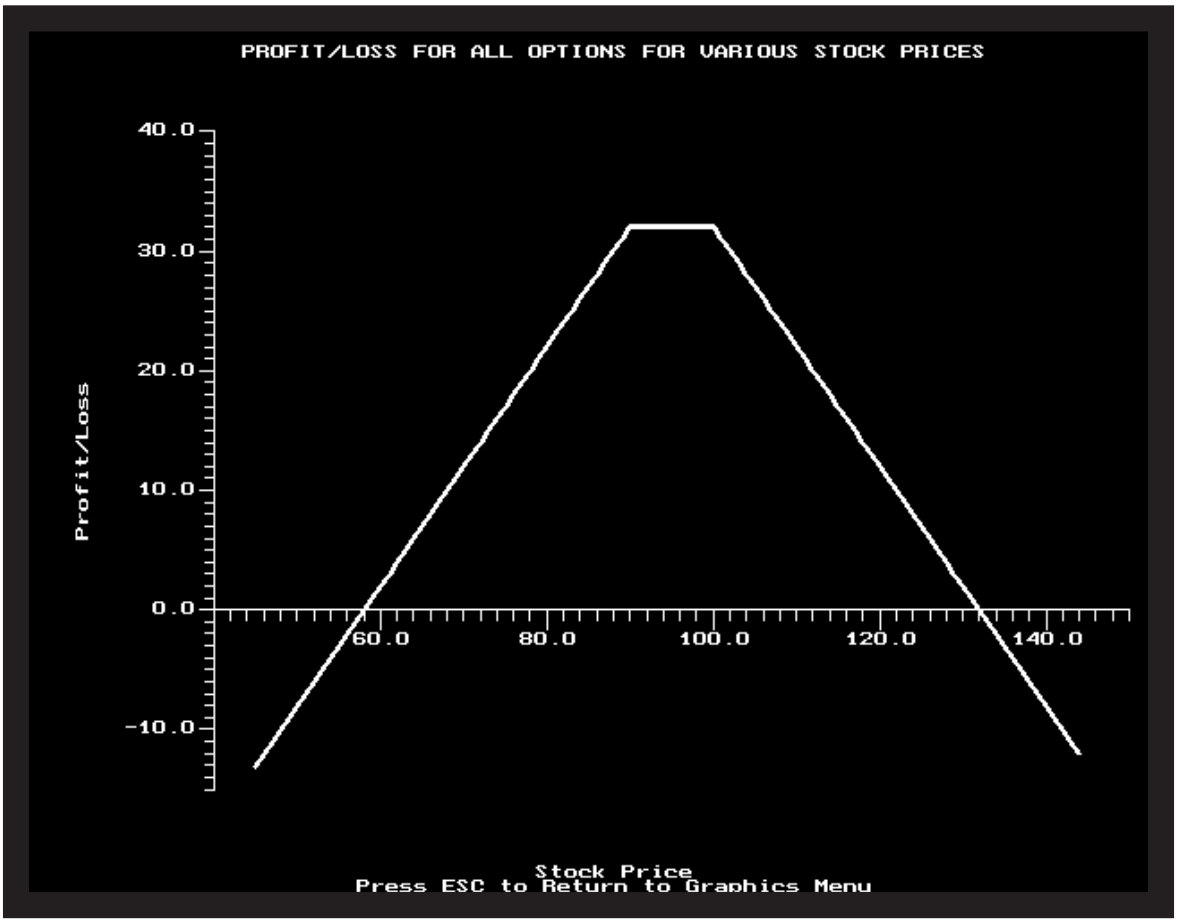
At expiration, the put price must equal the maximum of zero or the exercise price minus the stock price to avoid arbitrage. Therefore, the put price should be $20 in this situation, but it trades for $22.

This difference gives rise to an arbitrage opportunity, because the put is priced too high relative to its theoretical value. To exploit this, the trader should simply sell the put and receive $22. Now the option can be exercised against the trader or not. If it is not exercised, the put expires worthless, the obligation is complete, and the trader retains the $22 as total profit. However, the purchaser of the option may choose to exercise immediately. In this case, the seller of the put must buy the stock for the exercise price of $140. The trader then sells the stock for $120 in the market, giving a $20 loss on the exercise. But the put seller already received $22, so he or she still has a $2 profit. In summary, selling the put leads to a $22 profit if the put buyer foolishly fails to exercise. Alternatively, if the put buyer exercises, the put seller still has a $2 profit. This is an arbitrage opportunity, because selling the overpriced put gives a profit without investment in all circumstances.

6. If the stock trades for $120 and the expiring put with an exercise price of $140 trades for $18, how would you trade?

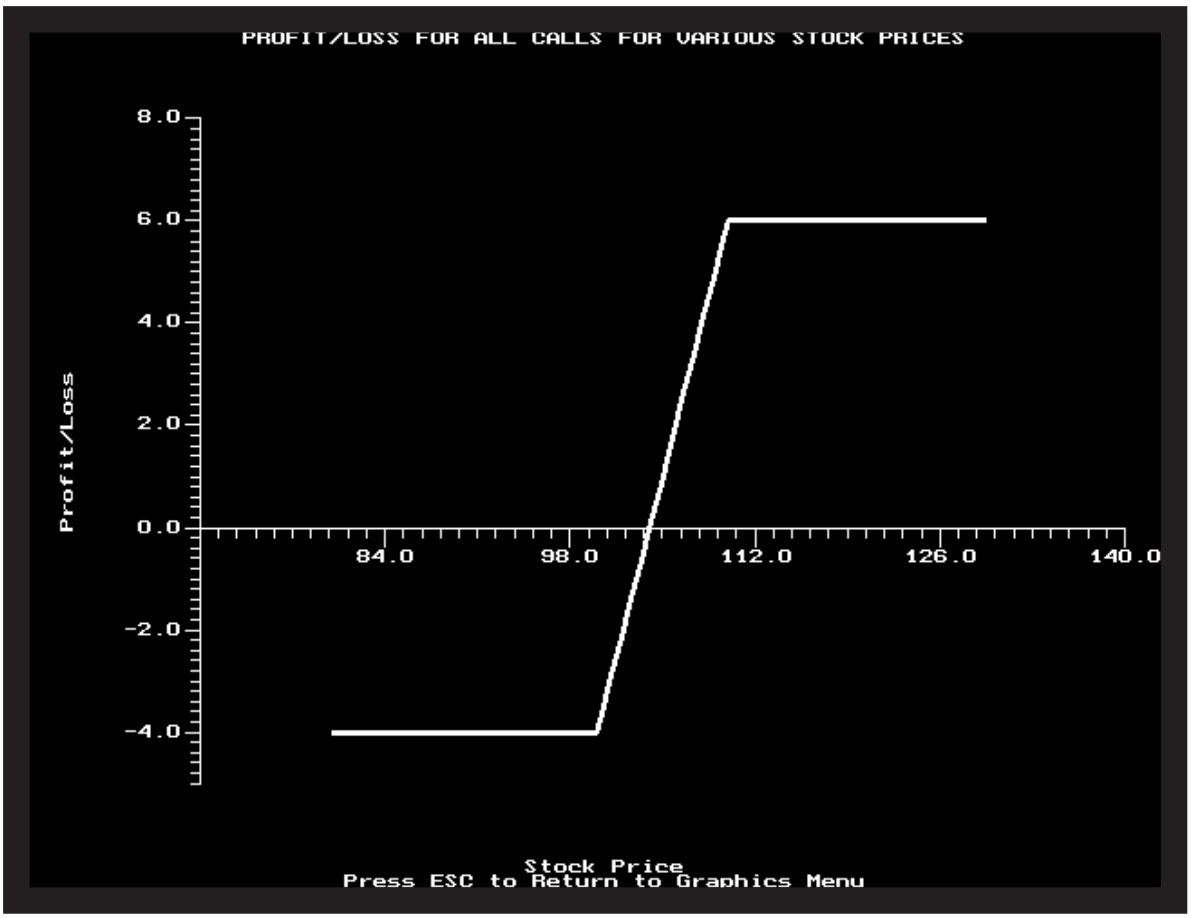
   As in the previous problem, these prices violate the no-arbitrage condition. Now, however, the put is underpriced relative to the other values. To conduct the arbitrage, the trader should buy the stock and buy and exercise the put. In this sequence of transactions, the trader pays $120 to acquire the stock, pays $18 to acquire the put, and receives $140 upon exercise of the put. These transactions yield a profit of $2 with no risk and no investment.

7. Consider a call and a put on the same underlying stock. The call has an exercise price of $100 and costs $20. The put has an exercise price of $90 and costs $12. Graph a short position in a strangle based on these two options. What is the worst outcome from selling the strangle? At what stock price or prices does the strangle have a zero profit?



The worst outcomes occur when the stock price is very low or very high. First, the strangle loses $1 for each dollar the stock price falls below $58. With a zero stock price, the strangle loses $58. If the stock price is too high, the strangle also loses money. Because the stock could theoretically go to infinity, the potential loss on the strangle is unbounded. For stock prices of $58 or $132, the strangle gives exactly a zero profit.

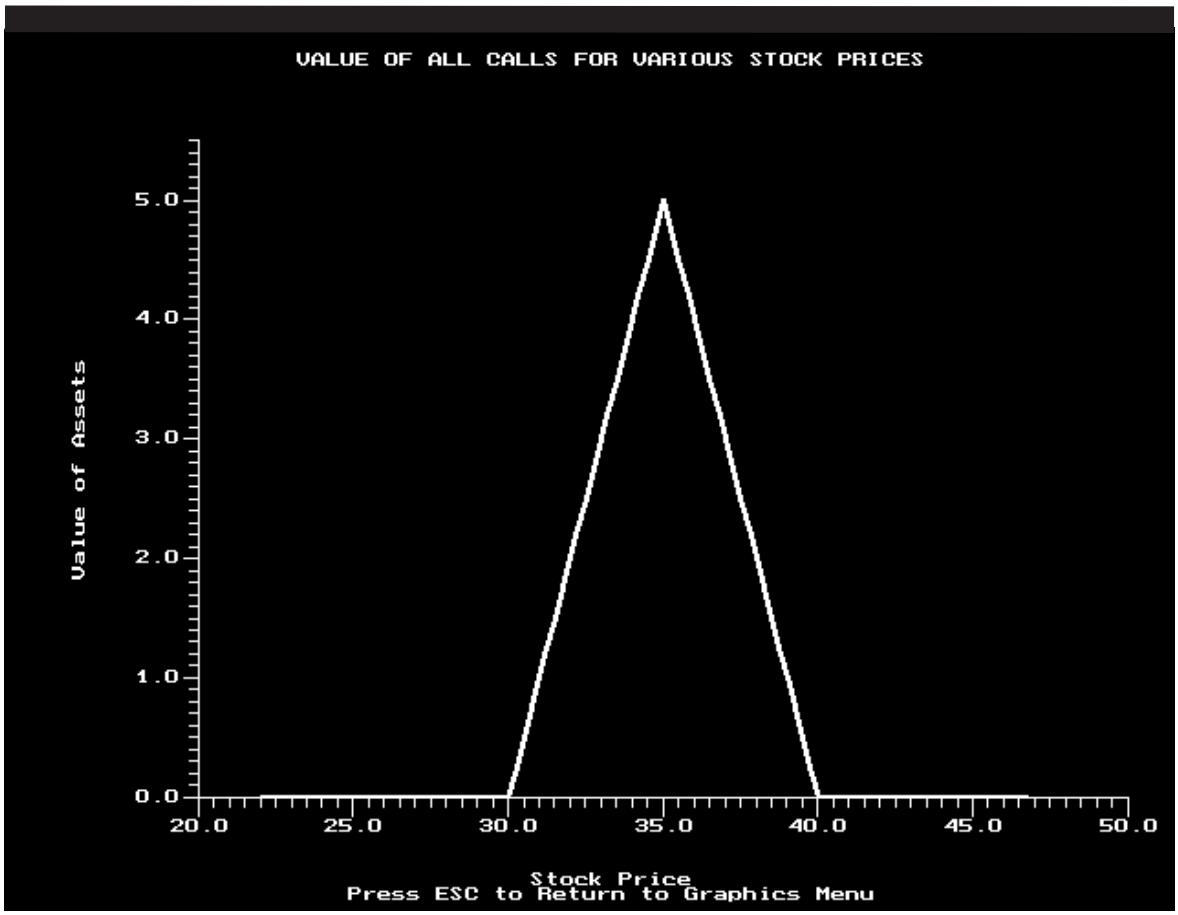
8. Assume that you buy a call with an exercise price of $100 and a cost of $9. At the same time, you sell a call with an exercise price of $110 and a cost of $5. The two calls have the same underlying stock and the same expiration. What is this position called? Graph the profits and losses at expiration from this position. At what stock price or prices will the position show a zero profit? What is the worst loss that the position can incur? For what range of stock prices does this worst outcome occur? What is the best outcome and for what range of stock prices does it occur?



This position is a bull spread with calls, because it is designed to profit if the stock price rises. The entire position has a zero profit if the stock price is $104. At this point, the call with the $100 exercise price can be exercised for a $4 exercise profit. This $4 exercise value exactly offsets the price of the spread. The worst loss occurs when the stock price is $100 or below, because the option with the $100 exercise price cannot be exercised, and the entire position is worthless. This gives a $4 loss. The best outcome occurs for any stock price of $110 or above and the total profit is $6.

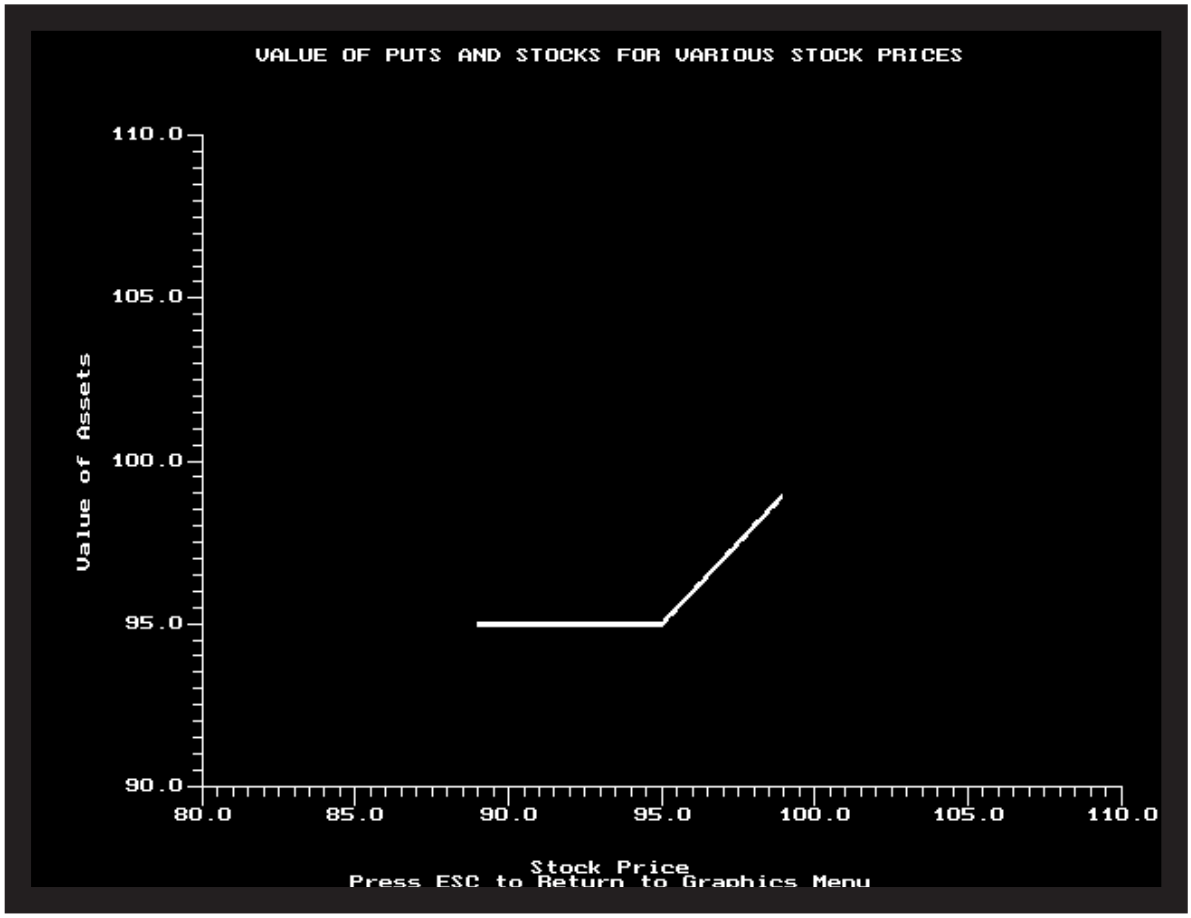
9. Consider three call options with the same underlying stock and the same expiration. Assume that you take a long position in a call with an exercise price of $40 and a long position in a call with an exercise price of $30. At the same time, you sell two calls with an exercise price of $35. What position have you created? Graph

the value of this position at expiration. What is the value of this position at expiration if the stock price is $90? What is the position's value for a stock price of $15? What is the lowest value the position can have at expiration? For what range of stock prices does this worst value occur?



This is a long position in a butterfly spread. If the stock price is $90, the value of the spread is zero. For a $15 stock price, the spread is worth zero. The entire spread can be worth zero at expiration. This zero value occurs for any stock price of $30 or below and $40 or above.

10. Assume that you buy a portfolio of stocks with a portfolio price of $100. A put option on this portfolio has a striking price of $95 and costs $3. Graph the combined portfolio of the stock plus a long position in the put. What is the worst outcome that can occur at expiration? For what range of portfolio prices will this worst outcome occur? What is this position called?



The worst result is a portfolio value of $95. The purchase of the put for $3 gives a loss of $8. This worst outcome occurs for a terminal stock portfolio value of $95 or less. This combined position is an insured portfolio. The position insures against any terminal portfolio value less than $95 or any loss greater than $8.

11. Consider a stock that sells for \$95. A call on this stock has an exercise price of \$95 and costs \$5. A put on this stock also has an exercise price of \$95 and costs \$4. The call and the put have the same expiration. Graph the profit and losses at expiration from holding the long call and short put. How do these profits and losses compare with the value of the stock at expiration? If the stock price is \$80 at expiration, what is the portfolio of options worth? If the stock price is \$105, what is the portfolio of options worth? Explain why the stock and option portfolio differ as they do.

No matter what stock price results, the option portfolio will have \$1 less profit than the stock itself. For example, the option portfolio costs \$1, but both options are worthless at a stock price of \$95. Therefore, at a stock price of \$95, the stock has a zero profit, and the option portfolio has a \$1 loss. Further, the option portfolio will be worth exactly \$95 less than the stock at every price. With a stock price of \$80, the call is worthless and the put will be exercised against the option holder for an exercise loss of \$15. Therefore, the option portfolio is worth −\$15 for an \$80 stock price. If the stock trades for \$105, the option portfolio will be worth \$10.

12. Assume a stock trades for \$120. A call on this stock has a striking price of \$120 and costs \$11. A put also has a striking price of \$120 and costs \$8. A risk-free bond promises to pay \$120 at the expiration of the options in one year. What should the price of this bond be? Explain.

A portfolio consisting of one long call, one short put, and a riskless investment equal to the common exercise price of the two options gives exactly the same payoffs as a share of the underlying stock on the common expiration date. This put-call parity relationship requires that this portfolio of long call, short put, plus riskless investment should have the same price as the stock. With our data, the riskless bond must therefore cost \$120 − \$11 + \$8 = \$117. The riskless interest rate must be 2.53 percent.

13. In the preceding question, if we combine the two options and the bond, what will the value of this portfolio be relative to the stock price at expiration? Explain. What principle does this illustrate?

    As the previous answer already indicated, the described portfolio (long call, short put, plus long bond) must have the same value as the stock itself. This illustrates the put–call parity relationship.

14. Consider a stock that is worth \$50. A put and call on this stock have an exercise price of \$50 and expire in one year. The call costs \$5 and the put costs \$4. A risk-free bond will pay \$50 in one year and costs \$45. How will you respond to these prices? State your transactions exactly. What principle do these prices violate?

    These prices violate put-call parity. The long call, plus short put, plus riskless investment of the present value of the exercise price must together equal the stock price:

$$S = C - P + Xe^{-r(T-t)}$$

    Instead, we have \$50 ⩾ \$5 − \$4 + \$45 = \$46. Therefore, the stock is overpriced relative to the duplicating right-hand side portfolio. Accordingly, we transact as follows, with the cash flows being indicated in parentheses: Sell stock (+\$50), buy call (−\$5), sell put (\$4), and buy the riskless bond (−\$45). This gives a positive cash flow of \$4 at the time of trading. To close our position, we collect \$50 on the maturing bond. If the stock price is above \$50, we exercise our call and use our \$50 bond proceeds to acquire the stock, which we can then repay to close our short position. The put cannot be exercised against us, so we conclude the transaction with our original \$4 profit. If the stock price is below \$50, the put will be exercised against us. If so, we lose \$50 − S on the exercise, paying our \$50 bond proceeds to acquire the stock. Now with the stock in hand, we close our short position and the call expires worthless. As a result, we still have our \$4 original cash inflow as profit. No matter what the stock price may be at expiration, our profit will be \$4.

15. A stock sells for \$80 and the risk-free rate of interest is 11 percent. A call and a put on this stock expire in one year and both options have an exercise price of \$75. How would you trade to create a synthetic call option? If the put sells for \$2, how much is the call option worth? (Assume annual compounding.)

    A synthetic call option consists of the following portfolio: long the stock, long the put, and short a risk-free bond paying the exercise price at the common maturity date of the call and put. Therefore, the following relationship must hold:

$$C = S + P - \frac{X}{(1+r)^t}$$

    Therefore, with the information given:

$$C = \$80 + \$2 - \$67.67 = \$14.43$$

16. A stock costs \$100 and a risk-free bond paying \$110 in one year costs \$100 as well. What can you say about the cost of a put and a call on this stock that both expire in one year and that both have an exercise price of \$110? Explain.

    Put–call parity implies:

$$S - \frac{X}{(1+r)^{T-t}} = C - P$$

    In this case, the stock and bond have the same price, so the left-hand side of the equation equals zero. For the right-hand side to equal zero, the call and put must have the same price as well. However, from the information given, we cannot determine what that price would be.

17. Assume that you buy a strangle with exercise prices on the constituent options of \$75 and \$80. You also sell a strangle with exercise prices of \$70 and \$85. Describe the payoffs on the position you have created. Does

this portfolio of options have a payoff pattern similar to that of any of the combinations explored in this chapter?

This position will profit for very low (say below \$70) or very high (say above \$85) stock prices at expiration. For prices in the range of \$75 to \$85, the position will lose. The exact breakeven points cannot be determined from the information given, however. In effect, the pair of strangles described is a short condor position.

18. If a stock sells for \$75 and a call and put together cost \$9 and the two options expire in one year and have an exercise price of \$70, what is the current rate of interest?

From the information given, and applying put–call parity, we know:

$$S - \frac{X}{1+r} = C - P = \$9$$

The stock price is \$75 and must exceed the bond price by \$9, so the price of the bond is \$66. Thus, the present value of the \$70 exercise price is \$66, implying an interest rate of 6.06 percent.

19. Assume you buy a bull spread with puts that have exercise prices of \$40 and \$45. You also buy a bear spread with puts that have exercise prices of \$45 and \$50. What will this total position be worth if the stock price at expiration is \$53? Does this position have any special name? Explain.

The bull spread with puts and these exercise prices implies buying the put with $X = \$40$ and selling the put with $X = \$45$. To buy the bear spread implies buying a put with $X = \$50$ and selling a put with $X = \$45$. If the stock price at expiration is \$53, all of the puts expire worthless. In making this trade, one has bought puts with $X = \$40$ and $X = \$50$, and sold two puts with $X = \$45$. This is equivalent to a long butterfly spread with puts.

20. Explain the difference between a box spread and a synthetic risk-free bond.

The box spread gives a riskless payoff, so it is equivalent to a synthetic risk-free bond.

21. Within the context of the put-call parity relationship, consider the value of a call and a put option. What will the value of the put option be if the exercise price is zero? What will the value of the call option be in the same circumstance? What can you say about potential bounds on the value of the call and put option?

If the exercise price of a put is zero, there can be no payoff from the put so the price must be zero. In this circumstance, the call must be worth the stock price. With respect to bounds on the option prices, the call price can never exceed the stock price and the put price can never exceed the exercise price.

22. Using the put-call parity relationship, write the value of a call option as a function of the stock price, the risk-free bond, and the put option. Now consider a stock price that is dramatically in excess of the exercise price. What happens to the value of the put as the stock price becomes extremely large relative to the exercise price? What happens to the value of the call option?

$$C = S + P - \frac{X}{(1+r)^{T-t}}$$

The put price declines as the ratio S/X becomes large. The call option must increase in value. As we will see, the call must always be worth at least the stock price minus the present value of the exercise price.

23. The CBOE is thinking of opening a new market center in London that will trade only European options and has hired you as a consultant. The board of directors for the CBOE believes that there is demand in the market for options contracts that can be used by investors to insure their portfolios. A unique feature of this market is the fact that only put options will be traded.

A. Explain why it will not be necessary to trade call options in this market.

Because the options that will be traded in this market are European options. Arbitrage, in the form of put-call parity, guarantees that the call options will be priced fairly. Any investor can use put-call parity to construct the corresponding synthetic call option $c = p + S - Xe^{-r(T-t)}$ that will have the same value as an exchange traded call option.

B. Discuss the expected clientele in this market and explain why these traders might not desire to use American options traded in Chicago on the CBOE.

The target clientele for this market is hedgers. Hedgers desire to protect the value of an asset that they currently own for a specified time period. That is, the hedger has an investment horizon and desires to protect the value of her investment over that investment horizon. The price of the option contract traded in Chicago on the CBOE includes the value of the early exercise premium. Hedgers do not expect to exercise their options prior to expiration, and they are not interested in paying for the right to exercise the option prior to expiration. The value of this early exercise premium on an American option effectively increases the cost of the insurance available to hedgers.

C. The CBOE has asked you as a consultant to identify the important characteristics of the market and the option contracts that will ensure the success of this new market. Identify those characteristics that you feel will contribute to the success of the market.

For this marketplace to be successful, the contracts and services offered must be demanded by the market. The target clientele for these contracts is hedgers. Hedgers have an investment horizon and desire to protect the value of their investment over that investment horizon. To do an effective job, you must understand the demographics of the marketplace. The following questions might be relevant in determining the nature of the marketplace. What types of assets are owned by the investors who are likely candidates to be the clients of this market? Who are the likely users of the market, individuals or professional funds managers? Should the option contracts be written on individual shares of stock or should they be written on stock indices (portfolios)? Should the market include equities traded globally or should they focus on a specific geographic location (e.g., the United States)? Should the contracts be settled in cash or call for delivery of the underlying security? Should the market be open continuously? How frequently should the options expire—weekly, bimonthly, monthly, and so on? Should the options have expiration dates longer than one year? What is the optimal size of the contract (e.g., 100 shares)? What is the optimal differential between the strike prices of the traded options? Should there be any restriction on the size of the options position held by an investor? What is the optimal structure of transactions costs for the market?

24. Suppose an investor purchases a call option on XYZ with a \$50 strike price and sells a put option on XYZ with the same strike price. Both options are European options that expire in one month. Describe the investor's position.

The investor has created a synthetic long position in a forward contract written on the underlying stock. The payoff on this position mimics the payoff on a long position in a share of the underlying stock ($S - X$, where X is the purchase price of the stock, and S is the spot price of the stock).

25. DRP is currently selling at \$58 per share. An American call option written on DRP with six weeks until expiration has a strike price of \$50.

A. If DRP does not pay a dividend, explain why it is not economically rational to exercise this American call option prior to expiration. Equivalently, explain why a call option is worth more alive than dead.

The price of an option consists of its intrinsic value ($S - X$), and its time value (the option's price – the intrinsic value of the option). When you exercise the call option, you capture its intrinsic value ($S - X$). (The option must be in-the-money for you to consider exercising it.) However, you lose the time value when you exercise. You can capture the time value by selling the option in the marketplace. For example, consider

the following information: $S = \$50$, $X = \$40$, and $C = \$11.5$. If the call were exercised, the payoff to the investor would be $(\$50 - \$40) \times 100 = \$1{,}000$. If the call were sold, the payoff to the investor would be $\$11.50 \times 100 = \$1{,}150$.

B. If DRP pays a quarterly dividend, but the dividend will not be paid for eight weeks, explain why it is not economically rational to exercise this American call option prior to expiration.

In this problem, the dividend is irrelevant to the early exercise decision. The dividend is to be paid after the option's expiration and will not affect your decision regarding the early exercise of the option.

26. Call prices are directly related to the volatility of the underlying stock. That is, the more volatile the underlying stock, the more valuable the call option. However, higher volatility means that the stock price may decrease by a large amount. That is, the probability of a large decrease in the stock price has increased. Explain this apparent paradox.

Examination of the components of the price of an option reveals the answer to this apparent paradox. The price of an option consists of its intrinsic value, and its time value. A call option pays off when the market price of the stock exceeds the exercise price (contracted purchase price). That is, the call option pays off in only one direction, when the stock price rises (the up state). An option that is out-of-the-money at expiration is worthless. In other words, it does not matter how far out-of-the-money the option is at expiration. A call option that is deep-out-of-the-money is just as worthless as a call option that is barely out-of-the-money. Increases in the volatility of the stock underlying the option contract increase the expected payoff on the option in the up state, which increases the value of the option. Thus, although increases in the volatility of the underlying stock imply that the stock is more likely to experience a large decrease in price, it is the corresponding increase in the probability of a large increase in the stock price that increases the value of a call option.

27. An investor has just obtained the following quotes for European options on a stock worth \$30 when the three-month risk-free interest rate is 10 percent per annum. Both options have a strike price of \$30 and expire in three months.

European Call:	\$3
European put:	$\$1\frac{3}{4}$

A. Given the information above, determine whether the prices conform to the put–call parity rule.

Put–call parity states $p = c - S + Xe^{-r(T-t)}$; however,

$$\$1.75 \neq \$3 - \$30 + \$30e^{-0.1 \times 0.25}$$

$$p \neq c - S + Xe^{-r(T-t)}$$

$$\$1.75 \neq \$2.2593$$

B. If there is a violation, suggest a trading strategy that will generate riskless arbitrage profits.

The synthetic put costs \$2.2593, and the exchange traded put is trading at \$1.75. To capture the potential profits from this arbitrage opportunity, we must simultaneously sell the synthetic put and purchase the traded put. Selling the synthetic put requires one to sell the call for \$3, purchase the stock for \$30, and sell the present value of \$30 of the risk-free bond, \$29.2593, resulting in a cash inflow of \$2.2593. Purchasing the put will cost \$1.75.

C. Indicate how much profit you will make from the arbitrage transactions, if such an opportunity exists.

The profit from the arbitrage transactions will be the difference between the cash inflow from selling the synthetic put and the cash outflow from purchasing the traded put, $\$2.2593 - \$1.75 = \$.5093$.

28. Suppose you are an information services professional with contracts throughout the industry. In conversations with colleagues, you get the feeling that Computer Associates International is likely to attempt to acquire Computer Sciences Corporation. You also remember from your finance courses in college that in the course of an acquisition, the share price of the target firm normally increases and the share price of the acquiring firm decreases. (You are confident that trading on the information gleaned in these conversations violates no law and is ethical.)

A. Discuss the advantages of using options to speculate on the expected stock price changes of the firms involved in an acquisition.

There are several factors favoring the use of options to speculate on expected changes in the prices of stocks involved in an acquisition. It is possible to replicate the payoffs to a particular stock position with the appropriate positions in option contracts. That is, one can replicate the payoffs to a long stock position by holding a call option and selling a put with the same strike price and expiration date. Additionally, you can replicate this payoff with options at a lower cost than purchasing the stock itself. However, options pay off in only one direction, calls in the up state, puts in the down state. Thus, if one expected the price of a particular stock to increase, one could hold a long call position and participate in the upside gains in the stock with a limited investment. A long call position offers the same action as a long stock position at a lower cost. Chapter 13 shows that a long call option is equivalent to a levered stock position. Since call options are cheaper than the stock underlying the option, an investor with a fixed amount to invest can increase his leverage by purchasing several call options. Consider an investor with $10,000 available to speculate in a $100 stock that has a call trading at $10. The investor could purchase 100 shares of stock, $100 \times \$100 = \$10{,}000$, or he could purchase 10 call options [(10 contracts $\times$ 100 shares per contract) $\times$ \$10 = \$10,000], significantly increasing his exposure to the market. The fact that an investor faces lower transactions costs in the options market is an important factor motivating investors to trade options. Additionally, there may be tax implications of trading options that induce particular investors to trade options rather than trading the underlying stock. The tax factor will be unique to the investor and will depend on the individual's tax obligations. Because it is possible to create synthetic securities with options, it may be possible to avoid stock market restrictions by trading options. In particular, there are specific regulations regarding the short selling of stock that may be avoided by creating a synthetic short position using the Treasury bill and traded options.

B. Based on your understanding of option payouts, discuss and explain the option positions that you would establish to speculate on these expected price changes. (Assume that traded options exist for both firms.)

Our expectations are the stock price of CSC will increase, and the stock price of CAI will decrease because of the expected acquisition. There are many strategies the investor could undertake to take advantage of the expected movements in the stock prices of CSC and CAI. One basic strategy would be to purchase call options written on CSC, and to purchase put options written on CAI. One could reduce the cost of this investment by writing puts on CSC, and writing calls on CAI. In addition, the investor must choose expiration dates and strike prices for the options. The decision regarding the maturity of the option contract will be influenced by expectations regarding the timing of the acquisition.

29. After watching a late night infomercial, a colleague comes to work professing the gospel of income enhancement via covered calls. The pitch man in the infomercial, Mr. Oracle, says that writing covered calls enhances the return on a stock investment with no cost. Discuss the sources of the apparent costless gains and the risks associated with writing covered calls.

When writing an out-of-the-money covered call, you contract to sell the stock you currently own at a price that is higher than the current stock price. As compensation for writing this contract, you receive premium income. The investor owns the underlying stock and is entitled to receive any distributions associated with owning the stock. These may include dividends, or additional shares of stock associated with a stock split or dividend. If the option is exercised against the investor, the investor sells the stock underlying the contract at the strike price. If the strike price is higher than the purchase price of the stock, then the investor will have earned a capital gain on the transaction. Thus, there are three potential sources of gains to the

investor: premium income received from writing the call options, dividends, and capital gains from the sale of the stock. These potential returns do not, however, come without corresponding risk. By writing covered calls, the investor is exposed to an opportunity cost. If the option goes into-the-money, the potential gains associated with owning the stock will be offset by the obligations associated with the call option. Writing the covered call has reduced the upside potential of the long stock position. By writing the call option, the investor has given the owner of the call the right to purchase the underlying stock according to the terms of the option contract. It is the owner of the call who determines if and when an option is exercised. If the investor is holding the stock as a long-term investment, having a call exercised against him forces the investor to sell an asset that he wished to hold in his portfolio. If the investor desires to continue to hold the asset in his portfolio, he must return to the market to purchase the stock at the market price bearing all the transactions costs associated with reestablishing an existing position in the stock. Besides limiting the upside potential associated with owning a stock, a covered call position provides no protection against stock price decreases.

30. Late one Friday afternoon in March, an investor receives a call from her broker. Her broker tells her that the March options on Microsoft will expire in ten minutes, that Microsoft is currently trading at $94.50, and that the March 85 call is selling at $8. The investor tells her broker that he is mistaken and must have read his screen incorrectly. Explain which individual, the broker or the investor, is correct. Supposing that the broker is in fact correct, state the transactions that the investor would undertake to take advantage of this situation. Suppose the broker tells the investor that these options are European options. Discuss the impact of the fact that the options are European options on the actions of the investor.

    Microsoft options are traded on the CBOE, and the stock options traded on the CBOE are American options. This means that the March 85 MSFT call option must trade at a price that is at least the intrinsic value of the option prior to the option's expiration. The intrinsic value of the call is $9.50 and the option is selling for $8.00, according to the broker. This mispricing represents a classic arbitrage opportunity. To capture the arbitrage profit of $1.50, the investor will simultaneously sell the appropriate number of Microsoft shares and purchase the appropriate amount of the March 85 MSFT call option. The investor will receive $94.50 from the sale of the stock and will pay $8.00 for the call (all on a per share basis). The investor will then exercise the call option paying $85 per share for MSFT. Exercising the option provides the investor with the shares necessary to cover her obligation from selling the stock (short position). The total cash outflows are $93 ($85 + $8), and the cash inflows are $94.5, resulting in $1.50 in riskless profit for the investor. While an investor cannot exercise a European call until the call option expires, the call options in question will expire in 10 minutes. The call should be trading at a price that is above the intrinsic value of the option, $9.50, at this point in the life of the European call option. That is, in ten minutes the call will be in-the-money and be worth $S - X$. The investor will undertake the same arbitrage transactions to profit from this mispricing.

31. As a finance major in college, you were taught the efficient market hypothesis. Because you believe that it is not possible for a mutual fund manager to consistently outperform the market, you hold a portfolio of the 30 stocks that make up the Dow Jones Industrial Average (DJIA). Your broker has just called you with the following offer. He can provide you with insurance that will guarantee that the value of your portfolio will not fall below an indexed level of 8,900. This insurance is evaluated at the end of each quarter and costs $500 per quarter. A quick search of the CBOE web site shows that the Dow is currently trading at an indexed level of 9,005, and that a three-month put option on the DJIA with a strike price of 89 is trading at $2\frac{1}{16}$. The DJIA options traded on the CBOE are quoted at strike prices that are $\frac{1}{100}$ of the level of the DJIA. Each premium point is multiplied by $100 to determine the total cost of the option.

A. Should you accept the insurance contract offered by your broker? Explain.

    It is possible to use the put option on the DJIA traded on the CBOE to create an insured portfolio. The cost of the traded put is less than the cost of the insurance offered by the broker. Therefore, a rational investor will use the option market to insure the value of his portfolio.

B. Explain how you can provide your own insurance for your portfolio. What is the cost of this insurance? What is the maximum loss on your insured portfolio?

The insured portfolio is constructed by combining your long position in the Dow with a long position in the DJIA put option with a strike price of 89 and an expiration in three months (the strike price of 89 translates to a Dow level of 8,900). The cost of this traded option is \$206.25 ($2.0625 \times \$100$). If the Dow drops below 8,900 at the end of three months, then the three-month DJIA put option with a strike price of 89 will be in-the-money. The investor will exercise the option at expiration, and be paid cash equal to the difference between the strike price of 89 (DJIA level of 8,900) and the level of the market at the expiration of the option contract. This payoff will create a price floor for the portfolio at an index level of 8,900. The maximum loss on the portfolio would be 105 index points, 9,005 − 8,900.

32. You own shares of AGH that are currently trading at \$100. A European put option written on AGH with a \$100 strike price that expires in three months is priced at \$4. The equivalent call option is priced at \$5.

A. What is the price of a three-month T-bill that pays par (\$100) implied by the prices given above?

$S = \$100$, $X = \$100$, $c = \$5$, $p = \$4$, $T - t = 0.25$ years

Given these prices, the put–call parity relationship implies that the price of the synthetic T-bill is \$99.

$$c - p = S - Xe^{-r(T-t)}, \$5 - \$4 = 100 - Xe^{-r(T-t)}, Xe^{-r(T-t)} = \$99.$$

B. What is the continuously compounded three-month interest rate implied by these prices?

The continuously compounded three-month interest rate implied by these prices is .0402.

$$\begin{aligned} \$99 &= \$100e^{-r(.25)} \\ \ln(99/100) &= \ln(e^{-r(.25)}) \\ -0.01005 &= -r(0.25) \\ r &= 0.0402 \end{aligned}$$

C. A quick check of the price of three-month T-bills on *The Wall Street Journal* web site reveals that this bill is trading at \$98.50. Explain your actions upon finding this information.

The synthetic bill is overpriced according to the prices given above. Thus, there is an arbitrage opportunity for the investor. To capture the \$.50 profit implied by these prices, the investor should simultaneously sell the synthetic Treasury bill and purchase the traded Treasury bill. The synthetic bill position is created by constructing a portfolio that consists of a long position in the stock, a short position in the call, and a long position in the put, $S - c + p = Xe^{-r(T-t)}$. To capture the profit opportunities, the investor must sell the stock short, buy the call, and sell the put resulting in a cash inflow of \$99 (+\$100 − \$5 + \$4 = \$99). As a result of these transactions, the investor has effectively borrowed at a low interest rate, the short synthetic bond position, and lent at a higher rate, the long traded T-bill position.

33. A protective put position is created by combining a long stock position with a long position in a put option written on the stock ($S + P$). Construct the equivalent position using call options, assuming all the options are European.

An equivalent position can be created by combining a long position in a call option with an investment (long position) in the appropriate amount of a Treasury bill. The call and put option must be written on the same stock and have the same strike price and expiration date. The appropriate investment in the T-bill is determined by calculating the present value of the strike price of the put option used to create the protective put position, $Xe^{-r(T-t)}$.

34. TMS is currently trading at \$40, which you think is above its true value. Given your knowledge of the firm, its products, and its markets, you believe that \$35 per share is a more appropriate price for TMS. One means

of purchasing TMS at $35 per share would be to write a limit order. A limit order is an order directed to a broker on the floor of the exchange that specifies, among other things, the purchase price of the stock. You also notice that TMS has traded options and that a $40 put option is selling for $5. An alternative investment strategy involves purchasing TMS for $40 and writing a TMS put option for $5.

A. If at the expiration of the option, TMS's stock is trading at a price above $40 per share, discuss the benefits of the buy stock/sell put investment strategy.

When TMS is trading above $40 per share at the put's expiration, the put option that you have written is worthless. In this environment, you will have effectively purchased TMS for $35 per share, $40 to purchase TMS in the spot market less $5 received from writing the $40 put option.

B. If at the expiration of the option, TMS's stock is trading at a price below $40 per share, discuss the benefits of this alternative investment strategy.

When TMS is trading below $40 per share at expiration, the short put option position is in-the-money and will be exercised against the investor. In this environment, the impact of the lower stock price on the cost of your position will vary linearly with the stock price. For prices between $35 and $40, the $5 premium income received from writing the put option will offset losses on the option position dollar for dollar. That is, at $39, the $1 loss on the option position will be offset by the $5 premium income resulting in an effective purchase price of $36 for a share of TMS. At stock prices lower than $35 per share, the losses from the option position will offset the gains from the premium income received resulting in an effective purchase price greater than $40 per share.

35. When you purchase property insurance, you must choose the dollar amount of the deductible on the policy. Similarly, when you construct a protective put position, you must choose the strike price on the put option. Discuss the similarities between the choice of the deductible on an insurance policy and the selection of a strike price for the protective put position.

When selecting the deductible on the insurance policy, the investor is deciding the dollar amount of losses that she is willing to self-insure. That is, the choice of the deductible determines how much of a loss must be covered by the resources of the insured prior to any payment by the insurer. In general, the greater the risk that is self-insured, the lower the cost of the insurance. A protective put position is constructed to limit an investor's exposure to losses arising from decreases in the price of the stock held by the investor. The investor must choose the strike price on the put option when constructing the position. If the investor chooses a strike price that is well below the current stock price, the investor is bearing more risk from holding the stock, and the price of the insurance will be less. That is, the investor has self-insured more of the risk of a stock price decline. Therefore, the price of a deep-out-of-the-money put should be less than the price of a near-the-money put.

36. A client has recently sold a stock short for $100. The short sale agreement requires him to cover the short position in one month. The client wants to protect his position against stock price increases. You notice that the stock has puts and calls traded on the CBOE. Explain how your client can use either the put or call option to construct a portfolio that will limit his risk exposure to stock price increases. Compare the two alternative strategies.

In this situation, increases in the stock price represent a risk to the investor. If at the end of the month the stock is selling above $100, the investor will have to cover the position and will incur a loss. In this hedging problem, the investor needs to construct a portfolio such that increases in the stock price will result in an increase in the value of the hedging instrument that can offset the losses incurred on the short stock position. The first strategy would be to combine the short stock position with a long call position. Increases in the price of the stock will increase the value of the call option. The increases in the value of the call option position will offset losses in the short stock position. One decision the investor must make is the selection of the strike price of the option—that is, the selection of the amount of loss (risk) the investor is willing to bear. The price of a call option with a $110 strike price will be less than the price of a call with a $100 exercise

price. The investor would choose to hold a call that expired in one month (the same time until the short position must be covered). For example, assume the investor held a $100 call that expired in one month. If the stock price at the end of the month was $105, then the investor would exercise the call and purchase the stock at $100 to cover the short position. The cost of the protection would be the price of the call option. The other strategy would combine a short put position with the short stock position. In this case, the investor would use the premium income received from writing the put to cover any losses associated with stock price increases. As happens with a covered call position, this premium income would provide limited protection against adverse stock price movements, and would limit potential gains associated with stock price decreases. For example, assume the investor wrote a $100 put that expired in one month. If the stock price at the end of the month was $95, then the owner of the put would exercise the put against your client, forcing your client to purchase the stock at $100. This loss of $5 on the option contract will offset the $5 gain in the value of the short stock position. At prices above $100, the put will expire worthless and the premium income will offset some of the losses in the short stock position.