7 Stock Index Futures: Introduction

Answers to Questions and Problems

1. Assume that the DJIA stands at 8340.00 and the current divisor is 0.25. One of the stocks in the index is priced at $100.00 and it splits 2:1. Based on this information, answer the following questions:

a. What is the sum of the prices of all the shares in the index before the stock split?

The equation for computing the index is:

$$\text{Index} = \frac{\sum_{i=1}^{N} P_i}{\text{Divisor}}$$

If the index value is 8340.00 and the divisor is 0.25, the sum of the prices must be 8,340.00(0.25) = $2,085.00.

b. What is the value of the index after the split? Explain.

After the split, the index value is still 8,340.00. The whole purpose of the divisor technique is to keep the index value unchanged for events such as stock splits.

c. What is the sum of the prices of all the shares in the index after the split?

The stock that was $100 is now $50, so the sum of the share prices is now $2,035.00.

d. What is the divisor after the split?

With the new sum of share prices at $2,035.00, the divisor must be 0.244005 to maintain the index value at 8340.00.

2. What is the main difference in the calculation of the DJIA and the S&P 500 index? Explain.

The S&P 500 index gives a weight to each represented share that is proportional to the market value of the outstanding shares. The DJIA simply adds the prices of all of the individual shares, so the DJIA effectively weights each stock by its price level.

3. For the S&P 500 index, assume that the company with the highest market value has a 1 percent increase in stock prices. Also, assume that the company with the smallest market value has a 1 percent decrease in the price of its shares. Does the index change? If so, in what direction?

The index value increases. The share with the higher market value has a greater weight in the index than the share with the smallest market value. Therefore, the 1 percent increase on the high market value share more than offsets the 1 percent decrease on the low market value share.

4. The S&P 500 futures is scheduled to expire in half a year, and the interest rate for carrying stocks over that period is 11 percent. The expected dividend rate on the underlying stocks for the same period is 2 percent of the value of the stocks. (The 2 percent is the half-year rate, not an annual rate.) Ignoring the interest that it might be possible to earn on the dividend payments, find the fair value for the futures if the current value of the index is 945.00.

   Assuming the half-year rate is 0.11/2, the fair value is:

   $$\text{Fair Value} = 945.00(1.055 - 0.02) = 978.075$$

   Assuming semiannual compounding, the interest factor would be 1.0536 and the fair value would be:

   $$\text{Fair Value} = 945.00(1.0536 - 0.02) = 976.752$$

5. Consider a very simple index like the DJIA, except assume that it has only two shares, A and B. The price of A is $100.00, and B trades for $75.00. The current index value is 175.00. The futures contract based on this index expires in three months, and the cost of carrying the stocks forward is 0.75 percent per month. This is also the interest rate that you can earn on invested funds. You expect Stock A to pay a $3 dividend in one month and Stock B to pay a $1 dividend in two months. Find the fair value of the futures. Assume monthly compounding.

   $$\text{Fair value} = 175.00(1.0075)^3 - \$3(1.0075)^2 - \$1(1.0075) = 174.91$$

6. Using the same data as in Problem 5, now assume that the futures trades at 176.00. Explain how you would trade with this set of information. Show your transactions.

   At 176.00, the futures is overpriced. Therefore, the trader should sell the futures, buy the stocks and carry them forward to expiration, investing the dividend payments as they are received. At expiration, the total cost incurred to carry the stocks forward is 174.91, and the trader receives 176.00 as cash settlement, for a profit of 1.09 index units. (This ignores interest on daily settlement flows.)

7. Using the same data as in Problem 5, now assume that the futures trades at 174.00. Explain how you would trade with this set of information. Show your transactions.

   At 174.00 the futures is underpriced. Therefore, the trader should buy the futures and sell the stocks short, investing the proceeds. The trader will have to borrow to pay the dividends on the two shares. At expiration, the total outlays, counting interest, have been:

   $$\$175.00(1.0075)^3 - \$3(1.0075)^2 - \$1(1.0075) = 174.91$$

   With the convergence at expiration, the trader can buy the stocks for 174.00 and return them against the short sale. This gives a profit of $.91.

8. For a stock index and a stock index futures constructed like the DJIA, assume that the dividend rate expected to be earned on the stocks in the index is the same as the cost of carrying the stocks forward. What should be the relationship between the cash and futures market prices? Explain.

   The cash and futures prices should be the same. In essence, an investment in the index costs the interest rate to carry forward. This cost is offset by the proceeds from the dividends. If these are equal, the effective cost of carrying the stocks in the index forward is zero, and the cash and futures prices should then be the same.

9. Your portfolio is worth $100 million and has a beta of 1.08 measured against the S&P index, which is priced at 350.00. Explain how you would hedge this portfolio, assuming that you wish to be fully hedged.

The hedge ratio is:

$$-\beta_P\left(\frac{V_P}{V_F}\right) = \text{number of contracts}$$

With our data we have:

$$-1.08\left(\frac{\$100{,}000{,}000}{(350.00)(500)}\right) = -617.14$$

The cash value of the futures contract is 500 times the index value of 350.00, or $175,000. Therefore, the complete hedge is to sell 617 contracts.

10. You have inherited $50 million, but the estate will not settle for six months and you will not actually receive the cash until that time. You find current stock values attractive and you plan to invest in the S&P 500 cash portfolio. Explain how you would hedge this anticipated investment using S&P 500 futures.

    Buy S&P 500 index futures as a temporary substitute for actually investing the cash in the stock market. Probably the best strategy is to buy the contract that expires closest in time to the expected date for receiving the cash. If the S&P 500 index value is 300.00, then the dollar value of one contract will be $150,000 (300.00 × $500). Therefore, you should achieve a good hedge by purchasing about 333 ($50,000,000/$150,000) contracts.

11. William's new intern, Jessica, is just full of questions. She is particularly inquisitive about stock index futures. She notices that the futures price is consistently higher than the current index level and that the difference gets smaller as the contracts near their expiration dates.

A. Explain the relationship between the futures price, the spot price, interest rates, and dividends.

The futures contract effectively allows one to commit to the sale or the purchase of the Dow index stocks at a specific point in the future at a price agreed upon today. There are costs and benefits related to using the futures market as opposed to the cash market to buy the Dow index stocks. Take for example an individual who wishes to own the Dow stocks in three months. The alternatives are to:

1. buy the stocks today and hold them, or
2. buy a futures contract with three months to delivery.

The benefit of using the futures contract is that there is no cash outlay today. The purchase price of the stocks can be invested for three months to earn interest. The downside of using the futures contract is that since the stocks are not held over the next three months, no dividends are received. This suggests the following relationship between the spot and the futures market prices:

$$F_{0,t} = S_0(1 + C) - \sum_{i=1}^{n} D_i(1 + r_i)$$

where:

$F_{0,t}$ is the futures price today for delivery at time t,
S_0 is the spot price for the index today,
C is the cost of carry,
D_i is the ith dividend paid between now and time t, and
r_i is the rate of return received on the ith dividend between the payment date and time t.

In general, the dividend yield is smaller than the cost of carry so the index futures markets are generally normal (futures index above the spot index). As the delivery date approaches the futures index converges to the spot index.

B. Jessica asks William to explain the Dow index to her. What type of index is the Dow? How is it constructed? How could she build a portfolio of stocks to replicate it?

The Dow is a price-weighted index. The stocks are represented in the index in proportion to their price. The index is computed as:

$$\text{Dow Index} = \frac{\sum_{i=1}^{30} P_i}{\text{Divisor}}$$

where the P_i are the prices of the stocks comprising the Dow, and the Divisor is a number used to compute the "average."

When the Dow first appeared with 30 stocks in 1928 (the Dow was first published in 1884 with 11 stocks), the divisor was 30. As stocks split or the components of the Dow changed, the divisor was adjusted to maintain continuity in the index. To form a portfolio that would replicate the Dow, Jessica should buy an equal number of shares of each stock in the Dow.

C. Jessica wants a numerical example of the relationship between a price weighted index and the futures contract based on that index. She supposes the following example. A futures contract is based on a price-weighted index of three stocks A, B, and C. The futures contract expires in 3 months. Stock A pays a dividend at the end of the first month, and Stock C pays a dividend at the end of month two. The term structure is flat over this time period with the monthly interest rate equal to 0.5%. The stock prices and dividends are summarized below

Stock	**Price**	**Dividend**
A	\$30	\$0.11 in one month
B	\$50	\$0
C	\$40	\$0.15 in two months

Compute the index assuming a divisor of 3. How many shares of each stock should be bought to replicate the index?

The index is computed as:

$$\text{Index} = \frac{\sum_{i=1}^{3} P_i}{\text{Divisor}} = \frac{30 + 50 + 40}{3} = 40$$

To replicate the index, you would buy 1/Divisor shares of each stock. In our example one-third share of each stock would replicate the index.

D. Suppose the divisor had been 0.5. Compute the index. How many shares of each stock must be bought to replicate the index?

With the divisor equal to 0.5 the index is:

$$\text{Index} = \frac{\sum_{i=1}^{3} P_i}{\text{Divisor}} = \frac{30 + 50 + 40}{0.5} = 240$$

The number of shares of each stock to buy to replicate the index is $\frac{1}{0.5} = 2$. Two shares of each stock will replicate the index.

E. Assuming the divisor is 0.5, compute the fair value for the 3-month futures contract.

The fair market value is computed using:

$$F_{0,t} = 240(1 + 0.005)^3 - 2(0.11)(1.005)^2 - 2(0.15)(1.005) = 243.09$$

F. Right now the Dow Jones Industrial Average is at 8,635. Its dividend yield is 1.76%. The 90-day T-bill rate is 5.6% bond equivalent yield. Compute a fair price today for the index futures contract expiring in 90 days.

For an index composed of a large number of securities it is sometimes helpful to express the relationship between the futures and spot indexes as:

$$F_{0,t} = S_0(1 + C - \text{DIVYLD})$$

where DIVYLD is the dividend yield on the index stocks.

The fair price for the futures contract expiring in 90 days is:

$$F_{0,t} = 8{,}717$$

12. Casey Mathers manages the $60 million equity portion of Zeta Corporation's pension assets. This past Friday, August 7, Zeta announced that it was downsizing its workforce and would be offering early retirement to many of its older employees. The impact on the portfolio Casey manages would be an anticipated $10 million withdrawal over the next 4 months. The stock market has been good for the past 5 years, but recently there have been signs of weakness. Casey is concerned about a drop in asset prices before the $10 million is withdrawn from the portfolio. Casey runs a fairly aggressive portfolio with a beta of 1.2, relative to the S&P 500 Index. Casey sees the following S&P 500 index futures prices:

Expiration	Contract Value $250 × Index
SEP	1088.50
DEC	1100.00
MAR × 1	1110.50

A. How much of the portfolio should Casey hedge? Justify your answer.

Casey anticipates $10 million being withdrawn from the portfolio over the next 4 months. Casey is concerned about the risk that falling stock prices will result in more shares having to be sold to raise the $10 million. It is just the anticipated withdrawal that should be hedged.

B. Design a hedge based on your answer to part A above.

To hedge the risk, Casey should use the futures contract with expiration as soon after the anticipated withdrawal as possible. This would be the December futures contract. Since Casey's portfolio has a beta of 1.2 relative to the S&P 500, Casey should sell $1.2 of future contract value per $1 of portfolio hedged. Then the number of contracts to trade is computed by:

$$N = -\beta_P \frac{V_P}{V_F} = -1.2 \frac{\$10{,}000{,}000}{(1{,}100)\,\$250} = -43.6 \text{ contracts}$$

To hedge the risk of the $10 million withdrawal, Casey should sell 44 contracts. As the assets are withdrawn from the portfolio, the hedge should be gradually unwound. The transactions would be as follows:

Date	Cash Market	Futures Market
Today	Anticipate the withdrawal of $10 million from the portfolio over the next four months.	Sell 44 December S&P 500 futures contracts.
Between now and December	Sell securities to meet portfolio withdrawal demands.	Enter reverse trades to unwind the hedge as assets are withdrawn from portfolio.

13. Byron Hendrickson manages the $30 million equity portion of Fredrick and Sons' pension plan assets. Byron has been trying to get the management of Fredrick and Sons to move more of their assets from their fixed income portfolio (market value of $60 million) to the equity portfolio in order to achieve the objectives

that management had set forth for growth of the plan. Management has decided to invest the proceeds of several bond issues that will be maturing over the next three months. The total proceeds from the bond issues will be \$10 million. It is now December 15. Byron believes that the January price runup will be particularly strong this year. Since his portfolio is not particularly aggressive, beta equal to 0.85, he would really like to have that \$10 million working for him in January. Design a hedge that will prevent Byron from missing the January action, based on the following current market prices for the S&P 500 index futures:

Expiration	Contract Value $250 × Index
MAR	1157.00
JUN	1170.80

Byron wishes to commit \$10 million to the stock market today, but the cash will not be available until March. Therefore, he is implicitly short in the cash market at present. Byron should make a long hedge using the March S&P 500 futures index. The number of contracts Byron should buy is computed as:

$$N = -0.85 \frac{-\$10{,}000{,}000}{(1{,}157)\,\$250} = 29.4 \text{ contracts}$$

Byron's implicit short position is reflected in the preceding equation by the −\$10,000,000 cash market position. Byron should buy 29 March S&P futures contracts. This will generate gains to replace the opportunity loss from not having the \$10 million invested between now and March. As the bonds mature and the proceeds are transferred to the equity side, the hedge should be unwound. Byron's transactions are summarized below.

Date	Cash Market	Futures Market
Today	Anticipate the investment of \$10 million into the equity portfolio over the next three months.	Buy 29 March S&P 500 futures index contracts.
March	Buy stocks using \$10 million plus any gains/losses from the futures position.	Unwind the futures position as the Investments are made into the equity portfolio.