6 Interest Rate Futures: Refinements

Answers to Questions and Problems

1. Explain the risks inherent in a reverse cash-and-carry strategy in the T-bond futures market.

   The reverse cash-and-carry strategy requires waiting to receive delivery. However, the delivery options all rest with the short trader. The short trader will initiate delivery at his or her convenience. In the T-bond market, this exposes the reverse cash-and-carry trader to receiving delivery at some time other than the date planned. Also, with so many different deliverable bonds, the reverse cash-and-carry trader is unlikely to receive the bond he or she desires. (These factors are fairly common for other commodities as well.) In the T-bond futures market, the short trader holds some special options such as the wildcard and end-of-month options. The reverse cash-and-carry trader suffers the risk that the short trader will find it advantageous to exploit the wildcard play or exercise the end-of-month option.

2. Explain how the concepts of quasi-arbitrage help to overcome the risks inherent in reverse cash-and-carry trading in T-bond futures.

   In pure reverse cash-and-carry arbitrage, the trader sells the bond short and buys the future. The trader thereby suffers risk about which bond will be delivered and the time at which it will be delivered. If the trader holds a large portfolio of bonds and sells some bond from inventory to simulate the short sale, these risks are mitigated. Receiving a particular bond on delivery is no longer so crucial to the trader's cash flows; after all, whichever bond is delivered will merely supplement the trader's portfolio. Further, the timing of delivery presents fewer problems to the quasi-arbitrage trader. In selling a bond from inventory, as opposed to an actual short sale, the trader did not need to worry about financing the short sale for a particular time. Therefore, the selection of a particular delivery date by the short futures trader is less critical. While quasi-arbitrage helps to mitigate the risks associated with the reverse cash-and-carry trade, risks still remain, particularly the risks associated with the short trader's options.

3. Assume economic and political conditions are extremely turbulent. How would this affect the value of the seller's options on the T-bond futures contract? If they have any effect on price, would they cause the futures price to be higher or lower than it otherwise would be?

   Generally, options are more valuable the greater the price risk inherent in the underlying good. This is certainly true for the seller's options on the T-bond futures contract. To see this most clearly, we focus on the wildcard option. Exploitation of the wildcard option depends on a favorable price development on any position day between the close of futures trading and the end of the period to announce delivery at 8 p.m., Chicago time. If markets are turbulent, there is a greater chance that something useful will occur in that time window on some day in the delivery month. The greater value of the seller's options in this circumstance would cause the futures price to be lower than it otherwise would be.

4. Explain the difference between the wildcard option and the end-of-the-month option.

   The wildcard option is the seller's option to initiate the delivery sequence based on information generated between the close of futures trading and 8 p.m., Chicago time, the time by which the seller must initiate the delivery sequence for a given day. The settlement price determined at the close of trading is the price that will be used for computing the invoice amount. Trading of the T-bond futures contract ceases on the eighth to last business day of the expiration month, and the settlement price on that day is used to determine the invoice amount for all deliveries. Any contracts not closed by the end of the trading period must be fulfilled by delivery. Even though the short trader must make delivery in this circumstance, the short trader still possesses an end-of-the-month option. The short trader can choose which day to deliver and can choose which bond to deliver. The short trader will deliver late in the month if the rate of accrual on the bond planned for delivery exceeds the short-term financing rate at which the bond is carried. Also, changing market conditions can change which bond will be cheapest-to-deliver, and the right to wait and choose a later delivery date has value to the short trader.

5. Some studies find that interest rate futures markets were not very efficient when they first began but that they became efficient after a few years. How can you explain this transition?

   The growing efficiency of these markets seems to be due to a market seasoning or maturation process. When these contracts were first initiated, it appears that some of their nuances were not fully appreciated. In particular, the complete understanding of the importance of the seller's options seems to have emerged only slowly.

6. Assume you hold a T-bill that matures in 90 days, when the T-bill futures expires. Explain how you could transact to effectively lengthen the maturity of the bill.

   Buy the T-bill futures that expires in 90 days. After this transaction, you will be long a spot 90-day bill, and you will hold (effectively) a spot position in a 90-day bill to begin in 90 days. The combination replicates a 180-day bill.

7. Assume that you will borrow on a short-term loan in six months, but you do not know whether you will be offered a fixed rate or a floating rate loan. Explain how you can use futures to convert a fixed to a floating rate loan and to convert a floating rate to a fixed rate loan.

   For convenience, we assume that the loan will be a 90-day loan. If the loan is to be structured as a floating rate loan, you can convert it to a fixed rate loan by selling a short-term interest rate futures contract (Eurodollar or T-bill) that expires at the time the loan is to begin. The rate you must pay will depend on rates prevailing at the time of the loan. If rates have risen you must pay more than anticipated. However, if rates have risen, your short position in the futures will have generated a profit that will offset the higher interest you must pay on the loan.

   Now assume that you contract today for a fixed interest rate on the loan. If rates fall, you will be stuck paying a higher rate than the market rate that will prevail at the time the loan begins. To convert this fixed rate loan to a floating rate loan, buy an interest rate futures that expires at the time the loan is to begin. Then, if rates fall, you will profit on the futures position, and these profits will offset the higher than market rates you are forced to pay on your fixed rate loan.

8. You fear that the yield curve may change shape. Explain how this belief would affect your preference for a strip or a stack hedge.

   If the yield curve is to change shape, rates on different futures expirations for the same interest rate futures contract may change by different amounts. In this case, it is important to structure the futures hedge so that the futures cash flows match the exposure of the underlying risk more closely. Thus, if the cash market exposure involves the same amount at regular intervals over the future, a strip hedge will be more effective against changing yield curve shapes.

9. A futures guru says that tailing a hedge is extremely important because it can change the desired number of contracts by 30 percent. Explain why the guru is nuts. How much can the tailing factor reasonably change the hedge ratio?

To tail a hedge, one simply reduces the computed hedge ratio by discounting it at the risk-free rate for the time of the hedge. For convenience, we assume that the untailed computed hedge ratio is 1.0. If the hedging period is one year, a 30 percent effect would require an interest rate of 43 percent, because 0.7 = (1/1.43). If the hedging horizon is long, say a full two years, the interest rate would still have to be 19.52 percent to generate the 30 percent effect, because $(1.1952)^2 = 1/0.7$. Thus, it seems extremely improbable that the tailing effect could be so large.

10. We have seen in Chapter 4 that regression-based hedging strategies are extremely popular. Explain their weaknesses for interest rate futures hedging.

    First, regression-based hedging (the RGR model) involves statistical estimation, so the technique requires a data set for both cash and futures prices. This data may sometimes be difficult to acquire, particularly for an attempt to hedge a new security. Second, the RGR model does not explicitly consider the differences in the sensitivity of different bond prices to changes in interest rates, and this can be a very important factor. The regression approach does include the different price sensitivities indirectly, however, since their differential sensitivities will be reflected in the estimation of the hedge ratio. Third, any cash bond will have a predictable price movement over time, and the RGR model does not consider this change in the cash bond's price explicitly. However, the sample data used to estimate the hedge ratio will reflect this feature to some extent. Fourth, the RGR hedge ratio is chosen to minimize the variability in the combined futures–cash position over the life of the hedge. Since the RGR hedge ratio depends crucially on the planned hedge length, one might reasonably prefer a hedging technique focusing on the wealth position of the hedge when the hedge ends. After all, the wealth change from the hedge depends on the gain or loss when the hedge is terminated, not on the variability of the cash–futures position over the life of the hedge.

11. You estimate that the cheapest-to-deliver bond on the T-bond futures contract has a duration of 10.2 years. You want to hedge your medium-term Treasury portfolio that has a duration of 4.0 years. Yields are 9.5 percent on the futures and on your portfolio. Your portfolio is worth \$120,000,000, and the decimal futures price is 68.91. Using the PS model, how would you hedge?

    From the text, the PS hedge ratio is:

$$N = -\left(\frac{P_i \, MD_i}{FP_F \, MD_F}\right) \mathrm{RYC}$$

    For this problem, we are entitled to assume that RYC = 1.0 since no other value is specified. Applying this equation to our data gives:

$$N = -\frac{\$120{,}000{,}000 \times 3.652968}{\$68{,}910 \times 9.315068} = -\,682.902707$$

    Therefore, the PS hedge would require selling about 683 T-bond futures.

12. Explain the relationship between the bank immunization case and hedging with the PS model.

    Both bank immunization and the PS model rely essentially on the concept of duration. A PS hedge finds the futures position to make the combined cash/futures position have a duration of zero. Similarly, in bank immunization with equal asset and liability amounts, the asset duration is set equal to the liability duration. For the combined balance sheet, the overall duration is effectively zero as well. Therefore, the two techniques are quite similar in approach, even if they use different instruments to achieve the risk reduction.

13. Compare and contrast the BP model and the RGR model for immunizing a bond portfolio.

    The BP model essentially is an immunization model that is suitable for the bank immunization case. The BP hedge ratio is found empirically, but it is the hedge ratio that gives a price movement on the futures position that offsets the price movement on the cash position. As such, it is effectively reflecting the duration of the two instruments. (Notice that the BP model does not really help with the planning period case, because it considers only the effect of a current change in rates, not a change over some hedging horizon.) The RGR Model does not really take duration into account in any direct fashion, so it is not oriented toward immunizing at all.

14. It was a hot day in August, and William had just completed the purchase of $20 million of T-bills maturing next March and $10 million of T-bills maturing in one month. The phone rang, and William was informed that the firm had just made a commitment to provide $30 million in capital to a client in mid-December. If William had known this 20 minutes earlier, he would have invested differently.

A. What risks does William face by using his present investments to meet the December commitment?

William faces several interest rate risks using his present investments to fund the anticipated cash outflow. These risks stem from the fact that the maturity of his present investments do not match the timing of the cash outflow. The proceeds of the $10 million of September T-bills must be reinvested from September to December. William faces the risk that interest rates may fall between now and September which would reduce the December proceeds from the reinvestment. The $20 million of March T-bills are subject to price risk. If interest rates rise between now and December, the proceeds from the sale of the March T-bills will be less than anticipated. Had he known about the firm's commitment earlier, he could have invested in $30 million of December maturity T-bills.

B. Using the futures markets, how can William reduce the risks of the December commitment? Show what transactions would be made.

William would like to lock in a reinvestment rate for September and a selling price for December. He can accomplish this by buying $10 million September futures contracts and selling $20 million December futures contracts. He is in effect lengthening the maturity of his September bills and shortening the maturity of his March bills. His transactions would be:

Date	Cash Market	Futures Market
Today		Buy $10 million September T-bill futures; sell $20 million December T-bill futures
September	Receive proceeds from $10 million September T-bills; reinvest $10 million into 90-day T-bills	Reverse September T-bill futures position by selling $10 million September T-bill futures
December	Receive proceeds from maturing December T-bills; sell March T-bills in cash market	Reverse December T-bill futures position by buying $20 million December T-bill futures

Gains and losses in the futures market will offset losses and gains in the cash market so that the total proceeds available to meet the December commitment will be as anticipated.

15. Handcraft Ale, Ltd. has decided to build additional production capacity in the US to meet increasing demand in North America. Uma Peele has been given the responsibility of obtaining financing for the project. Handcraft Ale will need $10 million to carry the firm through the construction phase. This phase will last two years, at which time the $10 million debt will be repaid using the proceeds of a long-term debt issue.

Ms. Peele gets rate quotes from several different London banks. The best quote is:

Variable	Fixed
LIBOR + 150 bp	8.5%

Each of these loans would require quarterly interest payments on the outstanding loan amount. Ms. Peele looks up the current LIBOR rate and finds that it is 5.60%. The variable rate of 7.1% (5.60 + 1.5) looks very attractive, but Ms. Peele is concerned about interest rate risk over the next two years.

A. What could Ms. Peele do to take advantage of the lower variable rate while at the same time have the comfort of fixed rate financing?

Ms. Peele could hedge each of the anticipated interest rate adjustments using the Eurodollar futures contract. This would be a strip hedge. She would sell $10 million 3-month Eurodollar futures contracts with expirations in the months when interest rates are adjusted. For longer term loans this can present a problem. Contracts may not be traded with the proper expiration date, or the market may be very thin making it difficult to find a counterparty. Alternatively, Ms. Peele could employ a stack hedge. In this case she would stack the hedges for all interest rate adjustments on a single contract whose expiration is not so far into the future. For example, she could sell $70 million Eurodollars futures contracts with expiration in the month of the first interest rate adjustment.

B. Consider the following 3-month Eurodollar quotes:

Delivery Month	**Rate**
AUGx0	94.32
SEP	94.34
OCT	94.31
NOV	94.33
DEC	94.35
JAN × 1	94.43
MAR	94.40
JUN	94.43
SEP	94.40
DEC	94.25
MAR × 2	94.30
JUN	94.27
SEP	94.23
DEC	94.15

Handcraft Ale takes out a floating rate note with the first interest rate adjustment coming in December. LIBOR at the time of loan initiation is 5.70%. Design a strip hedge to convert the Handcraft Ale floating-rate note to a fixed-rate note. What is the anticipated fixed rate?

Peele would hedge each of her interest payments by selling $10 million of 3 month Eurodollars in the following months:

Expiration	**Price**	**Anticipated Loan Rate**
DEC	94.35	7.15
MAR × 1	94.40	7.10
JUN	94.43	7.07
SEP	94.40	7.10
DEC	94.25	7.25
MAR × 2	94.30	7.20
JUN	94.27	7.23

The anticipated loan rate for each adjustment date is computed as:

$$\text{Anticipated Rate} = 100 - \frac{\text{Price}}{100} + 1.5$$

While the hedge is not the same as pure fixed rate financing, there is very little variation anticipated. The range of anticipated rates are from 7.07 to 7.25 percent. The average rate over the life of the loan is 7.15 percent, including the initial period.

C. Suppose that Handcraft Ale's quarterly interest payments were in November, February, May and August. Would a strip hedge be possible? Design a hedge that Ms. Peele could use in this case.

If Handcraft's interest payment cycle were November, February, May and August, Ms. Peele would have a problem. It would only be possible to match the timing of the nearest interest rate adjustment. This is a situation in which Ms. Peele could use a stack hedge. She would stack hedge all seven interest rate adjustments by selling $70 million in Eurodollar futures with November expiration. In November Handcraft would make its interest payment. At that time there will be a market for February Eurodollars, and Ms. Peele would stack the six remaining interest rate adjustments on the February contract. This process would continue until maturity with each interest rate adjustment reducing the size of the subsequent stack hedge by $10 million. Alternatively, Ms. Peele could hedge each rate adjustment using the Eurodollar futures contract that expires in the month just following the date the interest rate adjustment is made.

16. Jim Hunter is preparing to hedge his investment firm's decision to purchase $100 million of 90-day T-bills 60 days from now in June. The discount yield on the 60-day T-bill is 6.1%, and the June T-bill futures contract is trading at 94.80. Jim views these rates as very attractive relative to recent history, and he would like to lock them in. His first impulse is to buy 100 June T-bill futures contracts, but his recent experience leads him to believe that he should be buying something less.

A. Why is a one-to-one hedge ratio inappropriate in Jim's situation?

A one-to-one hedge ratio is not appropriate because the daily settlement gains and losses can be invested or must be financed. If no interest were earned on settlement cash flows then a one-to-one hedge would be appropriate because the settlement cash flows would exactly offset the change in value of the underlying instrument. Because interest can be earned on the settlement cash flows, the delivery date value of the settlement cash flows will not exactly equal the change in value of the underlying instrument. The terminal value of the settlement cash flows will exceed, in magnitude, the change in value of the underlying instrument. The difference will be the interest earned and/or assessed between the settlement date and the delivery date.

B. Compute an appropriate hedge ratio given the market conditions faced by Jim.

The hedge ratio is slightly reduced to account for the interest that can be earned between daily settlement and the delivery date. This is called tailing the hedge. The amount by which the hedge is reduced is called the tailing factor. The tailing factor is the present value factor between the settlement date and the delivery date. When Jim initiates the hedge the discount yield for the 60-day T-bill (his hedging time horizon) is 6.1%. Computing the tailing factor, which is the price per $1 of face value we have:

$$\text{Tailing Factor} = 1 - \left(\frac{.061 \times 60}{360}\right) = 0.9898$$

The tailed hedge for the $100 million of 90-day T-bills to be purchased in 2 months is:

$$\text{Tailed Hedge} = \$100\,(0.9898) = \$98.98 \text{ million}$$

Jim would buy 99 June T-bill futures contracts.

C. Under what conditions might Jim need to adjust his hedge ratio between now and June?

The tailing factor is a function of the time to delivery and the level of interest rates. If interest rates change significantly, then Jim might need to adjust the hedge. The longer the time to settlement, the more influential interest rate movements will be. Even if interest rates do not move, the passage of time will call for adjustment as the tailing factor will move toward the value of one on the delivery date.

17. Alex Brown has just returned from a seminar on using futures for hedging purposes. As a result of what he has learned, he reexamines his decision to hedge $500 million of long-term debt that his firm plans to issue in May. His current hedge is a short position of 5,000 T-bond futures contracts ($100,000 each). If the debt could be issued today, it would be priced at 119-22 to yield 6.5%. With its 8% coupon and 30 years to maturity, the duration of the debt would be 13.09 years. On the futures side, the futures prices are based on the cheapest-to-deliver bonds, which are trading at 124-14 to yield 5.6%. These bonds have a duration of 9.64 years.

A. List and briefly describe possible strategies Alex Brown could use to hedge his impending debt issue.

There are a number of different methods Alex could employ to hedge his impending debt issue:

Face Value Naive Model—In this method Alex would trade one dollar of nominal futures contract per one dollar of debt face value. The major benefit of this method is the ease of implementation. Unfortunately, it ignores market values and the differential responses of the bond and futures contract prices to interest rates.

Market Value Naive Model—In this method Alex would hedge one dollar of debt market value using one dollar of futures price value. That is, the hedge ratio is determined by the market prices instead of nominal and face values. Unfortunately, it does not consider the price sensitivities of the two instruments.

Conversion Factor Model—This model can be used when the hedging instrument is a T-note or T-bond futures contract. The conversion factor adjusts the prices of deliverable bonds and notes that do not have a 6% coupon to make them "equivalent" to the 6% coupon bond or note that is called for in the contract. The hedge ratio is determined by multiplying the Face Value Naive hedge ratio by the conversion factor. The appropriate conversion factor to use is the conversion factor of the cheapest to deliver T-bond or T-note. This model still ignores price sensitivity differences between the hedging and hedged instruments.

Basis Point Model—This model uses the price changes of the futures and cash positions resulting from a one basis point change in yields to determine the hedge ratio. It is calculated as:

$$HR = -\frac{BPC_S}{BPC_F}$$

This model works well if the cash and futures instruments face the same rate volatility. If they face different volatilities and that relationship can be quantified, then the basis point model can be adjusted to account for the differing volatilities.

Regression Model—In the regression model the historic relationship between cash market price changes and futures market price changes is estimated. This estimation is accomplished by regressing price changes in the cash market on futures price changes. The slope coefficient from this regression is then used as the hedge ratio. Alex may not find this model useful, as he is trying to hedge a new debt issue. Even if Alex had an historic price stream on 30-year corporate debt issues, the historic relationship with the futures price might prove to be an unreliable indicator of the present or future relationship. This stems from the fact that the price response of the futures contract is determined by the cheapest-to-deliver bond. The cheapest-to-deliver bond can vary in maturity from 15 years to 30 years. This means that the futures contract can have very different price responses to interest rates at different points in time.

Price Sensitivity Model—This may be a good model for Alex to use. It is designed for interest rate hedging, and it accounts for the differential price responses of the hedging and the hedged instruments. The model is duration-based so that it accounts for maturity and coupon rate differences of the cash and the futures positions. It is computed as:

$$N = -\left(\frac{P_i\,MD_i}{FP_F\,MD_F}\right)\text{RYC}$$

where:

FP_F and P_i are the respective futures contract and cash instrument prices; MD_i and MD_F are the modified durations for the cash and futures instruments, respectively, and RYC is the change in the cash market yield relative to the change in the futures yield.

B. What strategy is Alex Brown currently using? What are the strengths and weaknesses of this strategy?

Currently Alex has employed a Face Value Naive hedge. For each dollar of debt principal he plans to issue, he is short $1 of nominal T-bond futures. The benefit of the strategy is its ease of implementation. The drawback is that cash instrument and the T-bond futures may have differential price responses to interest rate changes.

C. Based on the knowledge Alex gained at the hedging seminar, he feels that a price sensitivity hedge would be most appropriate for his situation. Design a hedge using the price sensitivity method. Assume that the relative volatility between the corporate interest rate and the T-bond interest rate, RYC, is equal to one.

The price sensitivity hedge ratio is:

$$N = -\left(\frac{P_i\, MD_i}{FP_F\, MD_F}\right)\text{RYC}$$

$$FP_F = 124.4375\% \times 0.1 \text{ million} \qquad MD_F = 9.128788$$

$$P_i = 119.6875\% \times 500 \text{ million} \qquad MD_i = 12.29108$$

$$N = -\frac{\$598{,}437{,}500 - 12.29108}{\$124{,}437.50 - 9.128788} = -6{,}475 \text{ contracts}$$

To hedge the risk, 6,475 contracts should be sold.

D. At the time of refinancing, the T-bond futures price is 121-27 and B.I.G.'s new debt issue is priced at 116-08. Compute the net wealth change resulting from the naive hedge.

In the cash market, Alex suffers an opportunity loss because he anticipated issuing debt at 119-22, but he is only to get 116-08 for the debt. The opportunity loss is:

$$\text{Opportunity loss} = (1.1625 - 1.196875)\ \$500{,}000{,}000 = -\$17{,}187{,}500$$

In the futures market Alex realizes a gain because he was short T-bond futures. His gain is:

$$\text{Futures gain} = (1.244375 - 1.2184375)\ \$500{,}000{,}000 = \$12{,}968{,}750$$

$$\text{Net wealth change} = -\$17{,}187{,}500 + \$12{,}968{,}750 = -\$4{,}218{,}750$$

E. Compute the net wealth change resulting from the price sensitivity hedge.

Examining the price sensitivity hedge, the cash market loss will still be $17,187,500. The futures market gain will now be:

$$\text{Futures gain} = (1.244375 - 1.2184375)\ \$647{,}500{,}000 = \$16{,}794{,}531$$

$$\text{Net wealth change} = -\$17{,}187{,}500 + \$16{,}794{,}531 = -\$392{,}969$$

The price sensitivity hedge is much more effective than the face value naive hedge at reducing the risk.