SOLUTIONS TO TUTORIAL EXAMPLES

CHAPTER 19

Note: the standard formulae M = PL/4 and M = $wL^2/8$, applied in some of the following examples, are derived and explained in Chapter 16.

Question 1

M_{max} = PL/4 = 5 × 4/4 = 5 kNm = 5.0 × 10^6 Nmm.

Z = $bd^2/6$ = 75 × $300^2/6$ = 1 125 000 mm^3.

Bending stress σ = M/z = (5.0 × 10^6 Nmm) / (1.125 × 10^6 N/mm^2) = 4.44 N/mm^2.

Question 2

M_{max} = $wL^2/8$ = 25 × $3^2/8$ = 28.1 kNm = 28.1 × 10^6 Nmm.

$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$I = \frac{100 \text{ x } 200^3}{12} - \frac{85 \text{ x } 160^3}{12}$$

I = (66.67 × 10^6) – (29.01 × 10^6) = 37.66 × 10^6 mm^4.

Part (a)

Bending stress:

$$\sigma = \frac{My}{I} = \frac{28.1 \text{ x } 10^6 \text{ x } 100}{37.66 \text{ x } 10^6} = 74.6 \text{ N/mm}^2$$

Maximum bending stress in the beam = 74.6 N/mm^2.

Part (b)

Radius of curvature:

$$R = \frac{EI}{M} = \frac{205 \text{ x } 10^3 \text{ x } 37.66 \text{ x } 10^6}{28.1 \text{ x } 10^6}$$

R = 274 743.77 mm = 274.7 metres.

Part (c)

$$\sigma = \frac{My}{I} = \frac{28.1 \text{ x } 10^6 \text{ x } 80}{37.66 \text{ x } 10^6} = 59.7 \text{ N/mm}^2.$$

Note: top of web is 80 mm up from neutral axis position.

Question 3

M = 0.50 kNm = 0.50 × 10^6 Nmm.

$$I = \pi\left(\frac{D^4}{64} - \frac{d^4}{64}\right) = \frac{\pi}{64}\left(50^4 - 44^4\right) = 122{,}812 \text{ mm}^4.$$

$$\sigma = \frac{My}{I} = \frac{0.50 \text{ x } 10^6 \text{ x } 25}{122{,}812} = 101.8 \text{ N/mm}^2.$$

Question 4

M_{max} = $wL^2/8$ = 15 × $5^2/8$ = 46.9 kNm = 46.9 × 10^6 Nmm

$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$I = \frac{150 \text{ x } 350^3}{12} - \frac{90 \text{ x } 290^3}{12}$$

I = (0.536 × 10^9) − (0.183 × 10^9) = 353 × 10^6 mm^4.

Part (a)

$$\sigma = \frac{My}{I} = \frac{46.9 \text{ x } 10^6 \text{ x } 175}{353 \text{ x } 10^6} = 23.3 \text{ N/mm}^2.$$

Part (b)

$$R = \frac{EI}{M} = \frac{20 \text{ x } 10^3 \text{ x } 353 \text{ x } 10^6}{46.9 \text{ x } 10^6} = 150{,}533 \text{ mm} = 150.5 \text{ metres.}$$

Question 5

$M_{max} = wL^2/8 = 4 \times 4^2/8 = 8$ kNm $= 8 \times 10^6$ Nmm.

Part (a)

$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$I = \frac{100 \text{ x } 250^3}{12} - \frac{90 \text{ x } 230^3}{12}$$

$I = 10^6(130.2 - 91.3) = 38.9 \times 10^6$ mm^4.

Part (b)

Maximum bending moment = 8 kNm (see above).

Part (c)

$$\sigma = \frac{My}{I} = \frac{8 \text{ x } 10^6 \text{ x } 125}{38.9 \text{ x } 10^6} = 25.7 \text{ N/mm}^2.$$

Part (d)

Strain $\varepsilon = \sigma/E = (25.7 \text{ N/mm}^2) / (205 \times 10^3 \text{ N/mm}^2) = 1.25 \times 10^{-4} = 0.000125$.

Question 6

Zone	b	d	A	y	Ay	h	Ah^2 ($\times10^6$)	I = $bd^3/12$ ($\times10^6$)
1	200	20	4000	10	40 000	87.5	30.63	0.13
2	20	280	5600	160	896 000	62.5	21.88	36.59
Sum			9600		936 000		52.51	36.72

Part (a)

$$\overline{y} = \frac{\sum(Ay)}{\sum A} = \frac{936,000}{9,600} = 97.5 \text{ mm.}$$

So, using the top edge of the section as datum:

y_1 = 10 mm
y_2 = 160 mm
h_1 = (97.5 − 10) = 87.5 mm
h_2 = (160 − 97.5) = 62.5 mm

Part (b)

$$I = \sum(Ah^2) + \sum\left(\frac{bd^3}{12}\right)$$

$I = (52.51 + 36.72) \times 10^6 = 89.2 \times 10^6 \text{ mm}^4$.

Part (c)

y_{bottom} = (300 − 97.5) = 202.5 mm.

$$\sigma = \frac{My}{I}$$

$$\sigma_{TOP} = \frac{75 \text{ x } 10^6 \text{ x } 97.5}{89.2 \text{ x } 10^6} = 82.0 \text{ N/mm}^2.$$

$$\sigma_{BTM} = \frac{75 \text{ x } 10^6 \text{ x } 202.5}{89.2 \text{ x } 10^6} = 170.3 \text{ N/mm}^2.$$

The maximum bending stress is the greater of the above two figures: 170.3 N/mm^2.

Question 7

Note that the two flanges are different.

Part (a)

Zone	b	d	A	y	Ay	h	Ah^2 ($\times 10^6$)	I = $bd^3/12$ ($\times 10^6$)
1	100	10	1000	5	5000	80	6.40	0.008
2	10	160	1600	90	144 000	5	0.04	3.41
3	80	10	800	175	140 000	90	6.48	0.007
Sum			3400		289 000		12.92	3.425

$$\bar{y} = \frac{\sum(Ay)}{\sum A} = \frac{289{,}000}{3{,}400} = 85 \text{ mm}$$

So, using the top edge of the section as datum:

$y_1 = 5$ mm
$y_2 = 90$ mm
$y_3 = 175$ mm

$h_1 = (85 - 5) = 80$ mm
$h_2 = (90 - 85) = 5$ mm
$h_3 = (175 - 85) = 90$ mm.

Part (b)

$$I = \sum(Ah^2) + \sum\left(\frac{bd^3}{12}\right)$$

$I = (12.92 + 3.425) \times 10^6 = 16.345 \times 10^6$ mm^4.

Part (c)

$$\sigma = \frac{My}{I}$$

y_{TOP} = 85 mm
y_{BTM} = (180 − 85) = 95 mm.

$$\sigma_{TOP} = \frac{50 \text{ x } 10^6 \text{ x } 85}{16.345 \text{ x } 10^6} = 260 \text{ N/mm}^2$$

$$\sigma_{BTM} = \frac{50 \text{ x } 10^6 \text{ x } 95}{16.345 \text{ x } 10^6} = 290.6 \text{ N/mm}^2.$$

The maximum bending stress is the greater of the above two figures: 290.6 N/mm^2.