SOLUTIONS TO TUTORIAL EXAMPLES

CHAPTER 16

Note: The shear force and bending moment diagrams are shown later in this section.

In the diagrams:

SFD stands for shear force diagram, and the figures are in kN;
BMD stands for bending moment diagram, and the figures are in kNm.

Question (a)

Calculation of reactions

Vertical equilibrium:

$R_A + R_C = (16 \text{ kN/m} \times 5 \text{ m}) = 80 \text{ kN}.$

Taking moments about A:

$8 \text{ m} \times R_C = (16 \text{ kN/m} \times 5 \text{ m} \times 2.5 \text{ m})$
$8R_C = 200$
$R_C = 200/8 = 25 \text{ kN}.$

Taking moments about C:

$8 \text{ m} \times R_A = (16 \text{ kN/m} \times 5 \text{ m} \times 5.5 \text{ m})$
$8R_A = 440$
$R_A = 55 \text{ kN}.$

Calculation of moments at specific points

$M_A = M_C = 0$ kNm.
$M_B = (55 \text{ kN} \times 5 \text{ m}) - (16 \text{ kN/m} \times 5 \text{ m} \times 2.5 \text{ m}) = 275 - 200 = 75$ kNm.
Max moment occurs at (55/16) = 3.44 m from A (see shear force diagram).
M_{max} = (55 kN x 3.44 m) − (16 kN/m x 3.44 m x 3.44/2 m)
= 189.2 − 94.7
= 94.5 kNm.

Question (b)

Calculation of reactions

Vertical equilibrium:

$$R_A + R_C = 30 + 20 = 50 \text{ kN}$$

Taking moments about A:

$$5 \text{ m} \times R_C = (30 \text{ kN} \times 1 \text{ m}) + (20 \text{ kN} \times 8 \text{ m})$$
$$5R_C = 30 + 160 = 190.$$
$$R_C = 190/5 = 38 \text{ kN}.$$

Taking moments about C:

$$5 \text{ m} \times R_A = (30 \text{ kN} \times 4 \text{ m}) - (20 \text{ kN} \times 3 \text{ m})$$
$$5R_A = 120 - 60 = 60$$
$$R_A = 60/5 = 12 \text{ kN}.$$

Calculation of moments at specific points

$M_A = M_D = 0$ kNm.
$M_B = (12 \text{ kN} \times 1\text{m}) = 12$ kNm.
$M_C = (12 \text{ kN} \times 5 \text{ m}) - (30 \text{ kN} \times 4 \text{ m}) = 60 - 120 = -60$ kNm.

Question (c)

Calculation of reactions

Vertical equilibrium:

$$R_A + R_D = (10 \text{ kN/m} \times 4\text{m}) + 50 = 90 \text{ kN}$$

Taking moments about A:

$$8 \text{ m} \times R_D = (10 \text{ kN/m} \times 4 \text{ m} \times 2 \text{ m}) + (50 \text{ kN} \times 6 \text{ m})$$
$$8R_D = 80 + 300 = 380.$$
$$R_D = 380/8 = 47.5 \text{ kN}.$$

Taking moments about D:

$8\ m \times R_A = (10\ kN/m \times 4\ m \times 6\ m) + (50\ kN \times 2\ m)$
$8R_A = 240 + 100 = 340$
$R_A = 340/8 = 42.5\ kN$

Calculation of moments at specific points

$M_A = M_C = 0$ kNm.
$M_B = (42.5\ kN \times 4\ m) + (10\ kN/m \times 4\ m \times 2\ m) = 170 - 80 = 90$ kNm.
$M_C = (42.5\ kN \times 6\ m) + (10\ kN/m \times 4\ m \times 4\ m) = 255 - 160 = 95$ kNm.

Question (d)

Calculation of reactions

Vertical equilibrium:

$R_A + R_D = (40\ kN/m \times 3\ m) + (30\ kN/m \times 3\ m) = 210\ kN$

Taking moments about A:

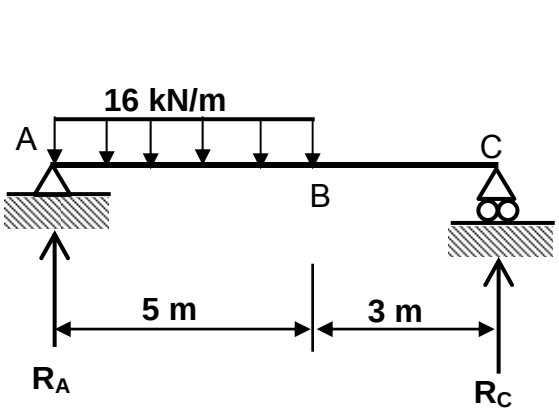
$9\ m \times R_D = (40\ kN/m \times 3\ m \times 1.5\ m) + (30\ kN/m \times 3\ m \times 7.5\ m)$
$9R_D = 180 + 675 = 855$
$R_D = 855/9 = 95\ kN.$

Taking moments about D:

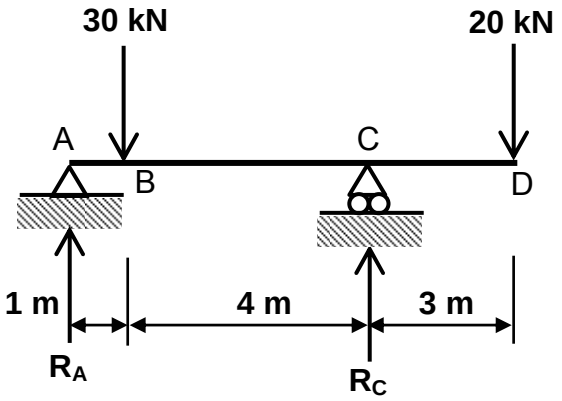
$9\ m \times R_A = (40\ kN/m \times 3\ m \times 7.5\ m) + (30\ kN/m \times 3\ m \times 1.5\ m)$
$9R_A = 900 + 135 = 1035$
$R_A = 1035/9 = 115\ kN.$

Calculation of moments at specific points

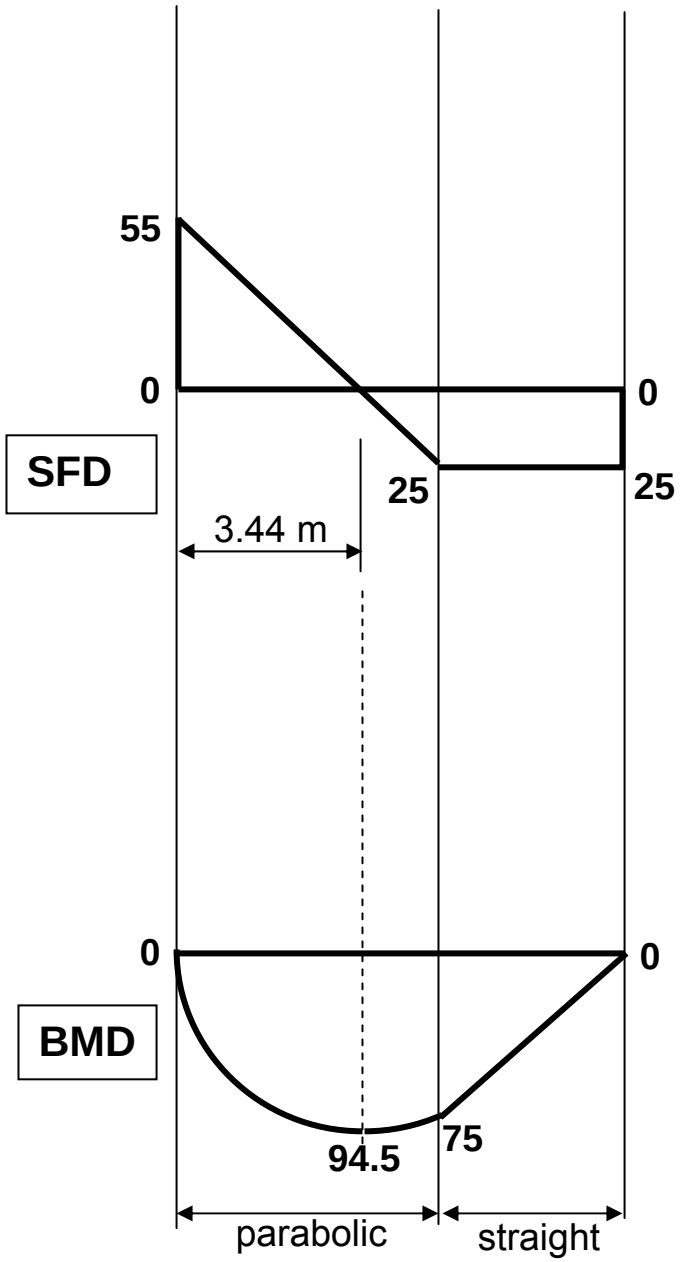
$M_A = M_C = 0$ kNm.
$M_B = (115\ kN \times 3\ m) - (40\ kN/m \times 3\ m \times 1.5\ m) = 345 - 180 = 165$ kNm.
$M_C = (115\ kN \times 6\ m) - (40\ kN/m \times 3\ m \times 4.5\ m) = 690 - 540 = 150$ kNm.
Max moment occurs at (115/40) = 2.875 m from A (see shear force diagram).
M_{max} $= (115\ kN \times 2.875\ m) - (40\ kN/m \times 2.875\ m \times 2.875/2\ m)$
$= 330.6 - 165.3$
$= 165.3$ kNm.

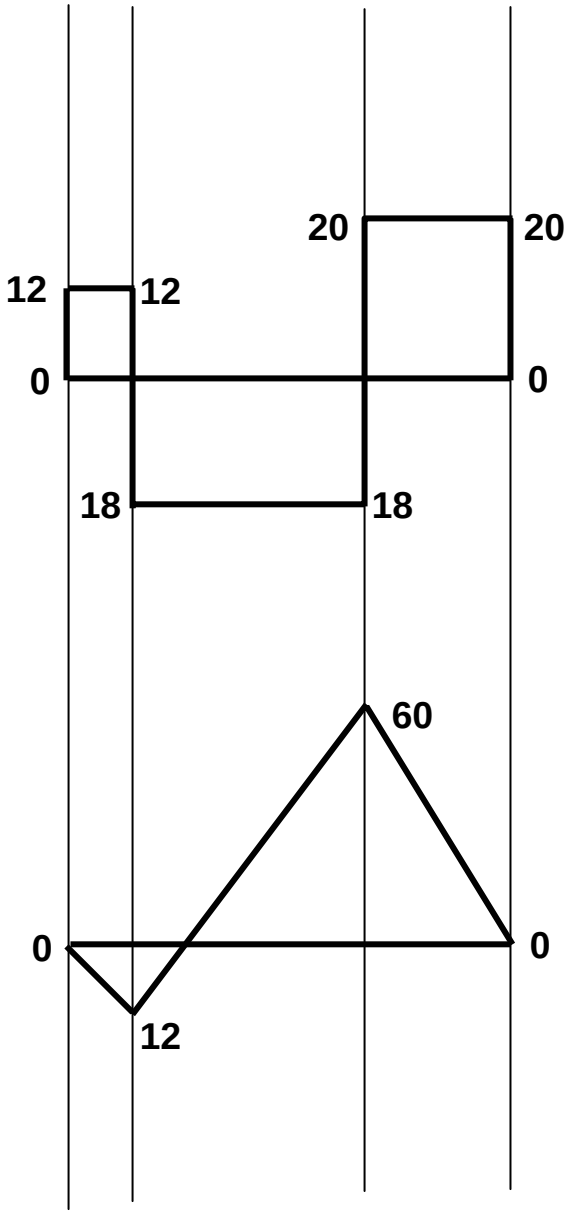

16 kN/m
A
B
C
5 m
3 m
R_A
R_C

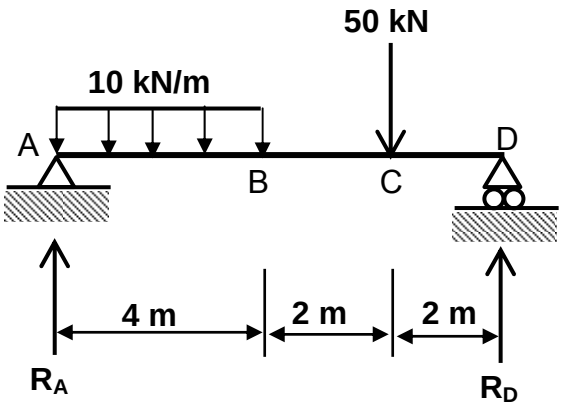
(a)

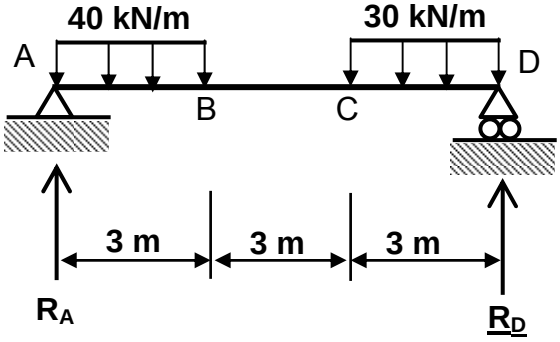

30 kN
20 kN
A
B
C
D
1 m
4 m
3 m
R_A
R_C

(b)

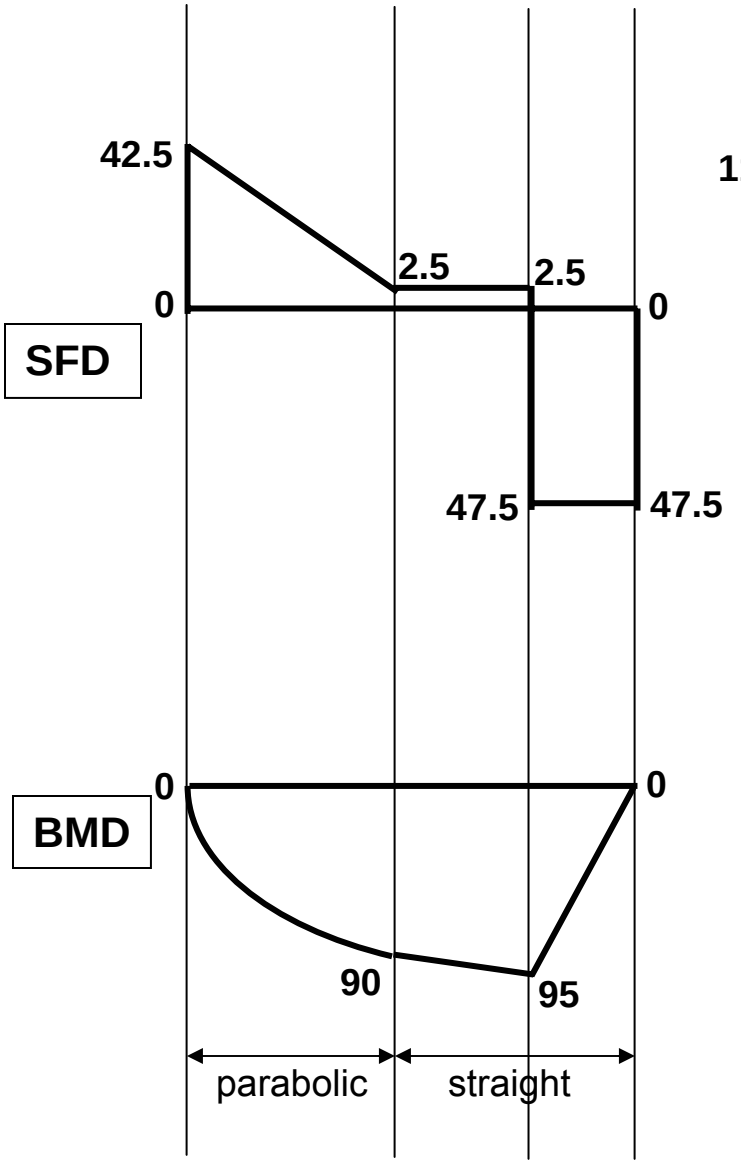

55
0
0
SFD
25
25
3.44 m
0
0
BMD
94.5
75
parabolic
straight

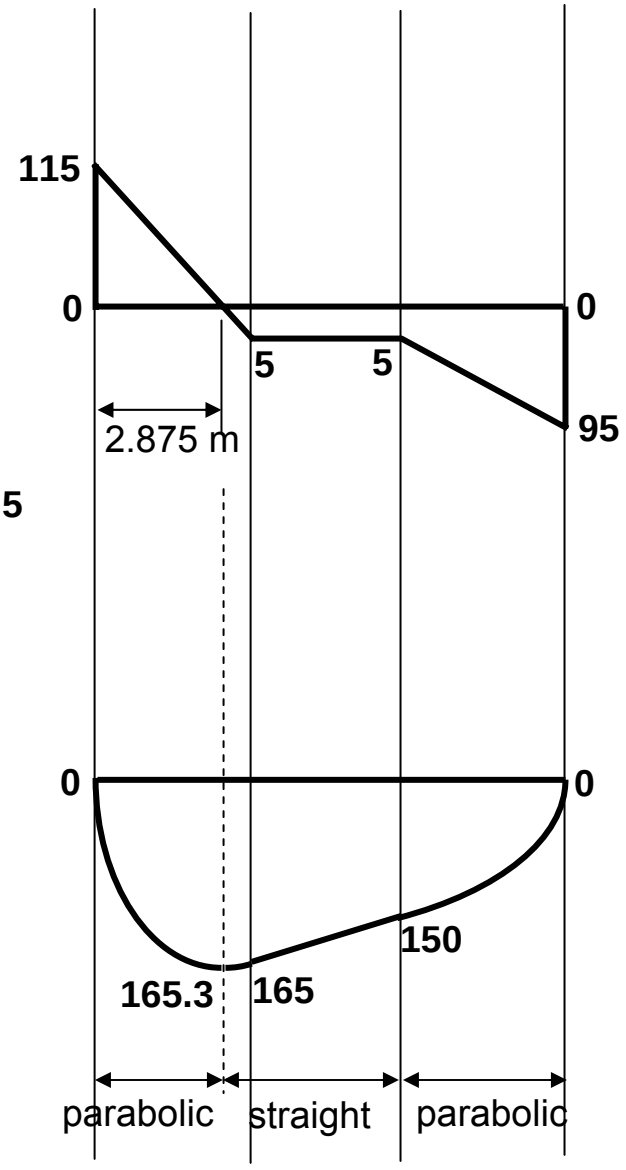

12
12
20
20
0
0
0
18
18
60
0
0
12


50 kN
10 kN/m
A
B
C
D
4 m
2 m
2 m
R_A
R_D


40 kN/m
30 kN/m
A
B
C
D
3 m
3 m
3 m
R_A
R_D

(c) **(d)**


SFD
42.5
0
2.5
2.5
0
47.5
47.5
BMD
0
0
90
95
parabolic
straight


115
0
0
5
5
95
2.875 m
0
0
165.3
165
150
parabolic
straight
parabolic