SOLUTIONS TO TUTORIAL EXAMPLES

CHAPTER 13

Note: The reader may find these solutions easier to follow if he/she marks the forces on a diagram of the frame as he/she proceeds through the calculations.

Question 1

Note the symmetry of both the frame and its loading. This means that only half of the members need to be analysed.

Calculation of reactions

Vertical equilibrium:

$R_A + R_J = 20 + 50 + 20 = 90$ kN.

Due to symmetry, $R_A = R_J = 90/2 = 45$ kN.

Determination of member forces

All angles are 45°. Sin 45° = 0.707, cos 45° = 0.707

Resolving vertically at A:

By inspection, force in member AB acts downwards.
$F_{AB} = 45$ kN (downwards).
So force in member AB = 45 kN (compression).

Resolving horizontally at A:

$F_{AC} = 0$ kN
So force in member AC = 0 kN.

Resolving vertically at B:

$F_{AB} = F_{BC} \times \sin 45°$
$45 = F_{BC} \times 0.707$
$F_{BC} = 45/0.707 = 63.6$ kN (downwards and to right)
So force in member BC = 63.6 kN (tension).

Resolving horizontally at B:

$F_{BD} = F_{BC} \cos 45° = 63.6 \times 0.707 = 45$ kN (to left).
So force in member BD = 45 kN (compression).

Resolving vertically at C:

Assume force in member CD is downwards at C.
$20 = F_{BC} \sin 45° - F_{CD}$.
$20 = (63.6 \times 0.707) - F_{CD}$
$F_{CD} = 45 - 20 = 25$ kN
So force in member CD = 25 kN (compression).

Resolving horizontally at C:

By inspection, force in member CE acts to the right.
$F_{AC} + F_{BC} \cos 45° = F_{CE}$.
$0 + (63.6 \times 0.707) = F_{CE}$
$F_{CE} = 45$ kN (to the right).
So force in member CE = 45 kN (tension).

Resolving vertically at D:

By inspection, force in member DE acts downwards to the right.
$F_{CD} = F_{DE} \sin 45°$
$25 = F_{DE} \times 0.707$
$F_{DE} = 25/0.707 = 35.4$ kN.
So force in member DE = 35.4 kN (tension).

Resolving horizontally at D:

By inspection, the force in member DF acts to the left (since the forces in members BD and DE act to the right).
$F_{BD} + F_{DE} \cos 45° = F_{DF}$.
$45 + (35.4 \times 0.707) = F_{DF}$.
$F_{DF} = 45 + 25 = 70$ kN (to the left).
So force in member DF = 70 kN (compression).

Resolving vertically at F:

$F_{FE} = 0$ kN.
So force in member FE = 0 kN.

Check: Resolving vertically at E:

$$50 = (2 \times 35.4 \sin 45°) + 0 = 50$$. This is correct.

Question 2

Calculation of reactions

Vertical equilibrium:

$$V_A + V_D = 20 + 30 = 50 \text{ kN}.$$

Taking moments about A:

$$9 \text{ m} \times V_D = (30 \text{ kN} \times 3 \text{ m}) + (20 \text{ kN} \times 6 \text{ m}) - (25 \text{ kN} \times 4 \text{ m})$$
$$9V_D = 90 + 120 – 100 = 110 \text{ kN}.$$
$$V_D = 110/9 = 12.2 \text{ kN}.$$

Taking moments about D:

$$9 \text{ m} \times V_A = (30 \text{ kN} \times 6\text{m}) + (25 \text{ kN} \times 4 \text{ m}) + (20 \text{ kN} \times 3 \text{ m})$$
$$9V_A = 180 + 100 + 60 = 340 \text{ kN}$$
$$V_A = 340/9 = 37.8 \text{ kN}.$$

Horizontal equilibrium:

$$H_D = 25 \text{ kN (to the right)}$$

Determination of member forces

All angles with horizontal are $\tan-1(4/3) = 53.1°$.
Sin $53.1° = 0.8$, cos $53.1° = 0.6$

Resolving vertically at A:

By inspection, force in member AB acts downwards and to the left.
$F_{AB} \sin 53.1° = 37.8$ kN.
$F_{AB} = 37.8/0.8 = 47.3$ kN.
So force in member AB = 47.3 kN (compression).

Resolving horizontally at A:

By inspection, force in member AF acts to the right.
$F_{AF} = F_{AB} \cos 53.1° = 47.3 \times 0.6 = 28.4$ kN.
So force in member AF = 28.4 kN (tension).

Resolving vertically at B:

By inspection, force in member BF acts downwards.
$F_{BF} = F_{AB} \sin 53.1° = 47.3 \times 0.8 = 37.8$ kN.
So force in member BF = 37.8 kN (tension).

Resolving horizontally at B:

Assume force in member BC acts to the left at B.
$F_{BC} = F_{AB} \cos 53.1° - 25 = (47.3 \times 0.6) - 25 = 3.4$ kN.
So force in member BC = 3.4 kN (compression).

Resolving vertically at F:

By inspection, force in member CF acts downwards and to left.
$F_{CF} \sin 53.1° = F_{BF} - 30 = 37.8 - 30 = 7.8$.
$F_{CF} = 7.8/0.8 = 9.75$ kN
So force in member CF = 9.75 kN (compression).

Resolving horizontally at F:

By inspection, the force in member FE acts to the right (since the forces in members AF and CF act to the left).
$F_{FE} = F_{AF} + F_{CF} \cos 53.1° = 28.4 + (9.75 \times 0.6) = 34.2$ kN.
So force in member FE = 34.2 kN (tension).

Resolving vertically at D:

By inspection, force in member CD acts downwards and to the right.
$F_{CD} \sin 53.1° = 12.2$ kN.
$F_{CD} = 12.2/0.8 = 15.25$ kN.
So force in member CD = 15.25 kN (compression).

Resolving horizontally at D:

By inspection, force in member ED acts to the left.
$F_{ED} = (F_{CD} \cos 53.1°) + 25 = (15.25 \times 0.6) + 25 = 34.2$ kN.
So force in member AF = 34.2 kN (tension).

Resolving vertically at E:

By inspection: $F_{CE} = 0$.

Check: Resolving vertically at C:

$20 = F_{CF} \sin 53.1° + F_{CE} + F_{CD} \sin 53.1°$
$20 = (9.75 \times 0.8) + 0 + (15.25 \times 0.8) = 20$. This is correct.

Question 3

Calculation of reactions

Vertical equilibrium:

$V_A + V_C = 18$ kN.

Taking moments about A:

$2 \text{ m} \times V_C = (6 \text{ kN} \times 2 \text{ m})$
$V_C = 6$ kN.

Taking moments about C:

$2 \text{ m x } V_A = (18 \text{ kN} \times 2 \text{ m}) - (6 \text{ kN} \times 2 \text{ m})$
$2V_A = 24$ kN.
$V_A = 12$ kN.

Horizontal equilibrium:

$H_A = 6$ kN (to the right)

Determination of member forces

All angles are 45°. Sin 45° = 0.707, cos 45° = 0.707

Resolving vertically at A:

By inspection, force in member AB acts downwards.
$F_{AB} = 12$ kN.
So force in member AB = 12 kN (compression).

Resolving horizontally at A:

By inspection, force in member AC acts to the left.
F_{AC} = 6 kN.
So force in member AC = 6 kN (compression).

Resolving horizontally at B:

By inspection, force in member BC acts upwards and to the left.
$F_{BC} \cos 45° = 6$ kN.
$F_{BC} = 6/0.707 = 8.5$ kN.
So force in member BC = 8.5 kN (compression).

Check: resolving vertically at C:

$F_{BC} \sin 45° = 6$ kN.
$F_{BC} = 6/0.707 = 8.5$ kN.
So force in member BC = 8.5 kN (compression) – as calculated before.

Question 4

Calculation of reactions

Vertical equilibrium:

$V_A + V_E = 50 + 30 + 20 = 100$ kN.

Taking moments about A:

$10 \text{ m} \times V_E = (25 \text{ kN} \times 5 \text{ m}) + (50 \text{ kN} \times 5 \text{ m}) + (30 \text{ kN} \times 10 \text{ m}) + (20 \text{ kN} \times 15 \text{ m})$
$10V_E = 125 + 250 + 300 + 300 = 975$
$V_E = 975/10 = 97.5$ kN.

Taking moments about E:

$10 \text{ m} \times V_A = (50 \text{ kN} \times 5 \text{ m}) - (25 \text{ kN} \times 5 \text{ m}) - (20 \text{ kN} \times 5 \text{ m})$
$10V_A = 250 - 125 - 100 = 25$
$V_A = 25/10 = 2.5$ kN.

Horizontal equilibrium:

H_A = 25 kN (to the left)

Determination of member forces

All angles are 45°. Sin 45° = 0.707, cos 45° = 0.707

Resolving vertically at A:

By inspection, force in member AB acts downwards.
F_{AB} = 2.5 kN.
So force in member AB = 2.5 kN (compression).

Resolving horizontally at A:

By inspection, force in member AC acts to the right.
F_{AC} = 25 kN.
So force in member AC = 25 kN (tension).

Resolving vertically at B:

By inspection, force in member BC acts downwards and to the right.
$F_{AB} = F_{BC} \sin 45°$.
$2.5 = F_{BC} \times 0.707$
$F_{BC} = 2.5/0.707 = 3.54$ kN.
So force in member BC = 3.54 kN (tension).

Resolving horizontally at B:

By inspection, the force in member BD acts to the left (since the external force at B (25 kN) and the force in member BC both act to the right).
$F_{BD} = 25 + F_{BC} \cos 45° = 25 + (3.54 \times 0.707) = 27.5$ kN
So force in member BD = 27.5 kN (compression).

Resolving vertically at C:

By inspection, the force in member CD acts downwards.
$F_{CD} = F_{BC} \sin 45° = (3.54 \times 0.707) = 2.5$ kN
So force in member CD = 2.5 kN (compression).

Resolving horizontally at C:

By inspection, the force in member CE acts to the right (since the forces in members BC and AC both act to the left).

$F_{CE} = F_{AC} + F_{BC} \cos 45° = 25 + (3.54 \times 0.707) = 27.5$ kN
So force in member CE = 27.5 kN (tension).

Resolving vertically at D:

By inspection, the force in member DE acts upwards and to the left.
$50 - F_{CD} = F_{DE} \sin 45°$
$50 – 2.5 = F_{DE} \times 0.707$
$F_{DE} = 47.5/0.707 = 67.2$ kN
So force in member DE = 67.2 kN (compression).

Resolving horizontally at D:

Assume the force in member DF acts to the right (this will be confirmed if the value for F_{DF} turns out to be positive).
$F_{DF} = F_{DE} \cos 45° - F_{BD} = (67.2 \times 0.707) - 27.5 = 20$ kN.
So force in member DF = 20 kN (tension).

Resolving vertically at H:

By inspection, force in member HG acts upwards.
$F_{HG} = 20$ kN.
So force in member HG = 20 kN (compression).

Resolving horizontally at H:

$F_{FH} = 0$ kN
So force in member FH = 0 kN.

Resolving vertically at G:

By inspection, force in member FG acts upwards and to the left.
$F_{GH} = F_{FG} \times \sin 45°$
$20 = F_{FG} \times 0.707$
$F_{FG} = 20/0.707 = 28.3$ kN (upwards and to left)
So force in member FG = 28.3 kN (tension).

Resolving horizontally at G:

By inspection, force in member EG acts to the right.
$F_{EG} = F_{FG} \cos 45° = 28.3 \times 0.707 = 20$ kN (to the right).
So force in member EG = 20 kN (compression).

Resolving vertically at F:

By inspection, the force in member EF at F must act upwards. This is because both the external force at F (30 kN) and the force in member FG act downwards.
$F_{EF} = 30 + F_{FG} \sin 45° = 30 + (28.3 \times 0.707) = 50$ kN.
So force in member EF = 50 kN (compression).

Check: resolving vertically at E:

$97.5 = F_{FE} + F_{DE} \sin 45° = 50 + (67.2 \times 0.707) = 97.5$ kN. Correct.

Check: resolving horizontally at E:

$F_{EG} + F_{CE} = F_{DE} \cos 45°$
$20 + 27.5 = (67.2 \times 0.707) = 47.5$ kN. This is correct.