PART I

Introduction to Monetary Economics

In the first part of this book we use standard monetary models to talk about the joint behavior of nominal and real variables. We start with the long-run relationship focusing on the relationship between money and inflation. The focus then shifts to the short-run relationship between money and output. Special attention is devoted to the choice of fiscal and monetary policy. This introductory part sets the stage for a less standard approach in the third part of the book.

CHAPTER 1

Overview

Monetary economics is about the relationship between real and nominal variables. Its aim is to develop and test models that can help in evaluating the effects of policy on inflation, employment, nominal and real interest rates and production. We start with some evidence on the relationship between nominal and real variables and then describe some of the questions that occupy monetary economics.

1.1 MONEY, INFLATION, AND OUTPUT: SOME EMPIRICAL EVIDENCE

In his Nobel lecture Lucas (1996) summarizes the long-run effects of changes in the money supply with the aid of two figures, taken from McCandless and Weber (1995). These figures are reproduced here. Figure 1.1 plots 30-year (1960–90) average annual inflation rates against average annual growth rates of M2 (currency + demand deposits + time deposits) over the same 30-year period, for a total of 110 countries. Figure 1.2 has the growth of real output on the vertical axis. We see a high correlation between average money growth and average inflation but no correlation between average money growth and average real growth.

Consider now the point of view of a government or a central bank that wants to choose a point in the figure 1.1. Should it choose a point of low inflation and low money supply growth or should it choose a point of high inflation and high money growth? Does this matter? These questions will occupy us in the first part of the book.

As was said before figure 1.2 suggests no long-run relationship between money and output. Is there a short run relationship? Figure 1.3 is a scattered diagram of the unemployment and (CPI) inflation in the US during the years 1959:1–1999:2. Figure 1.3 uses quarterly data and each point represents unemployment and inflation in a single quarter. The first four scatter diagrams are done for each decade separately. It is evident that in the 1960s there was a negative relationship between inflation and unemployment but this relationship disappeared later. The last scatter diagram uses all observations and shows no relationship between inflation and unemployment. Stockman (1996) and Lucas (1996) obtain similar conclusions for different time periods.

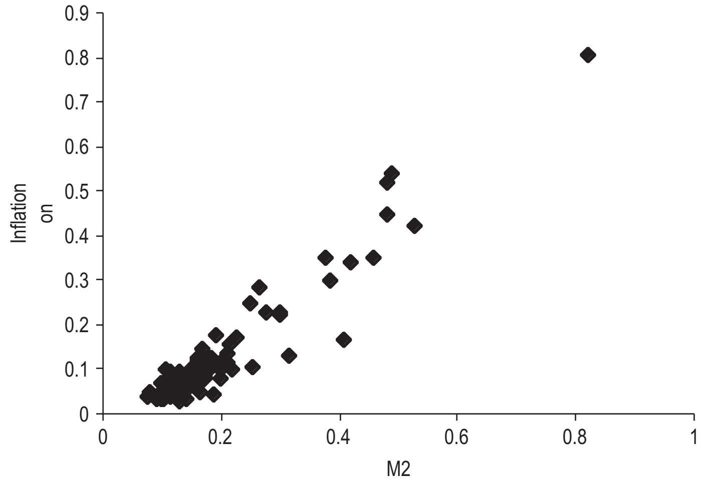

Figure 1.1 M2 growth and inflation

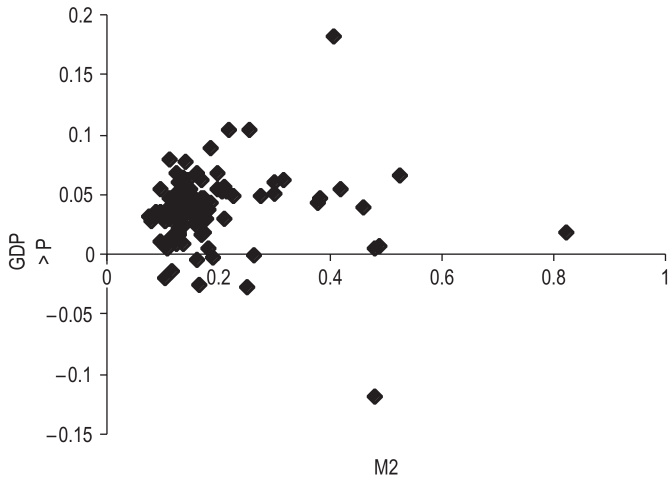

Figure 1.2 M2 and real GDP: growth rates

We may want to look at deviation of the rate of inflation from its expected value. The problem is that expected inflation cannot be directly observed. One possibility is to assume that the expected inflation is equal to the trend estimated by the Hodrick-Prescott (HP) filter (see the appendix to chapter 17). Baxter and King (1995) and Sargent (1997) use other filters and get similar results.

Figure 1.4 uses the difference of the variables from their trend value: detrended variables. As can be seen from figure 1.4, once we look at detrended variables we do get a strong and significant relationship between detrended inflation and detrended unemployment.

We now turn to some policy issues.

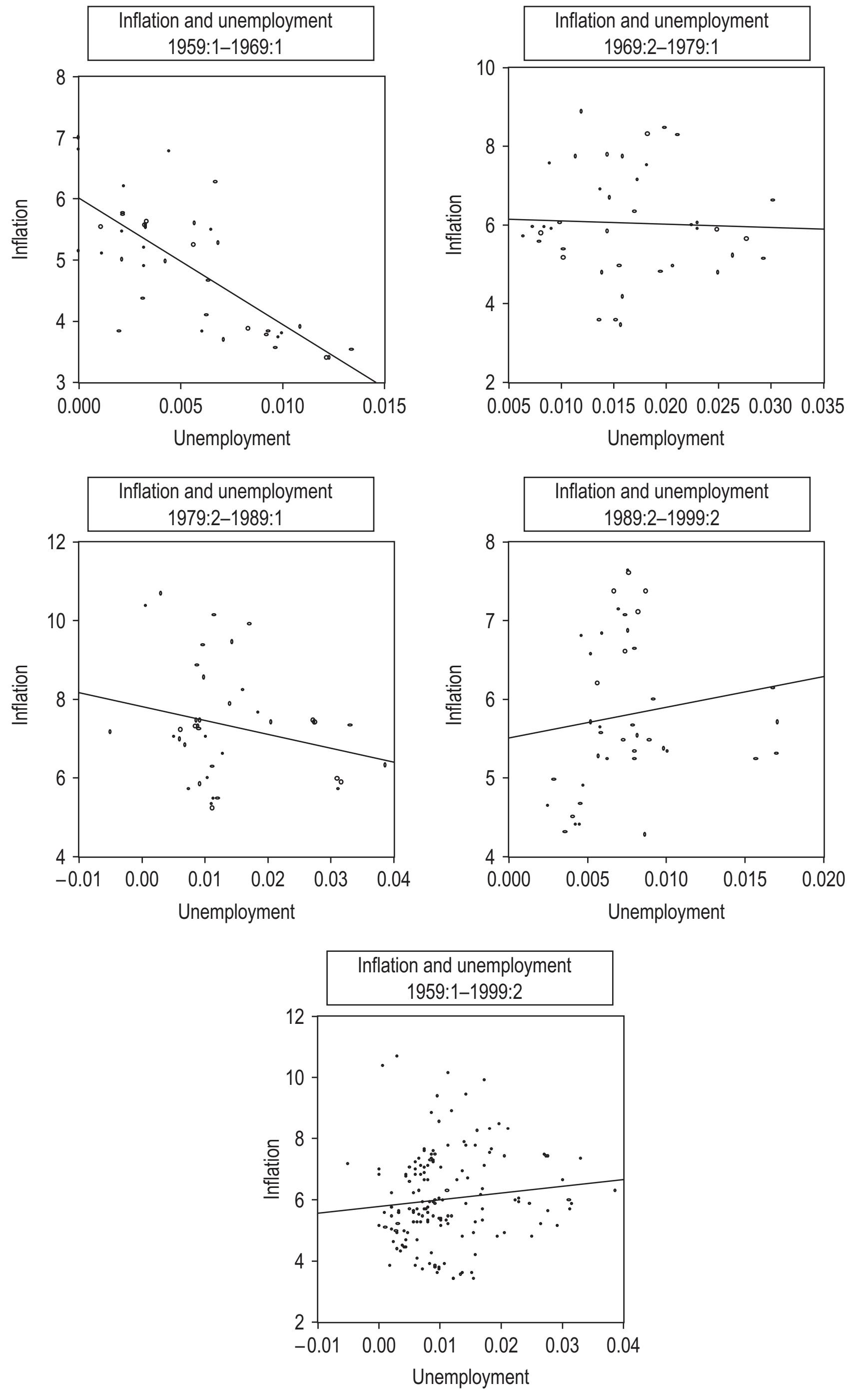

Figure 1.3 Phillips curves for various time periods

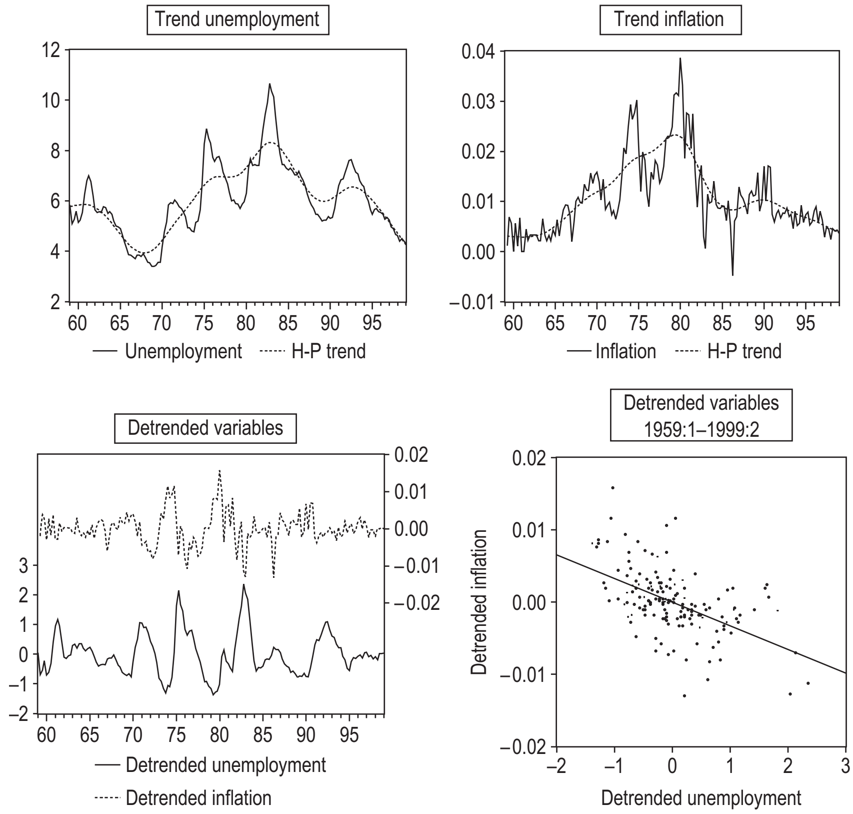

Figure 1.4 Using detrended variables

1.2 THE POLICY DEBATE

Can and should monetary policy be used to smooth output? In his essay "Of Money", David Hume observed that an increase in the money supply is "favourable to industry . . . At first, no alteration is perceived; by degrees the price rises, first of one commodity, then of another; till the whole at last reaches a just proportion with the new quantity of specie which is in the kingdom. In my opinion, it is only in this interval or intermediate situation, between the acquisition of money and rise of prices, that the encreasing quantity of gold and silver is favourable to industry" (1752, p. 38 in the 1970 edition). David Hume's conclusion with respect to the money supply is that: "The good policy of the magistrate consists only in keeping it, if possible, still encreasing; because by that means, he keeps alive a spirit of industry in the nation" (p. 39).

Figure 1.5 illustrates Hume's view by plotting the logs of money, prices and output against time. After the increase in the money supply prices rise gradually until they reach a new level proportional to the new money supply. During the period in which prices rise there is an

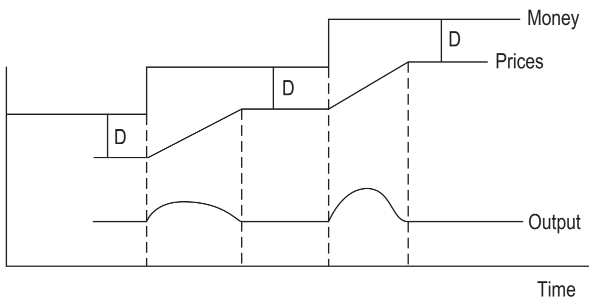

Figure 1.5 Hume's view

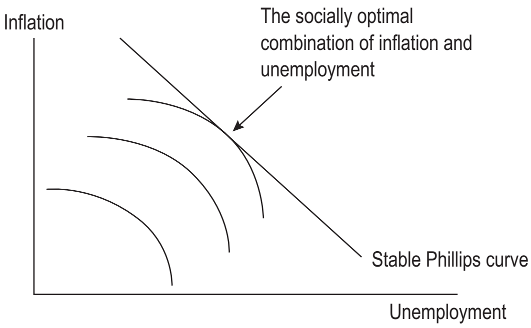

Figure 1.6 The stable Phillips curve view

increase in output but output is back to normal when prices reach the new equilibrium level. Hume suggests keeping the "spirit of industry in the nation" by increasing the money supply from time to time (or maybe increase it continuously).

In the twentieth century Phillips (1958) found a negative correlation between (wage) inflation and the rate of unemployment in British data. The Phillips curve – the relationship between inflation and unemployment – was initially accepted as a stable relationship along which policy makers could select a point. Figure 1.6 illustrates this view. The Phillips curve is drawn as a stable line. The policy-maker's indifference curves are concave to the origin and indifference curves that are closer to the origin are associated with a higher level of social welfare because both inflation and unemployment are harmful or "bads."

The stable Phillips curve view runs into difficulties when thinking about perfectly anticipated inflation. To illustrate, let us consider a hypothetical currency reform that promises to give 1.1 "new" dollars for an "old" dollar. It is expected that 1.1 "new" dollars will buy the same amount as 1 "old" dollar and therefore prices will go up by 10% immediately after the currency reform. This gimmick should have no effect on economic activity.

Friedman (1968) and Phelps (1968) make the distinction between expected and unexpected inflation. Their "natural rate hypothesis" says that in the long run, when expected inflation is the same as actual inflation, the rate of unemployment does not depend on the rate of

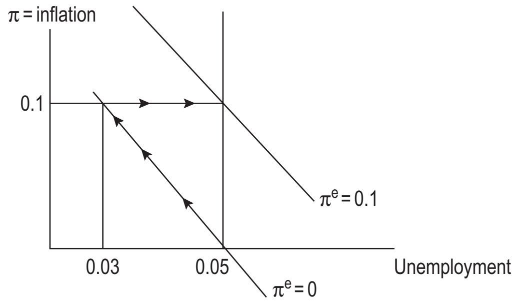

Figure 1.7 The natural rate hypothesis

inflation: The rate of unemployment when actual and expected inflation is 10% is the same as the rate of unemployment when actual and expected inflation is 20%.

The rate of unemployment which occurs when actual and expected inflation are the same is called the "natural rate of unemployment". The reasons for unemployment at the natural rate level are not in monetary policy. They are in various frictions in the process of matching workers to jobs. For example, a worker may be unemployed during the period in which he searches for a "more suitable job" because it takes time to find a new suitable job.

For the sake of concreteness let us assume that the natural rate of unemployment is 5% of the labor force, as in figure 1.7. When inflation is higher than expected workers are "excited" by the higher wages. For example, if workers expect zero inflation and the actual inflation is 10% they may be happy with their job even if they are promised a raise of less than 10% in their money wages. These workers are less likely to quit and search for a better job. Similarly firms that raise prices by 10% are less likely to fire workers who are willing to accept a less than 10% increase in their money wages. As a result, the rate of unemployment is reduced to say 3%. In terms of the graph of figure 1.7, we move on a short run Phillips curve that holds expectations fixed at $\pi^e = 0$. Eventually, however, workers learn that the rate of inflation is 10% and form correct expectations about their real wage. The rate of unemployment goes back to its normal 5% rate.

Now we are on a new short run curve that holds expectations constant at the higher level of $\pi^e = 10\%$. If we want to go back to zero inflation we must travel along this short run curve and endure unemployment which is higher than the natural rate: If when everyone expects 10% we have 5% inflation, workers who get a 5% raise will be disappointed and some will search for better jobs.

Note that the Friedman–Phelps natural rate hypothesis can be used to explain the strong negative correlation between detrended unemployment and detrended inflation (figure 1.4 in section 1.1). This explanation requires that: (1) the natural rate of unemployment is accurately measured by trend unemployment and (2) expected inflation is accurately measured by trend inflation.

Why do money surprises have real effects? This question is still not completely resolved. Friedman argued that it takes time for workers to learn about the general price level while the firm needs to know only the price of its own product. Therefore it is possible that immediately after a monetary injection the wage rate rises by less than prices but nevertheless workers

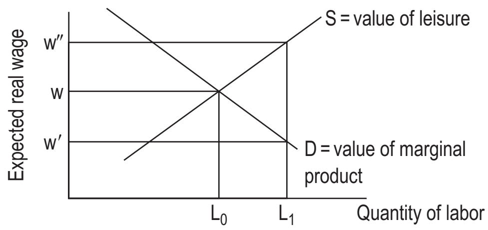

Figure 1.8 In the short run workers use the wrong deflator

think that their real wage went up. As a result workers are willing to work more and firms are willing to employ more labor.

To illustrate Friedman's argument, let us start from a long run equilibrium with $\pi^e = \pi = 0$; a nominal wage rate W, a price level P and a real wage $w = W/P$. At this real wage, workers supply L_0 units of labor as in figure 1.8. Suppose now that prices go up by 10% and the nominal wage rate goes up by 5%. At first, workers are "being fooled" and think that the real wage is $w'' = 1.05\,W/P = 1.05\,w$. As a result they are willing to supply a larger quantity of labor: L_1 in figure 1.8. The firm is perfectly informed about the price of its own product and knows that the actual real wage is $w' = 1.05\,W/1.1\,P < w$. It is therefore willing to employ a larger quantity of labor. We may even get market-clearing in the short run but this is not guaranteed. In the long run workers learn to deflate the nominal wage rate by the appropriate number and the labor market returns to the long run equilibrium in which the quantity of labor employed is L_0 and the real wage is w.

This story is not complete. Presumably the increase in money wages and prices is due to money injection. But why does the wage rate rise less than prices after an injection of money? Is it the assumption that wages are stickier than prices?

Lucas (1972) provides a complete description of the way money injections affect economic activity. A Walrasian auctioneer who knows the realizations of supply and demand announces the market-clearing price and agents use the information in this announcement. Lucas shows that even if workers know the model and form expectations by using the appropriate statistical procedures they may still respond to a purely nominal increase in their wage rate: They may attribute part of the change to real increase in the demand for their services and therefore expect that the price of the service they supply went up by more than the prices of other services that they buy (the price level).

For example, suppose that a teacher was getting 10 dollars an hour for private violin lessons. He now gets offers to teach for 20 dollars an hour. This increase may be explained as the result of doubling the money supply or as a result of an increase in the real demand for violin lessons or a combination of these factors. Optimal inference (signal extraction) will in general point to a combination of the two factors: say the price went up by 5 dollars because of monetary reasons and by 5 dollars because of real reasons. The violin teacher who makes this inference will conclude that his real wage went up by 33% and will therefore supply more lessons.

Thus, changes in the money supply may have real effects because agents in the economy are not perfectly informed about the reason for the change and attribute part of it to real changes in the demand for their product.

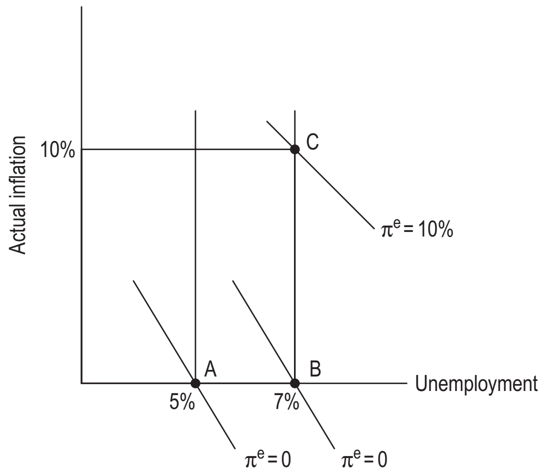

Figure 1.9 Perfectly anticipated feedback rule

An important implication is that only surprise changes in the money supply have real effects. For example, consider again the hypothetical case in which the central bank increases the money supply by 10% at the beginning of each year and as a result prices go up by 10% at the beginning of the year. In this hypothetical case, our violin teacher will know that the rise in the price of violin lessons at the beginning of the year is fully explained by the increase in the money supply. He will therefore not supply more lessons and in general economic activity will not be affected.

Similarly when the central bank increases the money supply whenever the rate of unemployment goes up, the increase in the money supply should be fully expected by agents who follow the unemployment statistics. Therefore such a transparent policy will only raise the inflation rate without affecting the probability distribution of the rate of unemployment.

To illustrate, assume that the natural rate of unemployment fluctuates randomly and may take two possible realizations: 5% and 7%. When the central bank adopts a consistent policy of zero inflation, the economy will be either at point A or at point B in figure 1.9. When it adopts a policy of creating 10% inflation whenever the rate of unemployment is 7%, agents will expect a 10% inflation whenever they observe a 7% unemployment rate. As a result we will move between points A and C in figure 1.9. Thus the central bank's attempt at reducing unemployment will lead only to inflation because the central bank's policy lacks the surprise element.

When this reasoning is generalized it leads to the conclusion that a feedback rule from unemployment to the rate of inflation will not work. Using a rather standard Keynesian type model, Sargent and Wallace (1975) show that if the central bank specifies the rate of change in the money supply as a function of the rate of unemployment, and the public understand this policy and has the same information as the central bank, then the central bank's policy will not affect the unemployment rate.

The impossibility of fighting unemployment by inflation (using a feedback rule) requires flexible prices, rational expectations and the absence of informational advantage. The debate over the effectiveness of a feedback rule is not over.

A crude summary of the main positions in the policy debate is as follows:

1 It is not possible to fight unemployment by inflation either in the short run or in the long run (Lucas, Sargent and Wallace);
2 It is possible to fight unemployment by inflation only in the short run. This is not desirable because the price of reducing the rate of inflation back to zero is too high and so is the cost of staying with high inflation rates (Friedman);
3 It is possible and desirable to fight unemployment by inflation.

1.3 MODELING ISSUES

In modeling monetary economies one often runs into the following questions: (1) why do agents hold money; (2) why do agents hold inventories and (3) how do prices adjust to various shocks?

Money

The first question arises because government bonds have the same risk characteristic as money and yield a positive nominal rate of return. Why do agents hold money when this dominating alternative is available? The answer to this question has to do with various frictions in the economy. But modeling friction is not simple. We want a model in which there are enough "frictions" so that agents choose to hold money and is still simple enough so that it can be used for analyzing the effects of alternative policies.

In the first few chapters of this book, we describe various ways of modeling money and ask whether the main policy conclusions are robust in the way in which the rate of return dominance question is solved. But we do not go into a deep discussion of the micro foundations of monetary models.

Inventories

There is wide agreement on the importance of changes in inventory investment in cyclical fluctuations. Blinder and Maccini (1991) found that the drop in inventory investment accounted for 87% of the drop in GNP during the average postwar recession in the United States (see their tables 1 and 2). There is much less agreement on how to model inventory behavior. According to the standard competitive model, inventories are held only when the expected increase in price covers storage and interest costs. This condition will not often hold but we do observe that inventories are almost always held.

The puzzle of why inventories are almost always held is similar to the money holding (rate of return dominance) puzzle. One possibility is that inventories yield convenience. This is analogous to assuming that money yields liquidity services (see chapter 2).

In the second part of the book we study the uncertain and sequential trade (UST) model. In this model sellers may fail to make a sale. When sellers fail to make a sale "undesired" inventories are carried to the next period. For this reason inventories are almost always held in the UST model and this result is obtained without having to assume convenience yield.

Prices

The short run determination of prices has strong implications for the way money affects real variables. We may distinguish between flexible prices, rigid prices and seemingly rigid prices.

Flexible prices: There is an important literature that is basically happy with the Walrasian model. This literature introduces frictions to get economic agents to hold money and to get a short run effect of money on output. Lucas's (1972) confusion model is a good example. To get money into the model Lucas assumes the overlapping generations frictions: Not all agents meet at the beginning of time. To get money non-neutrality he assumes that agents who belong to the same generation do not meet in a central Walrasian auction-place: They are distributed over disconnected islands. Another example is the limited participation model of Grossman and Weiss (1983), Lucas (1990), Fuerst (1992) and Christiano and Eichenbaum (1992). In these models money is not neutral because some agents cannot immediately adjust the amount of money balances they hold.

Rigid or sticky prices: This approach assumes that prices are set in advance. For example, Gray (1976, 1978), Fischer (1977, 1979) and Taylor (1980) develop models of the labor market in which wages are set in advance and labor unions supply the demand of the firm at the sticky wage. Phelps and Taylor (1977) and McCallum (1989, ch. 10.2) develop models in which firms set prices in advance at the level equal to the expected market-clearing price. They then supplied the quantity demanded at that price. Recently there has been a growing realization that some aspects of imperfect competition must be incorporated into a model that assumes price rigidity. Imperfect competition has become the trademark of the New Keynesian economics. See for example, Blanchard and Kiyotaki (1987), Ball and Romer (1991) and King and Watson (1996).

Seemingly rigid prices: In the uncertain and sequential trade (UST) model, prices may seem rigid but agents do not face any cost for changing prices. This approach which is developed in this book tends to yield Keynesian type behavior of inventories and prices but neoclassical policy implications.

1.4 BACKGROUND MATERIAL

In this section we introduce (review) some of the key ingredients of the monetary models used in this book: The Fisherian diagram, efficiency of the competitive outcome, the effect of distortive taxes and a non-stochastic version of the Lucas tree model (asset pricing). The last topic illustrates the working of an infinite horizon representative agent economy. The reader may choose to read this part after he or she masters the material in chapter 2.

1.4.1 The Fisherian diagram

We assume a consumer who lives for two periods ($t = 1, 2$) and consumes a single good (corn). The consumer's utility function is:

$$u(C_1, C_2) = U(C_1) + \beta U(C_2), \tag{1.1}$$

where C_t is corn consumption at time t and $0 < \beta < 1$ is a discount factor. The single period utility function U( ) is monotone and strictly concave ($U' > 0$ and $U'' < 0$).

The consumer can lend and borrow at the interest rate r. This means that if the consumer reduces his current consumption by one unit and lends it he will get $1+r$ units of consumption in the next period. If he borrows a unit he will have to pay $1 + r$ units in the next period. The price of first period's corn in terms of second period's corn is therefore $R = 1 + r$ where R is the gross interest rate.

The consumer gets an endowment of Y_t units of corn at time t. The value of the endowment in terms of second period consumption is: $RY_1 + Y_2$. The value of the consumption bundle cannot be larger than the value of the endowment and therefore the budget constraint is:

$$RC_1 + C_2 \leq RY_1 + Y_2 = W, \tag{1.2}$$

where W is the consumer's wealth in terms of future consumption (future value). To express the budget constraint in terms of present value we divide both sides of (1.2) by R.

The consumer maximizes (1.1) subject to (1.2) and non-negativity constraints. One way of solving the problem is by substituting $C_2 = W - RC_1$ from (1.2) into (1.1). This leads to:

$$\max_{C_1} F(C_1) = U(C_1) + \beta U(W - RC_1). \tag{1.3}$$

This is a maximization problem with one variable. We now take a derivative of F( ) and equate it to zero to find the first order condition:

$$F'(C_1) = U'(C_1) - \beta RU'(W - RC_1) = 0. \tag{1.4}$$

We may write (1.4) as:

$$U'(C_1) = \beta RU'(C_2). \tag{1.5}$$

Note that a small deviation from the optimal path does not change the level of the objective function: At the optimum $\hat{C}_1$ the slope of F( ) is zero and changing C_1 a bit does not change the value of F( ). Figure 1.10 illustrates this point.

This basic principle from calculus can be used to derive the first order condition (1.5) in an alternative way. We consider a small feasible deviation from the proposed optimal path ($\hat{C}_1$, $\hat{C}_2$). We cut current consumption by x units and lend it. This will result in increasing future consumption by Rx units. When x is small, the loss of current utility (the pain) is the marginal utility of current consumption times the change in current consumption: $xU'(C_1)$. Similarly the present value of the gain in future utility (the gain) is: $xR\beta U'(C_2)$. Since at the

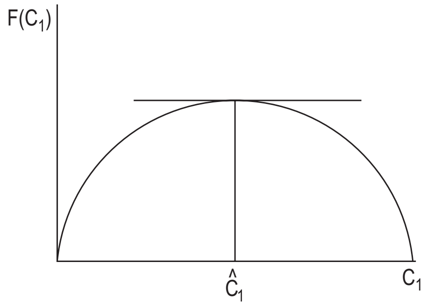

Figure 1.10 Small deviations from the optimum do not change the value of the problem

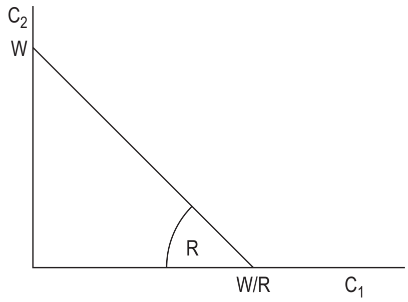

Figure 1.11 The budget line

optimum a small deviation should not change the level of the objective function the gain should equal the pain and this leads to (1.5).

The gain = pain method (sometimes refered to as calculus of variation) is more intuitive but difficult to implement. The difficulty is to think of an "easy feasible deviation" from the optimal path. Most economists proceed by deriving the first order condition in a technical way (using the substitution or the lagrangian method) and then provide an intuitive gain = pain explanation. To develop intuition we use, in this book the gain = pain method as a way of deriving first order conditions.

To solve the problem graphically we now draw the budget line: $RC_1 + C_2 = W$, in the (C_1, C_2) plane. The intersection with the horizontal axis is obtained by setting $C_2 = 0$. This leads to: $C_1 = W/R$ (as in figure 1.11). The intersection with the vertical axis is obtained by setting $C_1 = 0$. This leads to: $C_2 = W$. The slope of the budget line is obtained by taking the distance from zero to the intersection with the vertical axis and dividing it by the distance from zero to the intersection with the horizontal axis. This leads to: $W/(W/R) = R$. Thus the slope of the budget line is equal to the price of current consumption in terms of future consumption.

All points on the budget line are feasible. Points to the south west of the budget line (characterized by $RC_1 + C_2 < W$) are also feasible. Points to the north east of the budget line (characterized by $RC_1 + C_2 > W$) are not feasible.

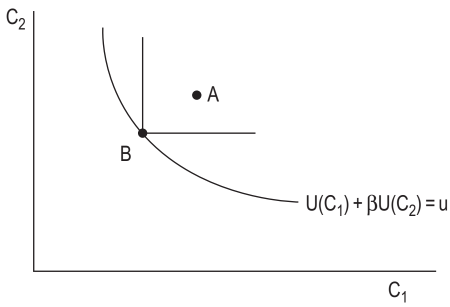

Figure 1.12 An indifference curve

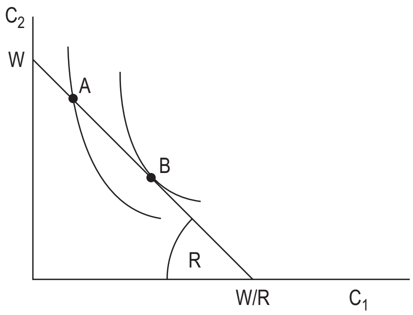

Figure 1.13 The optimal choice

We now define an indifference curve by the locus of points (C_1, C_2) that satisfy:

$$U(C_1) + \beta U(C_2) = u, \tag{1.6}$$

where here u is a given constant. All the points (C_1, C_2) that satisfy (1.6) promise the same utility level u and are illustrated by the curve that passes through point B in figure 1.12. Since the consumer is indifferent among all these points this curve is called an indifference curve. Since the utility function is strictly monotone the indifference curve is downward sloping: If we reduce C_2 we must compensate by increasing C_1 to get the same utility level. We can also verify that all points which are above the indifference curve are strictly preferred to points on the indifference curve: For each point like A, we can find a point B on the indifference curve that has less of both goods. Similarly, a point on the indifference curve is preferred to a point below the indifference curve.

At the optimal choice of consumption the indifference curve must be tangent to the budget line. Point A in figure 1.13 cannot be optimal because the consumer can get a point that is on his budget line and above the indifference curve that passes through point A. An improvement is not possible if we are at point B.

From a technical point of view, the tangency condition means that the slope of the budget line must be equal to the slope of the indifference curve that passes through the optimal point (B in figure 1.13). To find the slope of the indifference curve we take a full differential of (1.6).

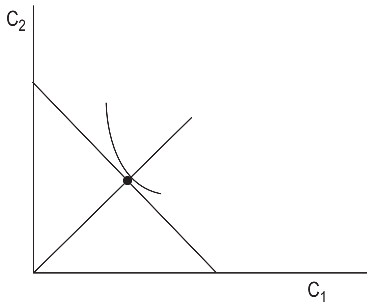

Figure 1.14 $\rho = r$

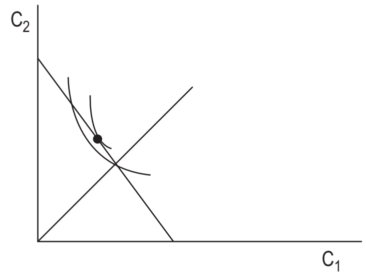

Figure 1.15 $\rho > r$

This yields:

$$U'(C_1)\,dC_1 + \beta U'(C_2)\,dC_2 = du = 0. \tag{1.7}$$

We can now solve for the slope:

$$-dC_2/dC_1 = U'(C_1)/\beta U'(C_2). \tag{1.8}$$

Equating the slope of the indifference curve (1.8) to the slope of the budget line (R) yields the first order condition (1.5).

The parameter β*:* To develop some intuition about the rate of discount β we define the subjective interest rate ρ by $\beta = 1/(1+\rho)$. For a consumer whose $\rho = r$ we have $\beta R = 1$ and therefore the first order condition (1.5) implies $U'(C_1) = U'(C_2)$ and $C_1 = C_2$. Figure 1.14 illustrates this case.

For a consumer whose subjective interest rate is greater than the market interest rate ($\rho > r$) we get $\beta R < 1$. In this case (1.5) implies $U'(C_1) < U'(C_2)$ and since $U'' < 0$, $C_1 > C_2$. Figure 1.15 illustrates this case. Similarly when $\rho < r$, $C_1 < C_2$.

1.4.2 Efficiency and distortive taxes

To make policy choices we want to know if the economy works well without intervention and if not can something be done about it. As a background material we now discuss an example that demonstrates the efficiency properties of the competitive outcome (or Adam Smith's invisible hand theorem). We then use the example to talk about tax distortion. Understanding tax distortion is important for its own sake because the tax system is an important policy choice. It turns out that understanding tax distortion is also important for understanding other cases in which the invisible hand theorem does not work and there are "market failures."

We consider an economy with many identical individuals who live for one period and consume a single good (corn). Each agent owns a firm but cannot work in his own firm.

The representative firm uses labor input l to produce corn (there is no capital). Production is done according to:

$$y = f(l), \tag{1.9}$$

where y is the quantity of corn produced, l is labor input and f( ) is a monotone and strictly concave production function ($f' > 0$ and $f'' < 0$).

The firm's profits are given by:

$$\pi = f(l) - wl, \tag{1.10}$$

where w is the wage rate (in units of corn per hour). The firm takes w as given and chooses l to maximize (1.10). The first order necessary condition that an interior solution ($l > 0$) to (1.10) must satisfy is:

$$f'(l) = w. \tag{1.11}$$

This says that at the optimum the marginal product must equal the wage rate.

The representative individual likes corn but does not like to work. We assume the special utility function:

$$y - v(L), \tag{1.12}$$

where y is the quantity of corn consumption, L is labor supply and v(L) is a standard cost function ($v' > 0$ and $v'' > 0$). The representative individual takes w and π as given and maximizes (1.12) subject to the budget constraint:

$$y = wL + \pi. \tag{1.13}$$

The first order condition for an interior solution ($L > 0$) to this problem is:

$$v'(L) = w. \tag{1.14}$$

This says that at the optimum the marginal utility cost must equal the wage rate.

In equilibrium the quantity of labor demanded by the firm is equal to the quantity of labor supplied by the individual. More formally,

Equilibrium is a vector (l, L, w) that satisfies the first order conditions (1.11) and (1.14) and the market clearing condition $l = L$.

We substitute (1.11) and $l = L$ in (1.14) and obtain the equilibrium condition:

$$f'(L) = v'(L). \tag{1.15}$$

Figure 1.16 illustrates the solution $\hat{L}$ to (1.15).

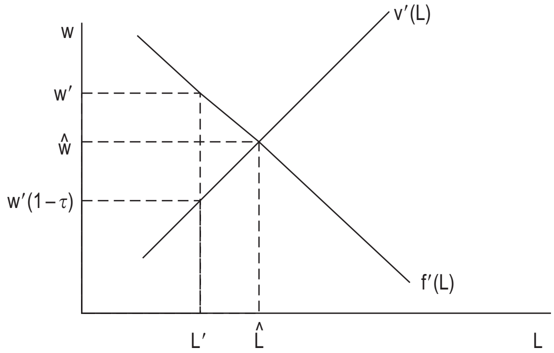

Figure 1.16 Distortive income tax

Efficiency: We now think of a hypothetical central planner who chooses L to maximize the welfare of the representative individual. The planner solves:

$$\max_{L} f(L) - v(L). \tag{1.16}$$

The first order condition for the problem (1.16) is (1.15) and therefore the equilibrium outcome $\hat{L}$ is a solution to (1.16). This means that the equilibrium outcome is efficient in the sense that a hypothetical planner cannot improve on it.

Tax distortion

We now introduce a government that collects income tax from the individuals and give it back to them as a transfer payment. The budget constraint (1.13) is now:

$$y = (1 - \tau)wL + \pi + g, \tag{1.17}$$

where $0 \leq \tau < 1$ is the income tax rate and g is the transfer payment from the government. The first order condition for the individual problem of maximizing (1.12) subject to (1.17) is:

$$v'(L) = (1 - \tau)w. \tag{1.18}$$

This says that at the optimum the marginal utility cost equals the net (after tax) wage rate.

Equilibrium with tax distortion is a vector (l, L, w) that satisfies the first order conditions (1.11) and (1.18) and the market clearing condition $l = L$.

We substitute (1.11) and $l = L$ in (1.18) and obtain the equilibrium condition:

$$v'(L) = (1 - \tau)f'(L). \tag{1.19}$$

The solution to (1.19) is $L' < \hat{L}$ in figure 1.16. It does not solve the planner's problem (1.16). The reason is in the discrepancy between the price of leisure from the individual's and the social point of view. From the individual point of view, a unit of leisure costs $w(1 - \tau) = f'(1 - \tau)$ units of corn. From the social point of view it costs $f' = w$ units of corn.

We elaborate on the choice of taxes in chapter 6. For now the reason for the inefficiency is important because it will help us understand the inefficiency caused by inflation (or more accurately by a positive nominal interest rate).

1.4.3 Asset pricing

In monetary economics we often want to abstract from distributional issues and some problem which occurs when the world ends at a finite date. We therefore often use a model in which many identical agents live for ever. In this infinite horizon representative agent economy there is no trade in equilibrium. We now demonstrate the working of such a model by considering a deterministic version of the Lucas' tree model. In chapter 13 we consider a more general version.

There are many infinitely lived identical individuals. Each individual owns a tree. All the trees are identical and promise the same path of fruits $\{d_t\}_{t=1}^{\infty}$, where d_t is the amount of fruits given by a tree at time t. At $t = 1$ the representative agent receives d_1 units of fruits as dividends. After receiving the dividends he can sell his tree for p_1 units of fruits. The total resources he has at the beginning of period 1 are thus: $p_1 + d_1$. He can use these resources for consumption (C_1) and for acquiring trees (A_1) that will give him fruits in period 2. His period 1 budget constraint is: $p_1 A_1 + C_1 = p_1 + d_1$. In general the asset evolution equation is: $p_t A_t = p_t A_{t-1} + d_t A_{t-1} - C_t$. The consumer's utility function is $\sum_{t=1}^{\infty} \beta^t U(C_t)$, where $0 < \beta < 1$ and the single period utility function is strictly monotone and concave: $U' > 0$ and $U'' \leq 0$.

The representative agent takes the stream of dividends per tree $\{d_t\}_{t=1}^{\infty}$ and the path of prices $\{p_t\}_{t=1}^{\infty}$ as given and chooses the path of tree ownership $\{A_t\}_{t=1}^{\infty}$. This is done by solving the following problem:[1]

$$\max_{A_t} \sum_{t=1}^{\infty} \beta^t U(C_t)$$

$$\text{s.t.} \quad p_t A_t = p_t A_{t-1} + d_t A_{t-1} - C_t$$

$$C_t,\ A_t \geq 0 \quad \text{and} \quad A_0 = 1 \text{ is given.} \tag{1.20}$$

Suppose that $\{\hat{C}_\tau\}_{\tau=1}^{\infty}$ is an interior solution ($\hat{C}_\tau > 0$) to (1.20). To derive the first order condition we consider the following feasible deviation: Cut consumption at t by x units, use it to buy x/p_t trees and never sell the additional trees. The additional trees yield the infinite stream of dividends $\{(x/p_t)d_\tau\}_{\tau=t+1}^{\infty}$ which are used to augment consumption. This deviation will change the consumption path from $\{\hat{C}_\tau\}_{\tau=1}^{\infty}$ to $\{C_\tau\}_{\tau=1}^{\infty}$, where

$$C_\tau = \hat{C}_\tau \quad \text{for } \tau \leq t-1; \qquad C_t = \hat{C}_t - x \quad \text{and} \quad C_\tau = \hat{C}_\tau + (x/p_t)d_\tau \quad \text{for } \tau > t.$$

For small x, the loss in utility (the pain) associated with the proposed deviation is: $\beta^t U'(\hat{C}_t)x$. The gain in utility is: $\sum_{\tau=t+1}^{\infty} \beta^\tau U'(\hat{C}_\tau)(x/p_t)d_\tau$. At the optimum a small deviation should not make a difference in the objective function and therefore:

$$\beta^t U'(\hat{C}_t)x = \sum_{\tau=t+1}^{\infty} \beta^\tau U'(\hat{C}_\tau)(x/p_t)d_\tau. \tag{1.21}$$

Rearranging (1.21) yields:

$$p_t = \sum_{\tau=t+1}^{\infty} \beta^{\tau-t}\{U'(\hat{C}_\tau)/U'(\hat{C}_t)\}d_\tau. \tag{1.22}$$

Since there is one tree per agent, consumption per agent is $C_\tau = d_\tau$ for all τ. Substituting this in (1.22) leads to the equilibrium condition:

$$p_t = \sum_{\tau=t+1}^{\infty} \beta^{\tau-t}\{U'(d_\tau)/U'(d_t)\}d_\tau. \tag{1.23}$$

When the price satisfies (1.23) everyone wants to hold one tree and consume the stream of dividends that it promises. To appreciate the equilibrium concept it is useful to consider cases in which (1.23) does not hold. For example, when the price is less than (1.23) the right hand side of (1.21) is greater than the left hand side of (1.21). This means that the gain from cutting corn consumption and buying trees is larger than the pain. Everyone wants to sell fruits for trees but no one wants to buy fruits. This excess supply of fruits works in the direction of reducing the price of fruits and increasing the price of trees.

Note that when U is linear and U' is a constant, the price of the asset is the expected discounted sum of the future dividends that it promises: $p_t = \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} d_\tau$. This will also be the case if $d_\tau = d_t$ for all τ and consumption is perfectly smooth.

Many different trees

We now assume n types of trees. A type i tree promises the stream of dividends $\{d_{ti}\}_{t=1}^{\infty}$ and its price at time t is p_{ti}. Each individual starts with a portfolio of n trees: one tree from each type. The individual's agent maximization problem is now:

$$\max_{A_{ti}} \sum_{t=1}^{\infty} \beta^t U(C_t)$$

$$\text{s.t.} \quad C_t + \sum_{i=1}^{n} p_{ti}A_{ti} = \sum_{i=1}^{n} (p_{ti}A_{t-1i} + d_{ti}A_{t-1i})$$

$$C_t, A_{ti} \geq 0 \quad \text{and} \quad A_{0i} = 1 \text{ are given.} \tag{1.24}$$

The derivation of the first order condition is similar to what we already did. We cut consumption at t by x units and invest it in x/p_{ti} trees of type i. We then use the infinite stream of dividends to augment consumption at $\tau > t$. Equating the gain to the pain yields:

$$p_{ti} = \sum_{\tau=t+1}^{\infty} \beta^{\tau-t}\{U'(\hat{C}_\tau)/U'(\hat{C}_t)\}d_{\tau i}, \tag{1.25}$$

which is similar to (1.22) except for the added index i. Since the endowment is a portfolio of n trees per agent the market clearing condition is:

$$\hat{C}_\tau = \sum_{i=1}^{n} d_{\tau i} \quad \text{for all } \tau. \tag{1.26}$$

We now substitute (1.26) into (1.25) to get the equilibrium condition:

$$p_{ti} = \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \left\{ U'\left(\sum_{i=1}^{n} d_{\tau i}\right) \Big/ U'\left(\sum_{i=1}^{n} d_{ti}\right) \right\} d_{\tau i}. \tag{1.27}$$

At the prices (1.27) each individual wants to hold his initial portfolio of n trees and consume their fruits.

Note that the dividend at time τ is multiplied by

$$S_\tau = U'\left(\sum_{i=1}^{n} d_{\tau i}\right) \Big/ U'\left(\sum_{i=1}^{n} d_{ti}\right). \tag{1.28}$$

In bad times, when $\sum_{i=1}^{n} d_{\tau i}$ is relatively small, the marginal utility of consumption is high and therefore S_τ is high. This means that fruits in bad times are weighted more heavily in the pricing formula than fruits in good times. We now turn to an example that illustrates the implication of this observation.

Example: There are $n = 3$ tree types. A type 1 tree yields 2 units in odd periods and 7 units in even periods. A type 2 tree yields 2 units in odd periods and no dividends in even periods. A type 3 tree yields 2 units in even periods and no dividends in odd periods. The single period utility function is: $U(C) = 2C^{0.5}$.

Here consumption is 4 units in odd periods and 9 units in even periods. The marginal utility of consumption is: $U'(C) = C^{-0.5}$. In odd periods this is: $U'(4) = 1/2$ and in even periods it is: $U'(9) = 1/3$.

Suppose we evaluate the assets in an even period t. Then $S_\tau = U'(4)/U'(9) = 1.5$ in odd periods and $S_\tau = U'(9)/U'(9) = 1$ in even periods. We now calculate the stream of dividends multiplied by the ratio of the marginal utilities:

$$\begin{aligned}
\tau &= t+1, t+2, t+3, t+4\ldots \\
d_{\tau 1} &= 2.0, 7.0, 2.0, 7.0\ldots \\
d_{\tau 2} &= 2.0, 0.0, 2.0, 0.0\ldots \\
d_{\tau 3} &= 0.0, 2.0, 0.0, 2.0\ldots \\
S_{\tau} &= 1.5, 1.0, 1.5, 1.0\ldots \\
S_{\tau} d_{\tau 1} &= 3.0, 7.0, 3.0, 7.0\ldots \\
S_{\tau} d_{\tau 2} &= 3.0, 0.0, 3.0, 0.0\ldots \\
S_{\tau} d_{\tau 3} &= 0.0, 2.0, 0.0, 2.0\ldots
\end{aligned}$$

We can now apply the asset pricing formula $p_{ti} = \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} S_{\tau} d_{\tau i}$ to show that the price of a type 2 tree is higher than the price of a type 3 tree.

PROBLEMS

For section 1.4.1:

1 Assume a consumer who comes to the market with an endowment of $Y_1 = Y_2$. The consumer's utility function is $U(C_1) + U(C_2)$.

(a) Show that this consumer will choose to consume his endowment if the interest rate is zero.
(b) Show that this consumer will choose to save if the interest rate is positive.

2 Consider an exchange economy in which all agents are the same and the representative agent's utility function is: $U(C_1) + U(C_2)$. Since everyone has the same endowment and the same preferences in equilibrium there is no trade. What can you say about the equilibrium real interest rate when

(a) $Y_1 = Y_2$.
(b) $Y_1 < Y_2$.
(c) $Y_1 > Y_2$.

3 How would you change your answer to 2 if storage is possible?

For section 1.4.2:

4 Solve for a competitive equilibrium vector for the special case: $f(l) = l^{0.5}$ and $v(L) = L^{1.5}$.

5 Answer (4) for the case in which there is an income tax rate of 50% ($\tau = 1/2$).

For section 1.4.3:

6 Assume an economy with three assets (trees). Asset 1 pays a unit of consumption in all odd periods and zero in even periods. Asset 2 pays a unit of consumption in all odd periods and zero in even periods. Asset 3 pays a unit of consumption in all even periods and zero in odd periods.

(a) Compute the value of each of the three assets in (i) odd periods (ii) even periods. Assume that $\beta = 0.95$ and $U(C) = \ln(C)$.

(b) Assume that we multiply the amount of dividends paid by each of the three assets. Will this change your answer to (a)?

(c) Assume that we add a fourth asset that yields 1 unit in all periods (even and odd). Will this change your answer to (a)?

NOTE

1 Here we assume that short sales are not possible and therefore $A_t \geq 0$.