Teaching Net Present Value Analysis: Returns-to-Assets versus Returns-to-Equity

Stephen E. Miller and Garnett L. Bradford

We survey textbooks from agricultural economics and other subject matter areas to determine their treatments of the returns-to-assets versus returns-to-equity methods for determining the net present value of investment projects. Textbook authors disagree about the appropriate method, often give conflicting advice as to how a given method should be applied, and do not show how the results of the two methods can be reconciled. We provide a consolidated discussion of the circumstances under which the two methods produce the same net present value.

Readers will agree that agricultural economics and agribusiness majors should have a basic competency in the evaluation of investment projects via net present value (NPV) analysis. But instructors and textbooks in agricultural economics do not agree on the appropriate method for conducting the analysis. In particular, our students are likely to be exposed to two alternative methods for conducting the analysis: the returns-to-assets (RTA) method and the returns-to-equity (RTE) method.1

The RTA and RTE methods start with a seemingly common expression for the NPV of a potential investment project that results in added cash flows over more than one year,

\[
\text{NPV} = -INV_0 + \sum_{t=1}^{N} \frac{CF_t}{(1+i)^t} + \frac{TERM_N}{(1+i)^N},
\]

where \(INV_0\) is the initial investment at year 0, \(CF_t\) is the cash flow attributable to the project in year \(t\), \(TERM_N\) is the terminal value of the project at year \(N\).

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(i.e., the end of the planning horizon), and $i$ is the discount rate. Both methods agree on some essential points, namely:

1. the NPV model is appropriate for evaluating a project that is of “average risk” in the sense that the “riskiness” of the proposed project is comparable to the firm’s current projects,
2. incremental direct and indirect after-tax cash flows from the project should be included and sunk costs excluded,
3. resources used in the project should be valued at their opportunity costs,
4. the cash flows should be discounted to obtain their present value,
5. the discount rate should measure the marginal cost of capital, and
6. the NPVs should measure the change in the present value of the firm (and the wealth of the firm’s owners) if the investment project is undertaken.

However, as shown in figures 1 and 2 and as discussed in detail below, the definitions of the terms in equation (1) differ between the RTA and RTE methods. The definitions differ because the two methods account for debt financing in different ways—the RTA method implicitly through the discount rate and the RTE method explicitly through the cash flows.

Figure 1. NPV definitions for the RTA method

\[
\text{NPV} = -INV_0 + \sum_{t=1}^{N} \frac{CF_t}{(1+i)^t} + \frac{TERM_N}{(1+i)^N}
\]

where:
- NPV = net present value of the project,
- INV$_0$ = initial total cash acquisition cost of the project,
- $INV_0$ = I$_0$,
- CF$_t$ = net cash flow attributable to INV$_0$ at the end of year $t$, $t = 1, 2, \ldots, N$,
- $R_t = (R_t - C_t - D_t)(1 - T) + D_t$,
- $C_t$ = pretax cash operating cost due to the project during year $t$,
- $D_t$ = additional depreciation due to the project during year $t$,
- $T$ = firm’s marginal income tax rate,
- TERM$_N$ = the after-tax terminal or salvage value of the project’s assets at the end of year $N$, the end of the planning horizon,
- $i$ = pertinent cost of capital, usually specified as $(1 - T)r_w + k(1 - w)$,
- $r$ = interest rate paid on debt capital,
- $k$ = required rate of return on the owner’s equity invested in the project, and
- $w$ = the optimal proportion of the firm’s financing from debt, $0 \leq w \leq 1$. 
Figure 2. NPV definitions for the RTE method

\[ \text{NPV} = -INV_0 + \sum_{t=1}^{N} \frac{CF_t}{(1+i)^t} + \frac{\text{TERM}_N}{(1+i)^N} \]

where:

- \( \text{NPV} \) = net present value of the project,
- \( INV_0 \) = owner’s equity invested in the project at year 0,
- \( CF_t \) = net cash flow attributable to \( INV_0 \) at the end of year \( t \), \( t = 1, 2, \ldots, N \),
- \( R_t \) = pretax cash operating revenue generated by the project in year \( t \),
- \( C_t \) = pretax cash operating cost due to the project during year \( t \),
- \( D_t \) = additional depreciation due to the project during year \( t \),
- \( T \) = firm’s marginal income tax rate,
- \( L_{t-1} \) = project debt outstanding during year \( t \), and \( L_0 \) is the initial amount of debt financing,
- \( \text{TERM}_N \) = the after-tax terminal or salvage value of the project’s assets net of any outstanding debt at the end of year \( N \), the end of the planning horizon,
- \( r \) = interest rate paid on debt capital, and
- \( i \) = pertinent cost of capital, which is the required rate of return on the owner’s equity invested in the project,
- \( k \).

Depending on the course they are taking and the accompanying text, students in agricultural economics are likely to learn that there is a “preferred” way to calculate NPVs, either by the RTA method or the RTE method. For example, a student in agricultural financial management may be taught that the RTE method “is applicable for smaller, non-corporate firms whose leverage fluctuates over time and that lack access to the national markets for debt and equity ... (and) is consistent with the smaller-scale, non-corporate structure of most farm businesses” (Barry et al., p. 286). On the other hand, a student in farm management may be taught that the RTA method is preferred and that the RTE method is “an alternative (and typically less accurate) procedure that may be used to reflect the source of financing (debt versus equity) and the tax deductibility of interest in the analysis is to specify the discount rate as the cost of equity funds only. The cash inflows and outflows ... would then include explicitly the downpayment, annual principal and interest payments, and the tax savings attributable to interest payments. We do not advocate this procedure” (Boehlje and Eidman, p. 323fn).

Our experience has been that students who first learn one of the methods, say the RTA, are confused when exposed to the other method, say the RTE. A typical classroom dialogue in this situation goes like this.

Student: I’m confused. I learned that the RTA method is the correct method for computing the NPV. Will I get the same value for NPV if I use the RTE method as I would if I use the RTA method?
Instructor: Only in special circumstances.
Student: If there are two different methods for implementing the analysis, and the two methods give different NPVs, why bother? The payback period approach doesn’t seem so bad: at least it gives you a single answer.

Exchanges like this motivated us to ask whether undergraduate students in agricultural economics are unique in being exposed to, and possibly confused by,
alternative methods for conducting an NPV analysis. To answer this question, we conducted a survey of texts from agricultural economics and from a number of other subject matter areas that cover NPV analysis. We found that disagreement about the appropriate method for conducting an NPV analysis is not confined to agricultural economics. In most texts, only one of the two methods is discussed and illustrated with numerical examples. Among textbooks recommending a common method, advice as to how the method should be applied is often conflicting. No text provides a comprehensive discussion of the circumstances under which the NPVs from the two methods can be reconciled.

The objective of this paper is to review textbook coverage of the RTA and RTE methods across subject matter areas. Our review draws attention to the often-conflicting advice that textbook authors give to students regarding how the RTA and RTE methods should be implemented, points out some important technical details of NPV analysis that are often missing from textbook treatments, and provides a consolidated discussion of the circumstances under which the NPVs of the two methods can be reconciled. We conclude with our recommendations for improving instruction in application of NPV analysis.

**Textbook Treatments of the RTA and RTE Methods**

Table 1 lists selected textbooks included in our survey. These texts are classified according to the subject matter area in which they most likely would be used, and in a column of notes we indicate the NPV analysis method(s) covered by the author(s). We exclude texts dealing with natural resource economics and public sector economics in which capital budgeting techniques are used to evaluate public investments. We learned several lessons from our survey.

**Lesson 1**

There are at least seven academic subject matter areas in which students encounter NPV analysis. Within some subject matter areas, students are exposed to the analysis in more than one course. For example, in agricultural economics, NPV analysis is covered in undergraduate farm management, agribusiness management, and agricultural financial management courses. Finance students encounter NPV analysis in corporate finance, financial management, and asset management courses.

**Lesson 2**

Fiske is incorrect when he says that the RTE method is confined for the most part to the agricultural economics literature. Students taking a course in real estate investment are likely to be exposed to the RTE method. Indeed, Okoruwa, Cox, and Thompson indicate that the RTE method predominates in the real estate discipline. Depending on the adopted textbook, students taking courses in corporate finance, financial management, or engineering economics may also encounter the RTE method.

**Lesson 3**

Even among authors discussing a common method, there are disagreements about how the terms in equation (1) should be defined.
Table 1. Summary of selected textbook treatments of net present value analysis, by subject area

<table>
<thead>
<tr>
<th>Subject Area</th>
<th>Selected Textbook</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finance</td>
<td>Bierman and Smidt</td>
<td>Recommends RTA, discusses RTE pitfalls (pp. 118–30)</td>
</tr>
<tr>
<td></td>
<td>Brigham and Gapenski</td>
<td>Focuses on WACC RTA, covers RTE in an appendix (pp. 278–80)</td>
</tr>
<tr>
<td></td>
<td>Copeland and Weston</td>
<td>Recommends WACC RTA</td>
</tr>
<tr>
<td></td>
<td>Moyer, McGuigan, and Kretlow</td>
<td>Recommends WACC RTA, is skeptical of deducting interest charges to calculate CFs (pp. 345–8)</td>
</tr>
<tr>
<td></td>
<td>Ross, Westerfield, and Jaffe</td>
<td>Covers WACC RTA and RTE, says either can be used if the firm’s target debt-to-value ratio applies to the project over the project life (pp. 457–63)</td>
</tr>
<tr>
<td></td>
<td>Van Horne</td>
<td>Recommends WACC RTA</td>
</tr>
<tr>
<td>Real estate</td>
<td>Brueggerman and Fisher</td>
<td>Recommends RTE for evaluating equity investment performance (pp. 323–7) and RTA for totally valuing property (pp. 438–46)</td>
</tr>
<tr>
<td>Accounting</td>
<td>Horngren, Foster and Datar</td>
<td>Covers RTA only</td>
</tr>
<tr>
<td></td>
<td>Morse, Davis, and Hartgraves</td>
<td>Covers RTA only</td>
</tr>
<tr>
<td>Economics</td>
<td>Salvatore</td>
<td>Covers RTA only</td>
</tr>
<tr>
<td></td>
<td>Morse, Davis, and Hartgraves</td>
<td>Covers RTA only</td>
</tr>
<tr>
<td>Industrial</td>
<td>Newman and Johnson</td>
<td>Covers RTA only</td>
</tr>
<tr>
<td>engineering</td>
<td>Park and Sharp-Bette</td>
<td>Covers both RTA and RTE</td>
</tr>
<tr>
<td>Agricultural</td>
<td>Boehlje and Eidman</td>
<td>Recommends RTA, notes that RTE is “typically less accurate” (p. 323)</td>
</tr>
<tr>
<td>economics</td>
<td>Kay and Edwards</td>
<td>Recommends RTA, indicates project financing should be omitted from the CFs (p. 289)</td>
</tr>
<tr>
<td></td>
<td>Beierlein, Schneeberger, and Osburn</td>
<td>Covers RTA only</td>
</tr>
<tr>
<td></td>
<td>Barry et al.</td>
<td>Recommends RTE “for smaller, non-corporate firms” (p. 286), discusses RTA as an alternative (pp. 285–6, 305–10)</td>
</tr>
<tr>
<td>Forestry</td>
<td>Lee et al.</td>
<td>Covers RTA only</td>
</tr>
<tr>
<td></td>
<td>Klemperer</td>
<td>Covers RTA only</td>
</tr>
<tr>
<td></td>
<td>Bullard and Straka</td>
<td>Covers RTA only</td>
</tr>
</tbody>
</table>

Defining the initial investment, \( \text{INV}_0 \)

Among proponents of both the RTA and RTE methods, there is near unanimity about the definition of the initial total cash outlay from debt \((L_0)\) and equity \((e_0)\) sources needed to acquire the investment assets, \(I_0\):

\[ I_0 = \text{the new project cost inclusive of installation, shipping, and training costs incurred in acquiring the asset and putting it into service} + \text{any increase in net working capital required at year 0 as a result of adopting the project} \]
the net proceeds from the sale or the trade-in allowance of existing assets when the investment project is a replacement for existing assets ± any taxes arising from the purchase of new assets and/or sale of existing assets

(Moyer, McGuigan, and Kretlow, pp. 344–5; Barry et al., p. 287). But without discussing their reasoning, Penson and Lins (p. 109) say that this total cash outlay should be gross of “the trade-in value of used machinery deducted from the purchase price at the time of the purchase.” They discuss this point in the context of an investment that is financed entirely from equity. Therefore, their statement can be interpreted to apply to both the RTA and RTE methods.

The RTA and RTE methods differ in how $INV_0$ is defined relative to $I_0$. The RTA method defines $INV_0$ as $I_0$, whereas the RTE method defines $INV_0$ as total cash acquisition cost less the net proceeds of the loan used to finance the project, $INV_0 = I_0 - L_0 = e_0$. In textbook applications of the RTE method, the initial debt financing level is taken as given; that is, $L_0$ is “exogenous to the project’s profitability” (Fiske, p. 50).

### Defining the cash flow, $CF_t$

The RTA method defines $CF_t$ as the after-tax net operating cash flow generated by the project in year $t$ so that debt service payments in year $t$ are excluded,

$$CF_t = (R_t - C_t - D_t)(1 - T) + D_t = A_t,$$

where $R_t$ is the pretax cash operating revenue generated by the project in year $t$, $C_t$ is the pretax cash operating cost due to the project in year $t$, $D_t$ is the additional depreciation due to the project in year $t$, and $T$ is the firm’s marginal income tax rate.

The RTE method requires a specific debt repayment schedule in order to compute the after-tax net equity flow,

$$CF_t = (R_t - C_t - D_t - rL_{t-1})(1 - T) + D_t - (L_{t-1} - L_t) = A_t - B_t,$$

where $r$ is the interest rate paid on debt capital, $L_{t-1}$ is the debt principal outstanding during year $t$, and $B_t$ is after-tax debt flow, $rL_{t-1}(1 - T) + (L_{t-1} - L_t)$. In typical applications of the RTE method, the debt repayment schedule is taken as given.

Most authors use equation (2b) when describing the RTE method (e.g., Chambers, Harris, and Pringle, pp. 25–6; Park and Sharp-Bette, p. 183; Fiske, p. 49; Okoruwa, Cox, and Thompson, p. 194; Brigham and Gapenski, p. 280). Penson and Lins (pp. 185–7, 195) distinguish between the explicit cost (the market interest rate on incremental borrowed funds) and implicit costs of debt capital. The latter cost “represents a cost . . . for not having sufficient liquid reserves” (p. 186). They advise that an adjustment should be made to the interest payment to account for the implicit cost of debt capital when there is less than perfect
certainty regarding the project’s cash flows. This adjustment would reduce the annual net cash revenue “by an amount equal to the implicit cost of debt capital multiplied by the remaining balance on the loan” (p. 186). Barry et al. (pp. 416–7) also refer to this implicit cost of debt capital, discuss ways that the credit liquidity premium could be estimated, and calculate a total cost of debt that incorporates this premium. However, they do not offer any guidance concerning whether or how the total cost of debt so calculated should be used to modify equation (2b).

**Defining the terminal value, \( \text{TERM}_N \)**

The terminal value of the project’s assets reflects their salvage value or trade-in value, net of any taxes on capital gains, at the end of the planning horizon under the RTA method. For the RTE method, only the equity portion of the after-tax salvage or trade-in value is included, as “any debt outstanding against the assets should be repaid when they are sold” (Barry et al., p. 289).

**Defining the discount rate, \( i \)**

The cost of equity capital, \( k \), is generally agreed to be the appropriate discount rate for the RTE method. In most applications of the RTA method, the weighted-average cost of capital, \( \text{WACC} = (1 - T)rw + k(1 - w) \) is used as the discount rate, where \( w \) is the firm’s target debt-to-value ratio, \( 0 \leq w \leq 1 \). Regardless of whether they are discussing the RTE method or the WACC version of the RTA method, most authors agree that the cost of equity capital should reflect the opportunity cost of the equity invested in the project and should be higher than the cost of debt capital (e.g., Lee et al., pp. 87–8; Boehlje and Eidman, p. 322; Barry et al., pp. 289–90). Beierlein, Schneeberger, and Osburn (p. 234) agree that the cost of equity capital should be greater than the cost of debt capital, “based on the premise that there is a higher risk cost of the owner’s money sunk in the business,” but go on to say that “the owner may view the opportunity cost at less than the actual interest cost of money in some cases.”

Although most authors discussing the RTA method specify the WACC as the appropriate discount rate, a few maintain that management can choose between the WACC and other rates. For example, Kay and Edwards (p. 289) advise that if “money will be borrowed to finance the investment, the discount rate can be set equal to the cost of borrowed capital,” while Beierlein, Schneeberger, and Osburn (p. 233) say that management can “use the interest rate attainable by ‘investing’ in lending institutions (deposits or securities) before taxes.”

Within the WACC framework, there are differences of opinion among authors regarding the specification of the debt rate, \( r \). According to Brigham and Gapenski (p. 168), the interest rate on debt capital should be the borrowing rate for long-term debt unless the firm uses a mixture of short-term and long-term debt in financing long-term investments, a practice that “is not common among well-managed firms.” If such a mixture is used, the value of \( r \) would be calculated as a weighted average of the short-term and long-term debt rates, with the weights given by the proportions of short-term and long-term debt used in long-term financing (Brigham and Gapenski, p. 189). But Beierlein, Schneeberger, and Osburn (p. 234) calculate a weighted average of short- and long-term interest rates on debt for usage in a WACC framework without specifying whether
short-term debt is used to finance long-term investments. Boehlje and Eidman (p. 322) specify $r$ as the cost of debt funds without specifying whether the debt is short or long term. Lee et al. (p. 86) say that “(t)he use of debt financing results in increased financial risk and reduced credit reserves. Thus an internal credit rationing premium should be added to the interest cost. The estimate of this will be somewhat subjective, depending upon the manager’s risk-return preference function.”

For the WACC version of the RTA method, the proportion of debt financing, $w$, is determined by the firm’s target debt-to-value ratio—the debt-to-value ratio that minimizes the WACC. A graph similar to figure 3 is often used to illustrate the relationships between the debt and equity rates of return and the debt-to-value ratio (e.g., Barry et al., p. 158; Block and Hirt, p. 307; Brigham and Gapenski, p. 407; Copeland and Weston, p. 450; Gitman, p. 451; Herbst, p. 45; Moyer, McGuigan, and Kretlow, p. 507; Penson and Lins, p. 188; Ross, Westerfield, and Jaffe, p. 395; Seo, p. 601). The usual assumption is that the prospective returns to both debt and equity sources increase as the debt-to-value ratio increases, but at different rates, and $w$ is chosen to minimize the WACC. Boehlje and Eidman (p. 321) say that “the cost of capital should be based on the combination of debt and equity capital used in the ‘long run’ to finance the operation, not the specific combination of debt and equity that may be used to finance a particular project.” However, their text, like other undergraduate texts in agricultural economics, does not caution students that the firm must maintain its target debt-to-value ratio in order for the WACC NPV to represent the change in the value of the firm and the change in the owners’ wealth if the project is adopted (Ross and Westerfield, p. 231; Golbe and Schachter; and Boudreaux and Long).

The firm’s value is usually specified as the firm’s capitalization: long-term debt plus the value of the owners’ equity, and thus excludes short-term debt (Brigham and Gapenski, p. 168; Moyer, McGuigan, and Kretlow, pp. 468–71).

![Figure 3. Relationship between the Cost of Capital and the Debt-to-Value Ratio](image-url)
Assuming that the firm’s current balance sheet reflects its target debt-to-value ratio, \( w \) would be calculated as the capitalization ratio:

\[
(3a) \quad w = \frac{\text{long-term debt}}{\text{long-term debt} + \text{owners’ equity}}.
\]

But Boehlje and Eidman (pp. 322–3) say that \( w \) would be calculated as the debt-to-asset ratio so that short-term debt would be added to the numerator and denominator of equation (3a). Students following this advice would compute a different value of \( w \) than would students reading other texts if the firm has any short-term debt.

In discussing the value of the firm, there appears to be agreement that assets already under the control of the firm should be assigned their market values (rather than their acquisition costs less accumulated depreciation) in calculating \( w \) when the firm’s current balance sheet reflects its target debt-to-value ratio. But there are differing views regarding the debt capacity of the proposed project, and as a consequence, the appropriate amount of initial debt financing of the project. These differing views arise because there are two ways to value the proposed project: (1) its “market value” or “reproduction value” equal to the incremental discounted cash flows of the project, \( I_0 + \text{NPV} \), and (2) its “book value” or “replacement value” equal to the total cash acquisition costs needed to acquire the investment assets, \( I_0 \) (Chambers, Harris, and Pringle, p. 29; Copeland and Weston, p. 446–8; Fiske, p. 50).

If the project is accepted, equation (3a) becomes

\[
(3b) \quad w = \frac{\text{old debt + new debt supported by new equity + new debt supported by NPV}}{\text{new value of the firm}}.
\]

The “market value” view recognizes the debt supported by the proposed project’s NPV in equation (3b), and determines the debt-financing ratio for the proposed project required to maintain the target debt-to-value ratio according to

\[
(4a) \quad w^* = \frac{w(I_0 + \text{NPV})}{I_0}.
\]

Note that \( w^* \) is greater than the target debt-to-value ratio for positive NPV projects and can be greater than unity. The “book value” approach ignores the new debt capacity provided by the project’s NPV, and determines the debt financing ratio for the proposed project as

\[
(4b) \quad w^* = \frac{w(I_0)}{I_0} = w.
\]

Fiske (p. 50) cites Beranek (1977) in saying that most writers agree that the “book value” view is appropriate for evaluating the debt capacity of the proposed investment. This is misleading. Brigham and Tapley (p. 49), Chambers, Harris, and Pringle (p. 29), Haley and Schall (pp. 353–4), and Ross, Westerfield, and Jaffe (p. 457), for example, indicate that the project’s financing mix should
be based on the project’s market value rather than its book value. Brigham and Tapley (p. 50) explain that while corporate finance textbooks may indicate that $w$ should be used in evaluating the debt capacity of a new project, those textbooks assume (implicitly) that “(i)t investors have correctly anticipated the total value that management expects to add through capital budgeting (the sum of all projects’ NPVs) during the budget period. If this condition holds, then positive NPV projects will already be reflected in the firm’s stock price. . . . With this scenario, the overall capital structure will remain on target if new projects are financed at the firm’s target capital structure, as most textbooks recommend.” This assumption would not be appropriate for firms that are not publicly traded, so equation (4a) should be used to determine the project’s financing mix for these firms.

**Lesson 4**

The presentations of both the RTA and RTE methods in agricultural economics texts often are incomplete and/or potentially misleading to our students. A hallmark of discussions of the RTA method in agricultural economics texts is the distinction between economic profitability and financial feasibility (e.g., Boehlje and Eidman, pp. 321–34; Lee et al., pp. 69–90; Kay and Edwards, pp. 289–95; Beierlein, Schneeberger, and Osburn, pp. 232–46). That is, students are instructed to first evaluate the project’s NPV using the RTA method, and then to compute the net equity flows for alternative financing plans. The textbook applications of this two-step process are incomplete on at least two counts. First, students are not warned that specific project financing schemes may result in substantial departures between the firm’s target and actual debt-to-value ratios, thus invalidating the RTA NPV (WACC version) as a measure of the change in the value of the firm. Second, no attempt is made to show students how project financing can be used to maintain the target debt-to-value ratio.

In their numerical illustration of the RTE method, Barry et al. (pp. 291–302) do use a higher cost of equity capital when debt financing is used than when the project is financed from equity. However, they do not account for the fact that the debt-to-value ratio of the project changes over the project life for their specific debt repayment schedule unless supplemental equity flows are used to maintain the debt-to-value ratio. If the relationship between the cost of capital and the debt-to-value ratio depicted in figure 3 holds, the cost of equity capital should vary over the life of the project absent those supplemental equity flows.

**Lesson 5**

Several texts discuss both methods, but usually one of the methods is touted as being superior to the other. The quotes from Boehlje and Eidman and Barry et al. in our introductory section illustrate this point for agricultural economics. However, the disagreements about the relative merits of the RTA and RTE methods are not confined to agricultural economics. Moyer, McGuigan, and Kretlow (p. 348) argue that “(o)ften the purchase of a particular asset is tied closely to the creation of some debt obligation, such as . . . a bank loan. Nevertheless, it is considered incorrect to deduct the interest charges associated with a particular project from the estimated cash flows. . . . The decision about
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how a firm should be financed can—and should—be made independently of
the decision to accept or reject one or more projects.” But Ross and Westerfield
(p. 231) say “(the) tax-adjusted WACC can be applied only to projects that are
financed in such a way that the firm’s debt-equity ratio remains constant. This
means that we cannot apply the WACC to a project whose financing will change
that of the firm, and when we do apply it, we are implicitly assuming that the
project’s financing is the same as the firm’s.” Brigham and Gapenski focus on
the WACC version of the RTA method and say that normally it is easier to use
than the RTE method, with an exception being merger analysis (pp. 279, 836–9).
But Van Horne (p. 165) says the WACC version of the RTA method, not the RTE
method, should be used in merger analysis.

**Lesson 6**

Only a handful of texts attempt to reconcile the NPVs from the RTA and RTE
methods. No textbook, or journal article for that matter, provides a consolidated
discussion of the circumstances under which the NPVs from the RTA and RTE
are equal. It turns out that there are at least five cases under which equivalency
is obtained. The RTA NPV equals the RTE NPV if:

I. there is no debt financing,
II. the after-tax cost of debt capital is used as the pertinent cost of capital,
III. the optimal debt-to-value ratio varies according to the specified debt
repayment schedule,
IV. debt repayment flows are used to maintain the debt-to-value ratio at its
target level, or
V. equity flows are used to maintain the debt-to-value ratio at its target level
according to the specified repayment schedule.

We use the example data from Barry et al. (p. 307) displayed in table 2 to illustrate
each of the cases. The total cash outlay required to obtain the project’s assets ($I_0$)
is $9,000. There is no depreciation and the project assets have no salvage value
($TERM_5 = 0$). The debt and equity rates are 0.10 and 0.20, respectively, and the
firm’s marginal income tax rate is 0.30.

**Case I—No debt financing**

When there is no debt financing, $w = 0$, $WACC = k$, $L_0 = 0$, $I_0 = INV_0$, $B_0 = 0$
for all values of $t$, and $CF_i = A_i$ for both the RTA and RTE methods. The RTE
and RTA formulations of equation (1) are identical and the NPVs from the two
methods are equal. For the example in table 2, the present value of the after-tax
equity flows in year 0 is computed as

$$E_0 = \sum_{t=1}^{N} \frac{A_i - B_i}{(1 + k)^t} + TERM_N = \frac{\sum_{t=1}^{N} A_i}{(1 + k)^N} + TERM_N = \frac{\sum_{t=1}^{N} A_i}{(1 + k)^N} + TERM_N = $10,467,$$

and the NPV for both methods is $-9,000 + 10,467 = 1,467$. The imputed
debt-to-value ratio is 0.00 over the life of the project.$^6$
Table 2. Example net present value calculations

<table>
<thead>
<tr>
<th>Item</th>
<th>Year</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$t = 1$</td>
<td>$t = 2$</td>
<td>$t = 3$</td>
<td>$t = 4$</td>
</tr>
<tr>
<td>Revenue less operating expense $R_i - C_i$</td>
<td>5000</td>
<td>5000</td>
<td>5000</td>
<td>5000</td>
<td>5000</td>
</tr>
<tr>
<td>Depreciation $(\Delta_i) = D_i^f$</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>After-tax operating cash flow $(\delta_i)$</td>
<td>3500</td>
<td>3500</td>
<td>3500</td>
<td>3500</td>
<td>3500</td>
</tr>
<tr>
<td>Case I—no debt financing</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Debt value $(\delta_i) = L_{i-1}$</td>
<td></td>
<td>4500</td>
<td>3600</td>
<td>2700</td>
<td>1800</td>
</tr>
<tr>
<td>After-tax interest expense $(\delta_i) = (1 - T)rL_{i-1}$</td>
<td>315</td>
<td>252</td>
<td>189</td>
<td>126</td>
<td>63</td>
</tr>
<tr>
<td>Principal payment $(\delta_i) = (L_{i-1} - L_i)$</td>
<td>900</td>
<td>900</td>
<td>900</td>
<td>900</td>
<td>900</td>
</tr>
<tr>
<td>Total after-tax debt flow $(\delta_i)$</td>
<td></td>
<td>1215</td>
<td>1152</td>
<td>1089</td>
<td>1026</td>
</tr>
<tr>
<td>Net after-tax equity flow $(\delta_i) = A_i - B_i = A_i$</td>
<td>2285</td>
<td>2348</td>
<td>2411</td>
<td>2474</td>
<td>2537</td>
</tr>
<tr>
<td>Equity flow present value $(\delta_i) = E_{i-1}$</td>
<td>9851</td>
<td>8255</td>
<td>6485</td>
<td>4528</td>
<td>2371</td>
</tr>
<tr>
<td>Total value $(\delta_i) = V_{i-1} = L_{i-1} + E_{i-1}$</td>
<td>14351</td>
<td>11855</td>
<td>9185</td>
<td>6328</td>
<td>3271</td>
</tr>
<tr>
<td>Imputed debt-to-value ratio $= L_{i-1}/V_{i-1} = w'$</td>
<td>0.314</td>
<td>0.304</td>
<td>0.294</td>
<td>0.284</td>
<td>0.275</td>
</tr>
<tr>
<td>Imputed WACC $= (1 - T)rw + k(1 - w')$</td>
<td>0.070</td>
<td>0.070</td>
<td>0.070</td>
<td>0.070</td>
<td>0.070</td>
</tr>
<tr>
<td>Case II—after-tax cost of debt capital is the pertinent cost of capital</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Debt value $(\delta_i) = L_{i-1}$</td>
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<td>3600</td>
<td>2700</td>
<td>1800</td>
<td>900</td>
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<td>63</td>
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<td>900</td>
<td>900</td>
<td>900</td>
<td>900</td>
</tr>
<tr>
<td>Total after-tax debt flow $(\delta_i)$</td>
<td>1215</td>
<td>1152</td>
<td>1089</td>
<td>1026</td>
<td>963</td>
</tr>
<tr>
<td>Net after-tax equity flow $(\delta_i) = A_i - B_i = A_i$</td>
<td>2285</td>
<td>2348</td>
<td>2411</td>
<td>2474</td>
<td>2537</td>
</tr>
<tr>
<td>Equity flow present value $(\delta_i) = E_{i-1}$</td>
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<td>6485</td>
<td>4528</td>
<td>2371</td>
</tr>
<tr>
<td>Total value $(\delta_i) = V_{i-1} = L_{i-1} + E_{i-1}$</td>
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<td>11855</td>
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<td>6328</td>
<td>3271</td>
</tr>
<tr>
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<td>0.304</td>
<td>0.294</td>
<td>0.284</td>
<td>0.275</td>
</tr>
<tr>
<td>Imputed WACC $= (1 - T)rw + k(1 - w')$</td>
<td>0.070</td>
<td>0.070</td>
<td>0.070</td>
<td>0.070</td>
<td>0.070</td>
</tr>
<tr>
<td>Case III—target debt-to-value ratio varies with the debt repayment schedule</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Debt value $(\delta_i) = L_{i-1}$</td>
<td>4500</td>
<td>3600</td>
<td>2700</td>
<td>1800</td>
<td>900</td>
</tr>
<tr>
<td>After-tax interest expense $(\delta_i) = (1 - T)rL_{i-1}$</td>
<td>315</td>
<td>252</td>
<td>189</td>
<td>126</td>
<td>63</td>
</tr>
<tr>
<td>Principal payment $(\delta_i) = (L_{i-1} - L_i)$</td>
<td>900</td>
<td>900</td>
<td>900</td>
<td>900</td>
<td>900</td>
</tr>
<tr>
<td>Total after-tax debt flow $(\delta_i)$</td>
<td>1215</td>
<td>1152</td>
<td>1089</td>
<td>1026</td>
<td>963</td>
</tr>
<tr>
<td>Net after-tax equity flow $(\delta_i) = A_i - B_i = A_i$</td>
<td>2285</td>
<td>2348</td>
<td>2411</td>
<td>2474</td>
<td>2537</td>
</tr>
<tr>
<td>Equity flow present value $(\delta_i) = E_{i-1}$</td>
<td>7143</td>
<td>6286</td>
<td>5195</td>
<td>3823</td>
<td>2114</td>
</tr>
<tr>
<td>Total value $(\delta_i) = V_{i-1} = L_{i-1} + E_{i-1}$</td>
<td>11643</td>
<td>9886</td>
<td>7895</td>
<td>5623</td>
<td>3014</td>
</tr>
<tr>
<td>Imputed debt-to-value ratio $= L_{i-1}/V_{i-1} = w'$</td>
<td>0.387</td>
<td>0.364</td>
<td>0.342</td>
<td>0.320</td>
<td>0.299</td>
</tr>
<tr>
<td>Imputed WACC $= (1 - T)rw + k(1 - w')$</td>
<td>0.150</td>
<td>0.153</td>
<td>0.156</td>
<td>0.158</td>
<td>0.161</td>
</tr>
</tbody>
</table>

Case II—After-tax cost of debt capital is the pertinent cost of capital

Bierman and Smidt (pp. 118–30) discuss this case. When the cost of equity capital, $k$, equals the after-tax cost of debt capital, $(1 - T)r$, the pertinent cost of capital is $(1 - T)r$ for both methods. The WACC in this case, $(1 - T)r + (1 -
Table 2. Continued

<table>
<thead>
<tr>
<th>Item</th>
<th>Year</th>
<th>( t = 1 )</th>
<th>( t = 2 )</th>
<th>( t = 3 )</th>
<th>( t = 4 )</th>
<th>( t = 5 )</th>
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<tbody>
<tr>
<td>Case IV—target debt-to-value ratio is maintained by debt flows</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Debt value ($) = ( L_{t-1} )</td>
<td>6081</td>
<td>5152</td>
<td>4097</td>
<td>2900</td>
<td>1542</td>
<td></td>
</tr>
<tr>
<td>After-tax interest expense ($) = (1 - T)rL_{t-1}</td>
<td>426</td>
<td>361</td>
<td>287</td>
<td>203</td>
<td>108</td>
<td></td>
</tr>
<tr>
<td>Principal payment ($) = L_{t-1} - L_t</td>
<td>929</td>
<td>1055</td>
<td>1197</td>
<td>1358</td>
<td>1542</td>
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<tr>
<td>Total after-tax debt flow ($)</td>
<td>1355</td>
<td>1415</td>
<td>1484</td>
<td>1561</td>
<td>1650</td>
<td></td>
</tr>
<tr>
<td>Net after-tax equity flow ($)</td>
<td>2145</td>
<td>2085</td>
<td>2016</td>
<td>1939</td>
<td>1850</td>
<td></td>
</tr>
<tr>
<td>Equity flow present value ($) = E_{t-1}</td>
<td>6081</td>
<td>5152</td>
<td>4097</td>
<td>2900</td>
<td>1542</td>
<td></td>
</tr>
<tr>
<td>Total value ($) = ( V_{t-1} = L_{t-1} + E_{t-1} )</td>
<td>12162</td>
<td>10304</td>
<td>8194</td>
<td>5800</td>
<td>3084</td>
<td></td>
</tr>
<tr>
<td>Imputed debt-to-value ratio = ( L_{t-1}/V_{t-1} = w' )</td>
<td>0.500</td>
<td>0.500</td>
<td>0.500</td>
<td>0.500</td>
<td>0.500</td>
<td></td>
</tr>
<tr>
<td>Imputed WACC = ((1 - T)r w' + k(1 - w'))</td>
<td>0.135</td>
<td>0.135</td>
<td>0.135</td>
<td>0.135</td>
<td>0.135</td>
<td></td>
</tr>
<tr>
<td>Case V—target debt-to-value ratio is maintained by equity flows</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Debt value ($) = ( L_{t-1} )</td>
<td>6081</td>
<td>4865</td>
<td>3649</td>
<td>2432</td>
<td>1216</td>
<td></td>
</tr>
<tr>
<td>After-tax interest expense ($) = (1 - T)rL_{t-1}</td>
<td>426</td>
<td>341</td>
<td>255</td>
<td>170</td>
<td>85</td>
<td></td>
</tr>
<tr>
<td>Principal payment ($) = ( L_{t-1} - L_t )</td>
<td>1216</td>
<td>1216</td>
<td>1216</td>
<td>1216</td>
<td>1216</td>
<td></td>
</tr>
<tr>
<td>Total after-tax debt flow ($)</td>
<td>1642</td>
<td>1557</td>
<td>1472</td>
<td>1386</td>
<td>1301</td>
<td></td>
</tr>
<tr>
<td>Net after-tax equity flow ($) = ( A_t - B_t )</td>
<td>1858</td>
<td>1943</td>
<td>2028</td>
<td>2114</td>
<td>2199</td>
<td></td>
</tr>
<tr>
<td>Equity flow present value ($) = E_{t-1}</td>
<td>5974</td>
<td>5311</td>
<td>4431</td>
<td>3289</td>
<td>1833</td>
<td></td>
</tr>
<tr>
<td>Total value ($) = ( V_{t-1} = L_{t-1} + E_{t-1} )</td>
<td>12055</td>
<td>10176</td>
<td>8080</td>
<td>5721</td>
<td>3049</td>
<td></td>
</tr>
<tr>
<td>Imputed debt-to-value ratio</td>
<td>( = L_{t-1}/V_{t-1} = w' )</td>
<td>0.504</td>
<td>0.478</td>
<td>0.452</td>
<td>0.425</td>
<td>0.399</td>
</tr>
<tr>
<td>Supplemental equity financing flow ($) = ( d_t )</td>
<td>-574</td>
<td>-246</td>
<td>81</td>
<td>412</td>
<td>740</td>
<td></td>
</tr>
<tr>
<td>Adjusted net after-tax equity flow ($)</td>
<td>( = A_t - B_t - d_t )</td>
<td>2432</td>
<td>2189</td>
<td>1947</td>
<td>1702</td>
<td>1459</td>
</tr>
<tr>
<td>Adjusted equity flow present value ($) = ( E_{t-1}^* )</td>
<td>6081</td>
<td>4865</td>
<td>3649</td>
<td>2432</td>
<td>1216</td>
<td></td>
</tr>
<tr>
<td>Total value with supplemental equity flows ($)</td>
<td>( = L_{t-1} + E_{t-1}^* )</td>
<td>12162</td>
<td>9730</td>
<td>7298</td>
<td>4864</td>
<td>2432</td>
</tr>
<tr>
<td>Imputed debt-to-value ratio with supplemental equity flows</td>
<td>( = L_{t-1}/V_{t-1} = w'' )</td>
<td>0.500</td>
<td>0.500</td>
<td>0.500</td>
<td>0.500</td>
<td>0.500</td>
</tr>
<tr>
<td>Imputed WACC with supplemental equity flows</td>
<td>( = (1 - T)r w'' + k(1 - w'') )</td>
<td>0.135</td>
<td>0.135</td>
<td>0.135</td>
<td>0.135</td>
<td>0.135</td>
</tr>
</tbody>
</table>

Notes:

- Parameters are \( T \), the marginal income tax rate = 0.30; \( k \), the cost of equity capital = 0.20 for Cases I and III–V and = 0.07 for Case II; and \( r \), the pretax cost of debt capital = 0.10.

\( T)(1 - w) = (1 - T)r \), is independent of \( w \). The RTE NPV is

\[
NPV = -e_0 + \sum_{t=1}^{N} \frac{A_t - B_t}{(1 + i)^t} + \frac{TERM_N}{(1 + i)^N},
\]

\[
= -e_0 - L_0 + \sum_{t=1}^{N} \frac{A_t}{(1 - (1 - T)r)^t} + \frac{TERM_N}{(1 + (1 - T)r)^N},
\]

\[
= L_0 + \sum_{t=1}^{N} \frac{A_t}{(1 + (1 - T)r)^t} + \frac{TERM_N}{(1 + (1 - T)r)^N},
\]
which is identical to the NPV from the RTA method. This equivalency holds for any arbitrary level of debt financing and any repayment schedule. For the numerical example in table 2, \( k = (1 - T)r = (1 - 0.30)0.10 = 0.07 \), \( e_0 = $4,500 \), \( L_0 = $4,500 \), and the debt repayment schedule requires equal principal payments of $900/year to amortize the loan over five years. The RTA NPV = $−9,000 + $14,351 = $5,351, the RTE NPV = $−4,500 + $9,851 = $5,351, and so RTA NPV = RTE NPV. Note that the imputed debt-to-value ratio varies over the life of the project, and so it must be assumed that \( k \) equals \((1 - T)r\) and \( k \) and \( r \) are invariant with respect to the debt-to-value ratio in order for the NPV formula values to be valid.

Case III—Target debt-to-value ratio varies with the debt repayment schedule

Fiske and Barry et al. (pp. 305–10) discuss this case. For an arbitrary debt repayment schedule, the RTA and RTE NPVs will be equal if the debt-to-value ratio \( w \) used to calculate the WACC under the RTA method varies according to the specified debt repayment schedule and \( e_0 + L_0 = I_0 \). For the numerical example in table 2, \( k = 0.20 \), \( e_0 = $4,500 \), \( L_0 = $4,500 \), and the debt repayment schedule requires equal principal payments of $900/year for five years. The present value of the after-tax equity flows at year 0 is

\[
E_0 = \sum_{t=1}^{N} \frac{A_t - B_t}{(1 + k)^t} + \frac{\text{TERM}_N}{(1 + k)^N} = \$7,143
\]

and the NPV from the RTE method is \(-e_0 + E_0 = −$4,500 + $7,143 = $2,643\). The imputed WACC in year \( t \), \( \text{WACC}'_t \), is calculated from the debt-to-value ratio generated by the specified debt repayment schedule. The NPV for the RTA method allowing the WACC to vary with the debt-to-value ratio is computed as

\[
\text{NPV} = −I_0 + \sum_{t=1}^{N} \frac{A_t}{\prod_{j=1}^{t}(1 + \text{WACC}_j)} + \frac{\text{TERM}_N}{\prod_{j=1}^{N}(1 + \text{WACC}_j)}
\]

\[
= −$9,000 + $11,643 = $2,643,
\]

and the NPVs from the two methods are equal. An alternative debt repayment plan would result in a different RTE NPV, a different pattern in the imputed debt-to-value ratios, and a different pattern in the WACCs. Because there are an infinite number of alternative debt repayment plans, there are an infinite number of patterns for the imputed debt-to-value ratios and WACCs. This is not consistent with the usual assumption that there is a unique debt-to-value ratio that minimizes the WACC (see figure 3).

Case IV—Target debt-to-value ratio is maintained by debt flows

Brigham and Gapenski (pp. 279) note that the NPVs of the RTE and RTA (WACC version) methods are equivalent provided that projected debt in any year equals a constant fraction of the present value of the future cash flows, a condition necessary to maintain the capital structure at the target level giving
consideration to the fact that taking on the project increases the value of the equity," but they do not show how debt repayments could be scheduled so that the optimal debt-to-value ratio is maintained. Ross, Westerfield, and Jaffe (pp. 457–60) provide a numerical example for a perpetuity in which debt is a constant proportion of the present value of the project and show the equivalency of the RTA (WACC version) and RTE NPVs. However, they do not show how the firm can use debt financing flows to maintain its target debt-to-value ratio for a project with a finite life. To maintain the target debt-to-value ratio, the ratio of the return of debt capital (i.e., the principal payment) to the return of equity capital must equal \( \frac{w}{1 - w} \) in each year. Although not illustrated in textbooks, it is simple to derive the required repayment schedule using the procedures of Golbe and Schachter for the special circumstance in which the after-tax operating cash flows are constant over the life of the project; or Linke and Kim for the general case in which the after-tax cash flows can vary over the life of the project. From Golbe and Schachter, the principal payment in year \( t \) that maintains a constant debt-to-value ratio is computed as

\[
L_{t-1} - L_t = w \left( \frac{A_t}{(1 + \text{WACC})^{N-t+1}} \right)
\]

when \( A_t \) is constant for all \( t \). From Linke and Kim, the principal payment in year \( t \) that maintains a constant debt-to-value ratio is given by

\[
L_{t-1} - L_t = w(A_t - (1 - T)rL_{t-1} - kE_{t-1}).
\]

For the numerical example in table 2, \( w = 0.5 \), so \( w/(1 - w) = 1 \), and the return of equity capital must equal the return of debt capital in each year. The WACC = 0.135, and the WACC NPV = $3,162. The debt financing ratio required to maintain the target debt-to-value ratio is \( w^* = \frac{w(I_0 + \text{NPV})}{I_0} = \frac{0.5(9,000 + 3,162)}{9,000} = 0.6756 \). Thus, \( L_0 = (0.6756)(9,000) = 6,081, e_0 = 9,000 - 6,081 = 2,919, \) and \( E_0 = (1 - w)(I_0 + \text{WACC NPV}) = 0.5 (9,000 + 3,162) = 6,081. \) The Golbe and Schachter formula returns \( L_0 - L_1 = 0.5(\frac{3,500}{1.135^5} - 6,081) = \$929 \) as the required principal payment in year 1. In year 1, the cost of debt capital is \( \$426 = (1 - T)rL_0 = 0.7 * 0.10 * \$6,081 \) and the cost of equity capital is \( \$1,216 = k * E_0 = 0.2 * \$6,081 \). Deducting these amounts from the after-tax operating cash flow in year 1, \( A_1 = \$3,500, \) leaves \( \$1,858 = \$3,500 - \$426 - \$1,216 \) available for the return of debt and equity capital. In agreement with the Golbe and Schachter formula, \$929 = w * \$1,858 = 0.5 * \$1,858 \) is the return of debt capital (i.e., \( L_0 - L_1 \)), and \$929 = (1 - w) * \$1,858 = (1 - 0.5) * \$1,858 \) is the return of equity capital (i.e., \( E_0 - E_1 \)) in year 1. Similar reasoning can be applied to derive the required principal payments in years 2 through 5. The RTE NPV = \(-\$2,919 + \$6,081 = \$3,162 = \) the WACC NPV, and the debt-to-value ratio is constant at the target level over the life of the project.

**Case V—Target debt-to-value ratio is maintained by equity flows**

For an arbitrary debt repayment schedule, the NPVs of the RTA (WACC version) and RTE methods will be equal if the target debt-to-value ratio is maintained by appropriate choice of equity flows and equation (4a) is used to determine the debt-financing ratio for the project. That is, the firm’s owners must supply or withdraw equity from the project as needed to maintain the
target debt-to-value ratio. Thus, the firm can maintain its target debt-to-value ratio by choice of an appropriate debt repayment schedule as in Case IV or by choice of appropriate “supplemental” equity flows for a specific debt repayment schedule as in Case V. The procedures of Linke and Kim can be adapted to determine the appropriate supplemental equity flows. The supplemental equity flow required to maintain the debt-to-value ratio in year \( t \) is given by

\[
d_t = \left[ w/(1 - w) \right] L_t - (1 + k)E_{t-1} - (A_t - B_t)
\]

where \( E_{t-1} \) is the present value of the equity flows adjusted for the supplemental equity flows, \( E_0^s = (1 - w)(L_0 + \text{WACC NPV}) \) and \( E_t^s = (1 + k)E_{t-1} - (A_t - B_t - d_t) \) for \( t > 0 \). A positive value of \( d_t \) indicates that the owners would have to provide additional equity in year \( t \), while a negative value of \( d_t \) indicates that the owners would have excess equity in year \( t \) that is available to support other projects.

For the example in table 2, \( w = 0.5 \), the WACC NPV = $3,162, \( L_0 = $6,081 \), and \( e_0 = $2,919 \) as in Case IV. We assume that the debt is retired by five principal payments of $1,216 each. For year 1, the Linke and Kim formula returns \( d_1 = [0.5/(1 - 0.5)] \ast 4,865 - 1.2 \ast 6,081 + 1,858 = -$574 \). Of the after-tax operating cash flow in year 1, \( A_1 - B_1 = $1,858 \), the return on equity is $1,216 (= \( kE_0^s = 0.2 \ast $6,081 \)) and the return of equity capital before the supplemental equity flow is $642 (= $1,858 - $1,216). The return of debt capital in year 1, \( L_0 - L_1 \), is $1,216. In agreement with the Linke and Kim formula, a supplemental equity withdrawal of $574 (= $642 - $1,216) is required to equate the returns of debt and equity capital in year 1. In year 2, \( A_2 - B_2 = $1,943 \), the return on equity is $973 (= 0.2 \ast $4,865), and the return of equity before the supplemental equity flow is $970 (= $1,943 - $973). A supplemental equity withdrawal of $246 (= $970 - $1,216) equates the returns of debt and equity capital in year 2. For year 3, \( A_3 - B_3 = $2,028 \), the return on equity is $730 (= 0.2 \ast $3,649), and the return of equity before the supplemental equity flow is $1,298 (= $2,028 - $730). To equate the return of equity capital and the return of debt capital in year 3, a supplemental equity contribution of $81 (= $1,298 - $1,216 with allowance for rounding) is required. Similar reasoning applies to the supplemental equity contributions in years 4 and 5. Obviously, alternative debt repayment plans would result in different patterns for the supplemental equity flows.

The NPV computed from the adjusted after-tax equity flows is

\[
-e_0 + \sum_{t=1}^{N} \frac{A_t - B_t - d_t}{(1 + k)^t} = -$2,919 + $6,081 = $3,162
\]

the same as the WACC NPV, and the target debt-to-value ratio is maintained over the life of the project. The present value of the supplemental equity flows at year 0 is

\[
\sum_{t=1}^{N} \frac{d_t}{(1 + k)^t} = -$107.
\]

Note that the NPV from the RTE method without the supplemental equity flows is

\[
-e_0 + \sum_{t=1}^{N} \frac{A_t - B_t}{(1 + k)^t} = -$2,919 + $5,974 = $3,055.
\]

Adjusting this value for the present value of the supplemental equity flows gives $3,055 - (-$107) = $3,162.
Concluding Remarks

Based on our reading of agricultural economics texts, we suspect that most of our majors learn that there is a “preferred” way to compute a project’s NVP, either by the RTA method or the RTE method. If students are exposed to both methods, they are likely to learn that one of the two methods is an imperfect substitute for the other. In our opinion, arguments about the relative merits of the RTA and RTE methods are not fruitful. The WACC version of the RTA method requires that the firm maintain its target debt-to-value ratio over the life of the project if the project’s NPV is to measure the change in the wealth of the firm’s owners. If the cost of equity capital varies with the firm’s debt-to-value ratio, as is the usual assumption, the use of a constant cost of equity capital as the discount factor in computing the RTE NPV is justified only if the firm maintains a constant debt-to-value ratio over the life of the project. But when the firm maintains a constant debt-to-value ratio, the WACC RTA and RTE methods produce the same NPV.

In our view, the WACC RTA and RTE methods should be viewed as complements, rather than substitutes. As illustrated in our discussions of Cases IV and V, the WACC RTA method can be used to determine the project’s NPV when the target debt-to-value ratio is maintained over the life of the project and the project financing recognizes the debt supported by the project’s NPV. The debt and/or supplemental equity flows required to maintain the target debt-to-value ratio can then be determined. Evaluation of the NPV of the resulting after-tax equity flows by the RTE method provides a useful check on whether the required debt and/or supplemental equity flows have been calculated correctly.

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Endnotes

1 The RTA method is also known as the Net Operating Cash Flow method and the Adjusted Discount Rate method; and the RTE method is also known as the Equity Residual method, the Flows-to-Equity method, the Equity Residual Income method, and the Equity Residual Value method.

2 Our textbook review is not comprehensive on at least two counts. First, we limit our discussion to implementation of the RTA and RTE methods by firms whose debt and equity are not traded publicly, and thus ignore complications that arise when firms have publicly traded securities (common and preferred stock, and bonds). Second, some texts cover other methods for calculating the NPV of an investment project, including the adjusted present value method (Ross, Westerfield, and Jaffe, pp. 455–63; Brigham and Gapenski, pp. 280–1; Van Horne, pp. 228–30). We ignore these other methods because we have a full plate in discussing the RTA and RTE methods.

3 Beranek (1980) shows that, in general, the NPVs calculated for mutually exclusive projects by the WACC approach should not be used to rank those projects when is used as the debt-financing ratio for the projects. In this circumstance, the firm’s owners must determine a debt redemption policy that maximizes the owners’ equity.

4 The imputed debt-to-value ratios in the financial feasibility examples used in agricultural economics texts range from 0 to 0.85 (Boehlje and Eidman, p. 333), 0 to 0.80 (Lee et al., p. 90), 0 to 0.88 (Beierlein, Schneeberger, and Osburn, p. 244), 0.88 to 0.99 (Beierlein, Schneeberger, and Osburn, p. 245), 0.72 to 0.83 (Kay and Edwards, Investment A, p. 294) and 0.36 to 0.83 (Kay and Edwards, Investment B, p. 294).

5 The imputed debt-to-value ratios in their example (pp. 298–9) range from 0.52 to 0.81.
The present values of after-tax equity flows for subsequent years are calculated as

\[ E_t = \sum_{i=2}^{N} \frac{A_t - B_t}{(1 + k)^i} + \frac{\text{TERM}_N}{(1 + k)^i}, \]

\[ E_2 = \sum_{i=2}^{N} \frac{A_t - B_t}{(1 + k)^i} + \frac{\text{TERM}_N}{(1 + k)^i}, \]

\[ \ldots, E_N = A_N - B_N + \frac{\text{TERM}_N}{(1 + k)^i}. \]

Beierlein, Schneeberger, and Osburn (pp. 244–5) use the after-tax cost of debt as the discount rate in their RTA analysis, so the NPVs of the after-tax equity flows in their financial feasibility analyses equal the RTA NPV when the after-tax equity flows are discounted at the same rate.

References


