On the Time Sequence of Unemployment Benefits When Search is Costly

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Abstract. Should we cut the level of unemployment benefits, or reduce their potential duration? The answer depends on the way the unemployed search behaviour and unemployment insurance schemes interact. In this paper, we consider that unemployment insurance funds can be used to improve search. Resulting hazards are increasing over the unemployment spell prior to the exhaustion of benefits, and plummet immediately after it. Turning to policy implications, we assume the public decision-maker aims to minimize the average duration of unemployment under a resource constraint. First, we show the stationary relationship between average unemployment duration and unemployment benefit is hump-shaped. Second, raising benefits over a short duration can reduce average duration. Finally, we demonstrate that most of the time, a declining (yet always positive) benefit scheme is optimal.

1. Introduction

This paper examines the effects of the level and potential duration of unemployment insurance (UI) benefits on unemployment durations. Most of the unemployment compensation systems of OECD countries deliver declining benefits according to the duration of the unemployment spell. Actually, benefits are proportional to the pre-unemployment wage for short unemployment spells, while they decrease to a common standard determined by the public assistance (PA) system for longer durations.

Conventional wisdom considers that such a declining sequence of unemployment compensation is desirable. Most of the time, this
view is sustained by a moral hazard argument. Risk-averse workers want to be insured against the risk of unemployment. However, search activity cannot be monitored easily. In this context, decreasing unemployment benefits are optimal, since they urge the unemployed to look for a job (see, for instance Cremer et al. 1996; Hopenhayn and Nicolini, 1997; Shavell and Weiss, 1979; Wang and Williamson, 1996; see also Cahuc and Lehmann, 2000 and Frederiksson and Holmlund, 2001 who take wage endogeneity into account).

All these papers use a utilitarian criterion. Actually, unemployment is minimal when there is no unemployment compensation. In contrast, we propose a macroeconomic argument based on the minimization of average unemployment duration and only examine the effects of the insurance scheme on individual hazards and duration. We thus consider the job search behaviour of an unemployed individual, and assume that search intensity depends on financial spending. Moreover, the unemployed have no access to the financial market and cannot save part of their earnings. Thus, unemployment benefits must be shared between consumption and search spending. In this framework, an increase in unemployment benefits, or in potential duration, exerts opposite effects on expected unemployment durations. First, a negative income effect through a rise in the global amount of resource available to search; second, a standard positive reservation wage effect.

The resulting hazard increases at each interval where the amount of benefits is constant, and decreases at each discontinuity of the duration–benefit function. From a normative perspective, we show that the stationary relationship between average unemployment duration and unemployment benefits is hump-shaped: benefits have no effects for very low replacement rates, negative effects for medium ones, and positive effects for higher rates. We also demonstrate that UI benefits can shorten the expected unemployment spell if the duration of benefits is sufficiently short.

We then turn to policy implications. To this extent, we assume that the public decision maker aims to minimize the average unemployment duration under a resource constraint — the discounted expected cost for the unemployment insurance is bounded above. Most of the time, a declining (yet always positive) benefit scheme is optimal.

There are three reasons why financial spending acts on search efforts. First, an applicant has to be well dressed. Second, he/she
must travel to interviews in different locations.\textsuperscript{1} The most sophisticated but convincing argument is provided by Machin and Manning (1999, p. 3121):

\begin{quote}
‘Many studies have documented the importance of the use of current workers to recruit friends and relatives. Something like a third of jobs in the UK are filled in this way. [...] There is other evidence that suggests that the unemployed lose social contacts as their spells lengthen and that what social contacts they do maintain come to be increasingly made up of other unemployed. Among the reasons given for this are that socializing costs money which the unemployed generally lack.’
\end{quote}

There is also some empirical evidence. Wadsworth (1991) has shown from UK data that the unemployed claiming benefits search more extensively than non-claimants. Wadsworth and Schmidt (1993) found that the level of benefit has no significant effect on the unemployed search behaviour; Hughes \textit{et al.} (1996) found that UI benefits incorporate the ability to improve search through the use of unemployment insurance funds. Any disincetive effect of an increase in benefits on the return to job search is offset by a positive stimulus to search because of increased income. Jones (1996) found that higher benefits increase search effort among benefit recipients.

The relevant theoretical literature is largely confined to the job search framework. Mortensen (1977, 1986) and Van den Berg (1990, 1995) investigated the movement of a job-seeking individual’s reservation wage over time in a context where unemployment benefits were declining. Both papers imply that the exit rate increases as the exhaustion date approaches. Unfortunately, another implication of the basic model is that the exit rate is maximized at (and forever after) the exhaustion date.\textsuperscript{2} This latter property is at odds with the data (see Meyer, 1990). Moreover, it has been largely documented that the long-term unemployed are more likely to stay unemployed, i.e. there is a globally negative state dependence (see Machin and Manning, 1999).

Two other papers have elaborated on the positive role of benefits when search is costly. Ben-Horim and Zuckerman (1987) have shown that the duration of unemployment may be decreasing in the level of benefits. Tannery (1983) assumes that search requires two inputs: time and money. An increase in UI benefits reduces the time devoted to search, but increases its budget. The overall effect on the duration of unemployment depends on the complementarity
between leisure and consumption in preferences. We extend these results by considering the role of the UI scheme on duration dependence and expected unemployment spells. Moreover, our paper differs in focus. It does not contest the almost universal findings that the higher the ratio of unemployment income to that of previous earnings, the longer the unemployment duration. Instead, we aim to contribute to the debate on the optimal level and duration of unemployment benefits.

The paper is organized as follows: the next section describes the behaviour of a representative unemployed person and his/her environment. Sections 3 and 4 are devoted to the positive analysis of the benefit system. Section 3 focuses on the level of benefits in the hypothetical case of a stationary environment. Section 4 investigates comparative dynamics. A normative analysis is developed in Section 5, which discusses the optimal design of the UI scheme.

2. The model

We consider the search behaviour of an infinite lifetime unemployed person whose unemployment spell is $s > 0$. The agent is risk averse, discounts time at rate $r > 0$ and does not have access to the financial market.

During the unemployment spell, the unemployed person faces a discontinuous path $B$ of the unemployment benefits, with $B(s) = B_1$ if $s \leq T$ and $B(s) = B_2$ if $s > T$; $T$ is the potential duration of UI benefits. Hereafter, $B_1$ will denote the level of UI benefits, while $B_2$ will denote the level of PA benefits. At each time the agent chooses the level of spending $b$ he/she invests in search. The risk $p(b)\equiv \mu b$ of leaving unemployment increases with the search effort. If $b$ is the efficiency of search, $\mu > 0$ is the flow probability to match per efficient unit of search. Then, a trade-off exists between the utility measure of the unemployment compensation and the probability to match.

The instantaneous felicity function $u$ obeys the following properties.

Assumptions. Let $I_a(c) \equiv -u''(c)/u'(c)$ and $I_r(c) \equiv -cu''(c)/u'(c)$.

(A1) $u$ is strictly increasing, strictly concave, satisfies $\lim_{c \to 0} u'(c) = \infty$ and $\lim_{c \to \infty} u'(c) = 0$, as well as $u(0) = 0$ and $\lim_{c \to \infty} u(c) = \infty$.

(A2) $I_a$ is strictly decreasing, and $I_r(c) < 1$ for all $c > 0$. 

The set of assumptions (A1) means $u$ is a standard neoclassical utility function, with decreasing marginal utility satisfying Inada conditions. These properties will be sufficient for most of the paper. But, to obtain more regular solutions, we will also need stronger restrictions, characterized by the set of assumptions (A2). Then, preferences exhibit strictly decreasing absolute risk aversion (the Arrow–Pratt index $I_a$ of absolute risk aversion is strictly decreasing), and relative risk aversion is not too large (the Arrow–Pratt index $I_r$ of relative risk aversion is lower than 1).

We denote by $x(s)$ the optimal present value of an unemployed person’s search whose unemployment spell is $s$. At each interval where the benefit is continuous, $x(s)$ is recursively defined by the Bellman motion:

$$rx(s) = \max_{\theta \geq 0} \left\{ u(B(s) - \theta) + \mu \theta [y - x(s)] + dx/\theta ds \right\}, \quad [1]$$

with $\lim_{t \to T} x(s) = \lim_{t \to T^-} x(s)$. In the case of a successful search, the worker is assumed definitely to have exited the search market and obtains a utility $y > u(B_1)/r$. For simplicity, we consider a unique wage offer. Our results also hold for a weakly dispersed wage offer distribution. The meaning of equation [1] is standard: $x$ must be seen as an asset value. The return of the asset is equal to the flow benefit, plus the expected matching gain, plus the appreciation of the asset. The optimal effort solves:

$$u'(B - \theta) = \mu [y - x]. \quad [2]$$

Let $\phi \equiv u'^{-1}$, that is the inverse of the marginal utility function. The optimal effort is $b(s) = B(s) - \phi(\mu [y - x(s)])$. Search spending increases with their marginal benefits. Hence, the higher the salary, the greater the spending. But the larger the value of search, the lower the search effort. After time $s = T$, the path of unemployment benefits is stationary. Therefore, both the value of search $x_F$ and the search effort $b_F$ are constant. Simultaneously they solve equation [2] and the following:

$$x_F = \frac{u(B_2 - b_F)}{r + \mu b_F} + \frac{\mu b_F}{r + \mu b_F} y. \quad [3]$$

We assume that the unique solution $(b_F, x_F)$ to these equations exists (see Section 3). To obtain further properties of the function $b$, one needs to solve (backwards) the following boundary value...
problem:

\[
\frac{dx(s)}{ds} = rx - u(\phi(y - x)) - \mu[B(s) - \phi(\mu(y - x))][y - x]
\]

\[x(T) = x_F.\]

Note that \( x \) monotonically decreases with \( s \) and increases with \( T \). Indeed, and since \( x \) is a purely forward value,\(^4 \) \( x(s - \Delta T, T) = x(s, T + \Delta T) \) for any \( s \geq \Delta T \), where \( x(s, T) \) denotes the value of search at duration \( s \) when the UI potential duration is \( T \). As \( B_1 > B_2 \), it follows that \( x(s, T) < x(s, T + \Delta T) \) for all \( s < T \). Put differently, as \( T \) increases, future changes in the level of unemployment benefits come further. As time is discounted at a positive rate, the value of search instantaneously rises. The following proposition sums up these results:

**Proposition 1:** State dependence on hazards. Search spending strictly increases with unemployment duration on \([0, T]\) from \( b(0) \) to \( b(T) \). It then decreases to \( b_F \) and remains constant thereafter.

The path of hazards is depicted in Figure 1. The continuous line is the hazard implied by our model — the costly search (CS) case — while the dotted line is the hazard foreseen by standard job search models à la Van den Berg (1990) — the VdB case. The

**Figure 1.** Duration dependence in hazards

\[\text{hazard}\]

\[\mu b(0)\]

\[\mu b_F\]

\[T\]

\[\text{duration}\]

\[\text{VdB case}\]

\[\text{CS case}\]

important point is that the hazard decreases after the exhaustion of UI benefits in the CS case, while it remains high in the VdB case. In this latter case, the reservation wage is lowest after the exhaustion date of UI benefits. Therefore, the exit rate increases monotonically from 0 to \( T \), and remains constant after this duration. In our model, the disposable resources to search decrease after the exhaustion date, so that search spending decreases. This result holds even if we consider a more complex trajectory of UI benefits. Actually, if the insurance design is piecewise constant, globally decreasing, the exit rate of unemployment increases at each interval where \( B \) is constant, and decreases at each discontinuity. Hence, the exit rate exhibits state dependence, is globally negative, but positive prior to the exhaustion date of UI benefits. This kind of duration dependence is in line with the empirical evidence reported by Meyer (1990). The two following sections deal with the positive analysis of policy experiments; Section 5 discusses the optimal unemployment insurance design.

3. Stationary environment

This section focuses on the case of a stationary environment. That is, we assume that unemployment benefits are forever constant. Then, we investigate the effects of unemployment benefits on hazards.

Proposition 2: Stationary benefit and hazard

Let \( r_y > \min_{c>0} \langle u(c) + ru'(c)/\mu \rangle \). The hazard \( b_F \equiv b_F(B) \) is continuous and

(i) under assumption A1, there are \( B, \tilde{B}, \) and \( \tilde{B} \), such that (a) \( b_F(B) = 0 \) if \( B < \tilde{B} \) or \( B > \tilde{B} \); (b) \( b_F'(B) > 0 \) and \( b_F'(\tilde{B}) < 0 \); (c) \( b_F(\tilde{B}) = 0 \);

(ii) under assumptions (A1) and (A2), \( B, \tilde{B}, \) and \( \tilde{B} \) are uniquely determined.

Proposition 2 has strong policy implications. First, note that the hazard is zero when the benefit is either below the threshold \( B \), or above the threshold \( \tilde{B} \). In the former case, individuals consume their whole income. This phenomenon reflects poverty states and is due to the Inada condition: the marginal utility of consumption is very large when consumption is low. In particular, it is far higher than the marginal return to search. When the benefit is above the
threshold $\bar{B}$, individuals have no incentives to search. Indeed, the expected matching gain is not sufficient to outweigh the loss of utility due to the loss of income. Job search takes place between these two bounds; the existence of a non-empty interval $[\underline{B}, \bar{B}]$ requires that the flow probability to match per efficient unit of search $\mu$ and/or the value of a job $y$ are sufficiently large, i.e. $ry > \min_{c > 0} (u(c) + ru'(c)/\mu)$.

Second, the relationship between benefit and hazard is hump-shaped (see Figure 2). Consider a marginal increase in benefit when $B \in [\underline{B}, \bar{B}]$. By taking the derivative of search intensity with respect to benefit, we have

$$\frac{db_F}{dB} = \frac{1}{\text{income effect}} - \frac{\mu}{r + \mu b_F} \frac{[I_a(b_F)]^{-1}}{\text{reservation wage effect}}.$$  \[5\]

There is a direct income effect, and an indirect reservation wage effect. The income effect is positive: an increase in benefit translates into a rise in the global amount of resources, which tends to raise the hazard. However, the same increase in benefit improves the value of search, so that the unemployed person tends to search less intensively. This is the negative reservation wage effect. Proposition 2 shows that the income effect dominates the reservation wage effect when the level of benefit is high.

Figure 2. Hazard as a function of benefits
Conversely, the income effect is dominated when the benefit is closer to the threshold $\bar{B}$. Under assumption (A2), there is a unique level of benefit $\overline{B}$ below (above) which an increase in benefit decreases (increases) the average search duration.

Hence, changes in the level of UI benefits have both quantitative and qualitative effects on the average duration of unemployment. Most of the studies consider that an increase in unemployment benefits will reduce search efforts, independently of the level of such benefits. However, it seems incredible that employment probabilities of the very long-term unemployed are sensitive to the level of benefit: these individuals are simply out of the labour force. Moreover, the fact that benefits can be used mainly as a search input for small replacement ratios seems credible. Conversely, the fact that the reservation wage effect dominates when the social system is very generous is highly intuitive.

To close this section, we consider an example. In this example, we assume that the value of a job is given by $\frac{y}{C_{17}}u(1)$; that is, the present value of discounted utility flows derived from the (normalized) wage $w$ is thus the replacement ratio.

Example. Let $u(c) = c^{1/2}$, and assume $\mu > 2r$.

(i) $\underB = \frac{1}{2} - \frac{1}{2}(1 - 2r/\mu)^{1/2}$
(ii) $\overline{B} = \frac{1}{2} + \frac{1}{2}(1 - 2r/\mu)^{1/2}$
(iii) $\overline{\overline{B}} = \frac{3}{4} - r/\mu$
(iv) $b_{F} = B - [1 - (1 - r/\mu - B)^{1/2}]^2$ for all $B \in [\underB, \overline{B}]$.

In this example, we can provide bounds for the replacement ratio. Letting $r/\mu$ vary over $[0, 1/2]$, we obtain $\underB \in (0, 1/4)$, $\overline{B} \in (1/4, 1)$ and $\overline{\overline{B}} \in (1/4, 3/4)$. These ranges seem reasonable.

4. Comparative dynamics

This section examines the effects of changes in the design of social security systems in the case of a non-stationary environment. Formally, we are interested in the effects of the level and duration of unemployment benefits. We first introduce the average unemployment duration $\bar{s} = \bar{s}(T, B_1, B_2)$:

$$\bar{s} = \int_{0}^{\infty} \exp \left[ - \int_{0}^{s} \mu b(\sigma)d\sigma \right] ds. \tag{6}$$
Definition [6] can also be written:
\[
\bar{s} = \int_0^T \exp \left[ -\int_0^s \mu b(\sigma) d\sigma \right] ds + \exp \left[ -\int_0^T \mu b(\sigma) d\sigma \right] \bar{s}_F. \tag{7}
\]
where \( \bar{s}_F \equiv 1/(\mu b_F) \) is the conditional expected duration at time \( s = T \). Equation [7] distinguishes events before the exhaustion of UI benefits, and after. The second term on the RHS is equal to the probability of reaching the duration \( T \) times the expected duration at that time. We analyse the effects of \( T, B_1 \) and \( B_2 \) separately.

4.1 Potential duration of benefits

Consider an increase in the duration of benefits from \( T \) to \( T + \Delta T, \Delta T > 0 \). Figure 3 depicts the resulting hazard.

An unemployed person who has spent the duration \( s \) in unemployment under the new scheme is in the same situation as an unemployed person who has spent \( s - \Delta T \) under the previous benefit design. Therefore, those two individuals behave identically. Formally, if we denote by \( b \) the hazard corresponding to the potential duration \( T \), and by \( b' \) the hazard corresponding to the potential duration \( T + \Delta T \), we have \( b'(s) = b(s - \Delta T) \) for all \( s \geq \Delta T \). Graphically, it implies that the locus \( b' \) is simply the locus

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure3.png}
\caption{An increase in potential duration of UI benefits}
\end{figure}
translated rightward for durations $s > \Delta T$. A direct consequence is that $b'$ is lower than $b$ for durations belonging to $[0, T]$, but higher for durations belonging to $[T, T + \Delta T]$.

What are the implications in terms of average unemployment duration? The trade-off is the following: on the one hand, the hazard is lower at short durations, due to the reservation wage effect. This tends to raise the duration of unemployment. On the other hand, the hazard is higher at medium durations, which tends to reduce the average unemployment duration. Taking the derivative of $\bar{s}$ with respect to $T$, we get

$$\frac{d\bar{s}}{dT} = 1 - \mu b(0)\bar{s}. \quad [8]$$

The global effect depends on the size of $\mu b(0)\bar{s}$; that is, the ratio of the actual average unemployment duration to the average duration that would prevail were the hazard constant and equal to the initial hazard $\mu b(0)$.

**Proposition 3: Potential duration of UI benefits. A marginal increase in the potential duration of benefits leads to a decrease in average unemployment duration if the potential duration of benefits is sufficiently short.**

The shorter the potential duration of benefits, the smaller the reservation wage effect. Consequently, increasing a short duration of benefit entitlement may well translate into lower average unemployment duration.

### 4.2 Unemployment insurance benefits

UI benefits directly favour search from [2], but indirectly lower search spending through induced improvement in the unemployed person’s economic situation. The closer the exhaustion date of unemployment benefits, the lower the latter effect. Consequently, although UI benefits have ambiguous effects on hazards at low durations, they unambiguously improve search spending at medium durations.

Taking the derivative of $\bar{s}$ with respect to $B_1$ yields

$$\frac{d\bar{s}}{dB_1} = - \int_0^T \left[ \psi(s) + \psi(T) \frac{\bar{s}_F}{T} \right] ds, \quad [9]$$

where $\psi(s) \equiv \int_0^s \mu (db(\sigma, \cdot)/dB_1) d\sigma \exp[-\int_0^s \mu b(\sigma) d\sigma]$. 

**Proposition 4:** Level of UI benefits. A rise in the level of UI benefits increases hazards before the exhaustion date if the potential duration of benefits is sufficiently short. Consequently, the average duration of unemployment can decrease.

When the potential duration of benefits is short, the reservation wage effect is low. The worker therefore invests more in search spending in the case of an increase in the level of UI benefits; this reduces the average unemployment duration. Importantly, the existence of such a phenomenon does not depend on the level of UI benefits. Hence, the expected duration can always be reduced by an increase in the replacement ratio, associated with a decrease in the exhaustion date. However, as we shall see below, that decrease is sustained by a corresponding rise in the mean actualized amount of resources devoted to the unemployed.

**4.3 Public assistance benefits**

Section 3 has shown that PA benefits have ambiguous effects on hazards after $T$ has elapsed. However, an increase in PA benefits makes the value of search higher at all durations, and more especially before the duration $T$ is reached. Consequently, PA unambiguously benefits lower search spendings before the exhaustion of UI benefits. This intuitive property is in accordance with the estimates of Van den Berg (1990), who has shown, using Dutch data, that the elasticity of expected duration with respect to PA benefits is much larger than the corresponding elasticity with respect to UI benefits.

**4.4 More complex schemes**

The insurance design may exhibit more than one jump. Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997) have indeed shown that UI benefits should display many jumps. Suppose there are $n$ jumps, and consider the two following sequences of positive real numbers \( \{B_i\}_{i=1,n+1} \) and \( \{T_j\}_{j=0,n} \) with $T_0 = 0$, so that the benefit level is

\[
B(s) = B_i \text{ if } s \in [T_{i-1}, T_i),
\]

where, by convention, $T_{n+1} = \infty$. The results established in this section are modified as follows. First, benefits prevailing after time $T_1$ exert negative effects on search spending in the previous
periods. Second, what happens above $T_n$ is given by Proposition 2 (simply replace $B_2$ with $B_{n+1}$). Third, Propositions 3 and 4 still hold if one replaces ‘UI benefits’ with $B_1$, and $T$ with $T_1$.

5. Optimal unemployment insurance design

In this section, the following question is addressed: what is the optimal time sequence of unemployment benefits? The social criterion under consideration usually plays a crucial role in such an evaluation. Here, we consider that the public decision maker aims to minimize the average unemployment duration, under a resource constraint. We more especially focus on the conditions that imply a decreasing sequence of unemployment benefits.

Let $\bar{C} = \int_0^\infty B(s) \exp\left[-\int_0^s (r + \mu b(\sigma)) d\sigma\right] ds$ be the expected discounted amount of resources spent per unemployed worker. We assume that policy instruments are restrained to $B_1$, $B_2$ and $T$. Moreover, we suppose that there is a maximal amount of resources $K$ spent per unemployed person. This ad hoc cost is supposed to reflect the government budget constraint or, more generally, the generosity of the social security system. Optimal commands result from

$$\min_{B_1, B_2, T} \left\{ \bar{\bar{s}} \equiv \int_0^\infty \exp\left[-\int_0^s \mu b(\sigma) d\sigma\right] ds \right\},$$

subject to the individual search behaviour summarized by equations [3] and [4], and the resource constraint $\bar{C} \leq K$.

We will not solve program [11]. Instead, we will analyse the naive stationary problem and provide conditions under which the naive solution can be improved by means of a declining sequence of unemployment benefits. We will proceed in two steps. First, we consider the optimal stationary level of unemployment benefits; that is, we restrict policy instruments to $B(s) = B$ for all $s \geq 0$. Then, we authorize a step function benefit, and demonstrate conditions under which such a function can perform better than the best stationary policy.

Throughout this section, we suppose that both assumptions (A1) and (A2) hold. As stated by Proposition 2, this implies the stationary relationship between hazard and benefit is hump-shaped.
5.1 The stationary problem

Let \( B(s) = B \) for all \( s \geq 0 \) The mean unemployment duration is worth \( \bar{s}_F(B) \equiv \bar{s}_F(B) \equiv (\mu b_F)^{-1} \), while the mean discounted cost is \( \bar{C}_F(B) \equiv \bar{C}_F(B) \equiv B(r + \mu b_F)^{-1} \). How does the cost vary with the level of benefits \( B \)? A simple answer can be given for sufficiently low and sufficiently high values of \( B \). Indeed, we know that search spending is zero below the threshold \( B \) as well as above the threshold \( \hat{B} \). In either cases, the mean discounted cost is worth \( \bar{C}_F(B) = B/r \). Over \([B, \hat{B}]\), the derivative of the cost with respect to benefit can be written as

\[
\frac{d\bar{C}_F}{dB} = \frac{r + \mu b_F - \mu B d_b/F dB}{(r + \mu b_F)^2},
\]

where \( d_b/F dB \) is given by [5]. As \( d_b/F dB \) is positive when \( B \in [B, \hat{B}] \), the sign of \( d\bar{C}_F/dB \) is a priori ambiguous. However, under decreasing absolute risk aversion and 'small' relative risk aversion, we can state the following lemma.

Lemma. The function \( \bar{C}_F \) is strictly increasing in \( B \).

Figure 4 depicts the expected discounted cost for the unemployment insurance as a function of benefits.

Proposition 5: Optimal stationary policy. Let \( B_c \) be the unique solution of \( \bar{C}_F(B_c) = K \). The optimal stationary benefit \( B^* \) is worth

\[
B^* = \begin{cases} \hat{B} & \text{if } K > \bar{C}_F(\hat{B}) \\ B_c & \text{if } B/r < K < \bar{C}_F(\hat{B}) \\ [0, B_c] & \text{if } K < B/r \end{cases}
\]

The objective is to minimize the average unemployment duration. Provided the benefit level \( \hat{B} \) achieves that goal, the optimal policy consists in offering \( \hat{B} \) as long as the resource constraint does not bind, that is as long as \( K > \bar{C}_F(\hat{B}) \). In that case, note some of the resources initially available to the unemployed are not spent. However, if the resource constraint is actually binding, the minimum unemployment duration is no longer feasible. The optimal policy consists in offering the highest benefit that makes the constraint bind, that is \( B_c \). Finally, if the maximum cost for the unemployment insurance is lower than \( B/r \), the unemployed give up the search. Any \( B \in [0, B_c] \) satisfies the resource constraint and leads to an infinite average unemployment duration.
5.2 Declining benefits

Is there room for declining benefits with the duration spent in unemployment? To discuss this issue, we suppose that the level of UI benefits is given. Let $B_1 = B_2 + \kappa$, $\kappa > 0$. Two cases are worth investigation. Suppose first that the resource constraint does not bind in the stationary case. Additional resources might be spent, but increasing the stationary benefit would lessen the job search, and ultimately raise the average unemployment duration. However, we know from Propositions 3 and 4 that an increase in UI benefits coupled with a sufficiently low duration of benefits can reduce the average unemployment duration. Formally, we have

\[
\frac{d\bar{s}}{dT} = 1 - \mu b(0)\bar{s} \tag{14}
\]

\[
\frac{d\bar{C}}{dT} = B_1 - (r + \mu b(0))\bar{C}. \tag{15}
\]

Taking the limit of both derivatives when $T$ tends to 0 yields

\[
\lim_{T\to 0} \frac{d\bar{s}}{dT} = -\mu \kappa \bar{F} \leq 0 \tag{16}
\]

\[
\lim_{T\to 0} \frac{d\bar{C}}{dT} = \kappa (1 - \mu \bar{C}_F) \leq 0. \tag{17}
\]
The average unemployment duration decreases unambiguously, while the mean cost of unemployment insurance can either decrease or increase — but always remains finite. Consequently, the stationary policy can be improved when the social security system is wealthy: simply provide a larger benefit at the beginning of the unemployment period.

Yet there is an open question: is there a limit to this strategy? Or, more accurately, are we sure the optimal policy does not consist in providing an infinite benefit over a very short period? The answer is definitely negative. Let us proceed by contradiction. Assume the policy is fruitful so that the average unemployment duration is zero, i.e. $\bar{s} = 0$ Since

$$\bar{s} > \int_0^T \exp[-\mu B_1 s] ds + \exp[-\mu B_1 T] \bar{s} r,$$  \hspace{1cm} [18]

this assumption implies that $B_1 T$ tends to infinity. But the value of search evaluated at the beginning of the unemployment period is such that

$$x(0) > \int_0^T \mu B_1 \exp[-(r + \mu B_1 s)] y ds.$$ \hspace{1cm} [19]

The RHS of the latter inequality is the utility which would be attained were the unemployed individual devoting all his/her resources to job search, and were PA benefits zero. When $B_1 T$ tends to infinity, $x(0) > y$. As the individual choice of search effort is

$$u'(B_1 - b(0)) = \mu (y - x(0))$$ \hspace{1cm} [20]

we have $b(0) = 0$, which is not compatible with $\bar{s} = 0$.

Now, consider the case where the resource constraint is binding in the stationary case. Simply setting higher benefits at the beginning of the unemployment period may be inappropriate. Indeed, and according to [16] and [17], the average unemployment duration still decreases, but the expected discounted cost spent per unemployed person may increase — which is not feasible. Does it mean nothing can be done to improve the stationary policy in that case? Another strategy may actually be under consideration: setting simultaneously higher UI benefits and lower PA benefits. Assume $B_1 = B + \kappa$, $B_2 = B - \varepsilon$ and $T = \varepsilon$, with $\kappa, \varepsilon > 0$. The
derivatives of \( \bar{s} \) and \( \bar{C} \) with respect to \( \varepsilon \) are as

\[
\frac{d\bar{s}}{d\varepsilon} = -\frac{\partial \bar{s}}{\partial B_2} + \frac{\partial \bar{s}}{\partial T} \tag{21}
\]

\[
\frac{d\bar{C}}{d\varepsilon} = -\frac{\partial \bar{C}}{\partial B_2} + \frac{\partial \bar{C}}{\partial T}. \tag{22}
\]

Taking the limit of both derivatives when \( \varepsilon \) tends to 0 yields

\[
\lim_{\varepsilon \to 0} \frac{d\bar{s}}{d\varepsilon} = -\frac{\partial \bar{s}_F}{\partial B_2} - \mu \kappa \bar{s}_F \tag{23}
\]

\[
\lim_{\varepsilon \to 0} \frac{d\bar{C}}{d\varepsilon} = -\frac{\partial \bar{C}_F}{\partial B_2} + \kappa \left( 1 - \frac{\mu B}{r + \mu b_F} \right). \tag{24}
\]

The sign of these derivatives is generally ambiguous. However, in the neighbourhood of \( \hat{B} \), we have \( d\hat{b}/dB = 0 \). In that case, the derivatives are reduced to

\[
\lim_{\varepsilon \to 0} \frac{d\bar{s}}{d\varepsilon} = -\mu \kappa \bar{s}_F < 0 \tag{25}
\]

\[
\lim_{\varepsilon \to 0} \frac{d\bar{C}}{d\varepsilon} = -\frac{1}{r + \mu \hat{b}_F} + \kappa \left( 1 - \frac{\mu \hat{B}}{r + \mu b_F} \right) \tag{26}
\]

For sufficiently small values of \( \kappa \), the cost strictly decreases. Consequently, it is possible to improve the stationary constrained solution by setting declining benefits when the amount of resources spent per unemployed is lower than but not too far from \( \hat{C}_F \).

The following proposition sums up our results.

**Proposition 6:** Optimal non-stationary policy. There is \( K_\ast \in [B/r, \hat{C}_F) \) such that declining benefits are optimal whenever \( K \geq K_\ast \).

Three points should be noticed. First, in order to minimize the average unemployment duration, it is necessary to provide a minimum level of unemployment benefits at all durations. Below that threshold, unemployed workers simply consume the benefit.
and give up searching. Second, most of the time the optimal policy consists in providing declining benefits over unemployment spells. Indeed, declining benefits allow the provision of a large amount of income available for both search and consumption without creating strong disincentive effects. Third, stationary benefits may perform better than declining benefits. Such a case arises when the size of resources available per unemployed individual is low, that is, slightly above \( B/r \). On the one hand, simply setting higher benefits at the beginning of the unemployment period would actually reduce unemployment spells, but it would also raise the cost of unemployment insurance. On the other hand, decreasing PA benefits would dramatically increase the average unemployment duration; this could not be counterbalanced by an increase in UI benefits.

6. Summary and conclusion

We consider a model of job search in which search intensity depends on financial spending. As unemployed workers have no access to the financial market, unemployment benefits must be shared between consumption and search spending. In this framework, an increase in unemployment benefits, or in potential duration, exerts opposite effects on expected unemployment durations. First, a negative income effect through a rise in the global amount of resources available to search. Second, a standard positive reservation wage effect. The resulting hazard increases on each interval where the amount of benefits is constant, and decreases at each discontinuity of the duration–benefit function.

This basic model is then used to examine the effects of policy experiments. We show that the effects of stationary benefits on duration depend on the replacement ratio: no effect for very low replacement rates, a negative effect for medium ones, and a positive effect for higher rates. We also demonstrate that UI benefits can shorten the expected unemployment spell if the duration of benefits is sufficiently short. We then turn to policy implications. To this extent, we assume that the public decision maker aims to minimize the average unemployment duration under a resource constraint. Most of the time, a declining benefit scheme is optimal.

APPENDIX: proofs

Proof of proposition 2

In a stationary environment, the worker maximizes the value of his/her search so that

\[ x = \max_{b \geq 0} \left( \frac{u(B - b)}{r + \mu b} + \frac{\mu b}{r + \mu b} y \right). \]  \[ \text{[A1]} \]

Equation [A1] is equivalent to equation [1]. As the objective function of equation [1] is strictly concave, first-order conditions are necessary and sufficient. Consider the Lagrangean

\[ \ell(b, \lambda) = \frac{u(B - b)}{r + \mu b} + \frac{\mu b}{r + \mu b} y + \lambda b. \]  \[ \text{[A2]} \]

First-order conditions are

\[ \frac{\partial \ell}{\partial b} = \frac{u'(B - b)}{r + \mu b} - \frac{\mu}{r + \mu b} \frac{u(B - b)}{r + \mu b} + \frac{\mu}{r + \mu b} y + \lambda = 0. \]  \[ \text{[A3]} \]

\[ \lambda \geq 0, \quad b \geq 0. \]  \[ \text{[A4]} \]

Manipulating [A3], we have

\[ \frac{u'(B - b)}{r + \mu b} = \frac{\mu}{r + \mu b} [y - x] + \lambda. \]  \[ \text{[A5]} \]

Consider an interior solution. Such a solution solves

\[ (r + \mu b_F)u'(B - b_F) - \mu y + \mu u(B - b_F) = 0. \]  \[ \text{[A6]} \]

Let \( F(B, b) = u(B - b) + (r + \mu b)u'(B - b)/\mu - ry \).

(i) Under assumption A1, the function \( F \) is strictly increasing in \( b \), with \( F(B, 0) = G(B) = u(B) + ru'(B)/\mu - ry \) and \( F(B, B) = (r + \mu B)u'(0)/\mu - ry \gg 0 \). Therefore, \( b_F > 0 \) if and only if \( G(B) < 0 \). Due to Inada conditions and the boundary property of the felicity function, we have \( \lim_{B \to 0} G(B) = \lim_{B \to \infty} G(B) = \infty \). Consequently, there are several \( B > 0 \) such that \( G(B) = 0 \) if and only if
ry > \min_{c > 0} (u(c) + ru'(c)/\mu), which is true by assumption. Let $V = \{B > 0; G(B) = 0\}$, $B = \min_{B \in V} B$, $\bar{B} = \max_{B \in V} B$. Thus $G(B) > 0$ if $B < B$ or $B > \bar{B}$, i.e. (a) is proved. Moreover, $G'(\bar{B}) < 0$ and $G''(\bar{B}) > 0$. Since $F$ is strictly increasing in $b$, we can apply the implicit function theorem. We obtain

$$\text{sign} \left\{ \frac{db_F}{dB} \right\} = \text{sign} \left\{ 1 + \frac{\mu}{r + \mu b} u'(B - b_F) \right\}.$$  \[A7\]

Hence $b'_f(B)$ has the sign of $-G'(\bar{B}) > 0$ and $b'_f(\bar{B})$ has the sign of $-G'(\bar{B}) < 0$. This shows (b). Finally, by continuity of $b_F$ with respect to benefit there is $B \in (\bar{B}, \bar{B})$ such that $b'_f(B) = 0$, i.e. (c) is proved.

(ii) From (i) we know there are $B$ and $\bar{B}$ such that $G(B) = G(\bar{B}) = 0$. Taking the derivative of $G$, we get

$$G'(B) = u'(B)[1 - ri(B)/\mu],$$  \[A8\]

which has the sign of $H(B) = 1 - ri(B)/\mu$. Under assumption A2, $H$ is strictly increasing. This implies there is a unique $B_{min} = \arg \min_B G(B)$ such that $G'(B) \leq 0$ if and only if $B \leq B_{min}$. Consequently, $G$ has only two roots, i.e. $B$ and $\bar{B}$ are uniquely determined.

On $[\bar{B}, \bar{B}]$, the derivative of the hazard with respect to benefit is worth

$$\frac{db_F}{dB} = 1 + \frac{\mu}{r + \mu b} u'(B - b_F).$$  \[A9\]

Differentiating twice,

$$\frac{d^2b_F}{dB^2} = \left( \frac{\mu}{r + \mu b} \right)^2 I_a(B - b_F) \left\{ \frac{db_F}{dB} - [I_a(B - b_F)]^{-1} \right\}.$$  \[A10\]

From (i) we know there is $\hat{B} \in (\bar{B}, \bar{B})$ such that $b'_f(\hat{B}) = 0$. But

$$\frac{d^2b_F}{dB^2} \bigg|_{B = \hat{B}} = - \left( \frac{\mu}{r + \mu b} \right)^2 I_a(\hat{B} - b_F)[I_a(\hat{B} - b_F)]^{-1} < 0.$$  \[A11\]
since $I_0$ is strictly decreasing from assumption A2. It follows that $\hat{B}$ is unique. This closes the proof. \(\square\)

**Proof of Proposition 3**

Taking the derivative of $\tilde{s}$ with respect to $T$, we get

$$
\frac{d\tilde{s}}{dT} = 1 - \mu b(0)\tilde{s}. \tag{A12}
$$

Without loss of generality, let $B_1 = B_2 + \kappa$, $\kappa > 0$ and finite. When $T$ tends to 0, then $\tilde{s}$ tends to $\tilde{s}_F$. As

$$
u'(B_2 - b_F) = \mu [y - x_F] \tag{A13}
$$

while

$$
u'(B_1 - b(0)) = \mu [y - x_F] \tag{A14}
$$

we have $b(0) = b_F + \kappa$. Therefore,

$$
\lim_{T \to 0} \frac{d\tilde{s}}{dT} = -\mu \kappa \tilde{s}_F < 0. \tag{A15}
$$

The result follows from the continuity of $\tilde{s}$ with respect to $T$. \(\square\)

**Proof of Proposition 4**

Let $\hat{B}_1 > B_1$. In the remainder of the proof, a hat over a variable denotes a variable associated with $B_1$. In other cases, the variable is linked to $B_1$. For all $s < T$,

$$
\hat{b}(s) - b(s) = \hat{B}_1 - B_1 - [u'^{-1}(\mu(y - \hat{x})) - u'^{-1}(\mu(y - x))]. \tag{A16}
$$

As $T$ tends to zero, $\chi(s)$ and $\hat{\chi}(s)$ both tend to $x_F$. Therefore,

$$
\hat{b}(s) - b(s) = \hat{B}_1 - B_1 > 0. \tag{A17}
$$

The fact that $\hat{s} > \bar{s}$ follows directly from stochastic dominance. \(\square\)
Proof of the lemma

As \( \bar{C}_F = B/r \) when \( B \in [0, B] \cup [B, \infty) \), \( \bar{C}_F \) is strictly increasing on \( [0, B] \cup (B, \infty) \). For \( B \in [\underline{B}, \bar{B}] \), we have

\[
\frac{d\bar{C}_F}{dB} = \frac{r + \mu b_F - \mu B d b_F / dB}{(r + \mu b_F)^2},
\]

the sign of which depends on the numerator. But

\[
\frac{d b_F}{dB} = 1 + \frac{\mu}{r + \mu b_F} \frac{u'(B - b_F)}{u''(B - b_F)}.
\]

Strictly decreasing absolute risk aversion \((I'_a(c) < 0)\) implies that

\[
\frac{u'(B - b)}{u''(B - b)} < \frac{u'(B)}{u''(B)}, \text{ for all } b < B.
\]

It follows that

\[
\frac{d b_F}{dB} < 1 - \frac{\mu B}{r + \mu b_F} [I_a(B)]^{-1}
\]

\[
< 1 - \frac{\mu B}{r + \mu b_F}
\]

because of \( I_a(c) < 1 \) for all \( c \geq 0 \). Consequently,

\[
\frac{d\bar{C}_F}{dB} > \frac{1}{r + \mu b_F} - \frac{\mu B}{(r + \mu b_F)^2} \left( 1 - \frac{\mu B}{r + \mu b_F} \right).
\]

Therefore \( d\bar{C}_F/dB > 0 \) for all \( B \in [\underline{B}, \bar{B}] \). This completes the proof. \( \square \)

Proof of Proposition 5

The optimization problem consists in choosing \( B \) so as to maximize \( b_F \) subject to the resource constraint \( \bar{C}_F \leq K \). Due to the lemma, this constraint is equivalent to \( B \leq B_c \).

(i) Under \( I'_a(c) < 0 \) and \( I_a(c) < 1 \) for all \( c \geq 0 \), we know from Proposition 2 that \( b_F = 0 \) if \( B \in [0, B] \). Therefore, if \( B_c \leq B \), which

is equivalent to $K \leq B/r$, we have $b_F = 0$ for all $B$ such that $C_F(B) \leq K$. The optimal stationary policy is indeterminate in that case and $B^* \in [0, B]$.

(ii) Now assume that $K > B/r$ and consider the Lagrangean

$$l(B, \eta) = b_F(B) + \eta(C_F(B) - K).$$

[A24]

Under $I_0(c) < 0$ and $I_1(c) < 1$ for all $c > 0$, Proposition 2 ensures first-order conditions are necessary and sufficient — $b_F$ is hump-shaped on $[B, \bar{B}]$. Thus

$$b_F'(B^*) + \eta^*(C_F'(B^*)) = 0$$

[A25]

$$\eta^*(C_F(B^*) - K) = 0.$$  

[A26]

(iia) Suppose first $\eta^* = 0$. Then $b_F'(B^*) = 0$. From Proposition 2, this is equivalent to $B^* = \bar{B}$. It also implies that $C_F(B) \leq K$.

(iib) Now assume that $\eta^* < 0$. This implies $C_F(B^*) = K$ so that $B^* = B_c$. Moreover, $b_F'(B^*) > 0$.

It follows from (i), (iia) and (iib) that $B^* = \bar{B}$ if $C_F(B) \leq K$, and $B^* = B_c$ if $C_F(B) > K > B/r$. The claim follows. \hfill \square

**Proof of proposition 6**

The proof is in the text. It shows that the optimal stationary policy can marginally be improved by means of declining benefits. It is proved first that if $K > \bar{C}_F(B)$, then it is always possible to reduce the average duration of unemployment by setting UI benefits $B_1 > \bar{B}$ over a short period, while still satisfying the resource constraint. Second, it is shown that if $K$ is lower than $\bar{C}_F(B)$, but still lies in the neighbourhood of $\bar{C}_F(B)$, then it is possible to improve the stationary solution by setting some $B_1 > B_c$ and $B_2 < B_c$. \hfill \square

**Proof of the example.**

Condition $ry > \min_{c > 0} (u(c) + ru'(c)/\mu)$ implies that $\mu > 2r$. Then $G(B) = \frac{1}{2} B^{-1/2} - \mu[1/r - B^{1/2}/r]$. Points (i) and (ii) follow directly from solving $G(B) = 0$. Moreover,

$$F(b) = (B - b)^{-1/2}(r + \mu b)/2 - \mu(1 - (B - b)^{1/2}/(r + \mu))$$

[A27]

Multiplying by $\lambda \equiv (B - b)^{1/2}$, the optimal search intensity solves:

$$\lambda^2 - 2\lambda + r/\mu + B = 0.$$  \[A28\]

Under condition (E), the discriminant is positive. Two real roots exist, the greater of which checks $\lambda > B^{1/2}$. Point (iv) follows directly:

$$b_F = B - [1 - (1 - r/\mu - B)^{1/2}]^2.$$  \[A29\]

Taking the derivative of $b_F$ with respect to $B$, we have

$$\frac{db_F}{dB} = 2 - (1 - r/\mu - B)^{-1/2}.$$  \[A30\]

Hence, $db_F/dB = 0$ is equivalent to $B = \bar{B}$, which illustrates point (iii).

**Notes**

1. Holzer *et al.* (1994), for instance, show from US data that travel costs increase unemployment durations.
2. Mortensen (1977) obtained the correct property after the exhaustion date by assuming that (i) search is time consuming, and (ii) income and leisure are substitues.
3. We use the subscript ‘$F$’ — Final — to make $b_T \neq \lim_{s \to T_+} b(s)$ different from $b_F$.
4. However, the crucial point is that agents have no access to the financial market.
5. Both papers consider a discrete time model and show that benefits should change in each period. In our continuous time framework, the equivalent principle would lead us to analyse continuous alterations in the benefit sequence. This is is not only very difficult to address, but also unrealistic from an empirical point of view.
6. In more standard models, this objective would not make sense: as benefits would systematically deteriorate hazards, zero benefits would result.

**References**


The Time Sequence of Unemployment Benefits


