Option Value to Waiting Created by a Control Problem

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ABSTRACT

We study a principal-agent model in which there is an option to defer a capital project approval decision. A control (incentive) problem makes the option to wait valuable when it would not have been valuable otherwise. Deferring the project approval decision has both a cost and a benefit. The cost of waiting is that the agent’s uncertainty regarding future project cost realizations cannot be exploited. However, by delaying the first project’s approval decision, the principal can condition its approval on the agent’s cost report of the second project. Such conditioning can be valuable in the provision of incentives because of a diversification effect.

1. Introduction

In this paper, we study a principal-agent model in which there is an option to defer a capital project approval decision. An option to wait (defer) can be valuable because waiting reduces uncertainty. For example, a project with an initial positive expected net present value (NPV) may later be found to have a negative NPV. An option to wait can also be valuable because of the possibility of an improvement in the environment. For example, a project
with an initial negative NPV can have a positive NPV if interest rates decline.\textsuperscript{1} We present a different reason for an option to wait to be valuable: a control (incentive) problem makes an option to wait valuable when it would not have been valuable otherwise.

The control problem we study involves a tradeoff between production and information rents (slack) in a model of capital budgeting. Our model is closely related to Antle and Eppen [1985], Antle, Bogetoft, and Stark [1999], and, in particular, Antle and Fellingham [1990].\textsuperscript{2} In Antle and Eppen, a risk-neutral principal sometimes rejects (rations) a positive NPV project in order to curb a risk-neutral agent’s incentives to pad his budget. Antle and Fellingham [1990] show how the production-slack tradeoff can be fine tuned in a two-period extension of the basic resource allocation model. In Antle and Fellingham, the agent has a new project idea in each of the two periods. The project NPVs are uncorrelated. Gains to fine tuning arise because the principal can elicit truthful budgets from the agent regarding the NPV of the first period’s project by exploiting his uncertainty regarding the NPV of the second period’s project. In their model, the timing of project approval decisions is fixed: a project approval decision is made at the beginning of each period.

In our model, the principal has the additional choice of deferring the first project approval decision until period two. The disadvantage of waiting is that the agent now knows the second project’s NPV at the time he submits his budget regarding the NPV of the first project. The principal is no longer able to fine tune the production-slack tradeoff in the same manner as in Antle and Fellingham.

The advantage of waiting is that the principal’s information is improved. She can base the first project’s approval decision on the agent’s budget of the second project. Such conditioning allows the principal to better trade off production and rents because of a diversification effect. As uncorrelated projects are added, the probability distribution of NPVs becomes less dispersed. In response, the principal optimally chooses a cutoff such that production is lost only in the lower thin tail of the distribution and the agent earns significant rents only in the upper thin tail of the distribution. In a proposition, we provide conditions under which deferring the project approval decision is optimal.

In practice, while individual managers come up with project ideas at different dates, final board approval for large projects typically occurs on an annual basis. One commonly discussed advantage of grouping investment decisions is that it enables the firm to compare alternatives and to fund only the best positive NPV projects if the total resources required to fund proposed positive NPV projects exceeds the funds the board is willing to spend.

\textsuperscript{1} For an overview of the literature on deferment options and real options in general, see Dixit and Pindyck [1994] or Trigeorgis [1996].

\textsuperscript{2} For an overview of resource allocation models of this type, see Antle and Fellingham [1997].
on capital projects in a given year. This explanation suffers from the stan-
dard criticism that a firm should accept all positive NPV projects in order
to maximize firm value.

Another explanation for the grouping of investments decisions is that
projects often complement each other, particularly in “modern manufac-
turing environments” (Milgrom and Roberts [1995]). It is only by bundling
project decisions that the complementarities characteristic of such environ-
ments can be fully taken advantage of. The discussion usually focuses on
technological complementarities. Our paper points out that, even when
there are no technological complementarities, there may still be an incen-
tive complementarity that makes grouping investment decisions optimal.

2. Model

2.1 EARLY PROJECT APPROVAL DECISION

Our model is a variant of the model in Antle and Fellingham [1990]. A risk-
neutral principal (for example, a budget center) contracts with a risk-neutral
agent (for example, a division manager) for two periods. At the beginning of
each period, the principal has an opportunity to invest in a project. These
projects are referred to as project 1 and project 2, respectively. (In this
subsection of the paper, the sequential timing of the investment decisions
is taken as given; later, we will allow for a delay in the project 1 approval
decision.) If undertaken, each project produces a cash inflow of X at the
end of the period and requires a cash outflow (cost) of cX at the beginning
of the period, where \( c \in \{c_L, c_H \} \), \( c_L < c_H \). Denote the principal’s one-period
discount factor by \( c^0 \), \( c^0 < c^H < 1 \). The lower bound on the discount factor
implies \( c^0 X - c^H X > 0 \), i.e., even a high cost project has a positive NPV. The
upper bound on the discount factor implies the principal’s cost of capital
(discount rate) is positive.

Let \( p_i, p_i > 0 \), denote the common knowledge marginal probability that
a project’s cost is \( c_i \), \( i = L, H \). The joint probability of project 1’s cost be-
ing \( c_i \) and project 2’s cost being \( c_j \) is \( p_i p_j \), i.e., the two projects’ costs are
uncorrelated.

At the beginning of each period, the agent privately learns the cost of that
period’s project. Prior to making the project approval decision, the principal
asks the agent for a report on the project’s cost. The principal makes the
project 1 approval decision at the beginning of period 1 and the project 2
approval decision at the beginning of period 2. All funds for investment are
provided by the principal.

If the agent reports the project 1 cost is \( c_k \), the principal either accepts
\( (x_1^k = X) \) or rejects \( (x_1^k = 0) \) the project. The amount of resources transferred
to the agent at the beginning of period 1 is \( y_1^k \); the principal receives cash
inflow \( x_1^k \) at the end of period 1. If the agent reports the project 1 cost is
\( c_k \) and the project 2 cost is \( c_m \), project 2 is either accepted \( (x_2^{km} = X) \) or
rejected \( (x_2^{km} = 0) \). The amount of resources transferred to the agent at the
beginning of period 2 is $y^2_{km}$; the principal receives cash inflow $x^2_{km}$ at the end of period two.

The agent consumes organizational slack (funding above cost). The agent’s preferences are assumed to be additive over time, and utility is discounted using the factor $c_0$. The agent’s utility function is: $y^1_k - c_i x^1_k + c_0(y^2_{km} - c_j x^2_{km})$, where $c_i$ and $c_j$ are the true costs and $c_k$ and $c_m$ are the reported costs. The principal consumes the residual (cash inflow less transfer to the agent). The principal’s utility function is: $c_0 x^1_k - y^1_k + c_0(c_0 x^2_{km} - y^2_{km})$.

The sequence of events is as follows:

<table>
<thead>
<tr>
<th>Time Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agent privately learns project 1’s cost.</td>
</tr>
<tr>
<td>Contract $(x^1_i, y^1_i)$, $x^2_{ij}, y^2_{ij}, i = L, H$ is signed.</td>
</tr>
<tr>
<td>Agent submits report $c_k$ on project 1’s cost.</td>
</tr>
<tr>
<td>The agent is paid $y^1_k$ at the beginning of period 1.</td>
</tr>
<tr>
<td>The principal receives $x^1_k$ at the end of period 1.</td>
</tr>
<tr>
<td>Agent privately learns project 2’s cost.</td>
</tr>
<tr>
<td>Agent submits report $c_m$ on project 2’s cost.</td>
</tr>
<tr>
<td>The agent is paid $y^2_{km}$ at the beginning of period 2.</td>
</tr>
<tr>
<td>The principal receives $x^2_{km}$ at the end of period 2.</td>
</tr>
</tbody>
</table>

**Fig. 1.**—Timeline when project 1 approval decision is early.

The principal’s contracting problem is to maximize her expected utility subject to the following constraints. First, the individual rationality constraints (IR) require the contract be sufficiently attractive to the agent. In particular, whether the project 1 cost is low or high, the contract must provide the agent with an expected utility of at least his reservation utility, denoted $\bar{U}$. Second, the self-selection constraints (SS1 and SS2) ensure the agent has incentives to report each project’s cost truthfully. Imposing these constraints simplifies the search for an optimal contract. By the Revelation Principle (for example, Myerson [1979]), this simplification does not reduce the principal’s (or the agent’s) expected utility. Third, the resource feasibility constraints (RF1 and RF2) require the contract provide the agent with non-negative slack in each period if he is being truthful. The agent is assumed to be able to carry resources from the first to the second period. Resources carried over are assumed to grow at the rate $r_0$, where $c_0 = 1/(1 + r_0)$. Fourth, the output feasibility constraints (OF) require the project either be accepted or rejected.3

For simplicity, assume $\bar{U} = 0$. This ensures the resource feasibility constraints dominate the individual rationality constraints. The principal’s program (P1) is as follows:

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3 The only difference between the model presented in this subsection of the paper and the model in Antle and Fellingham [1990] is that they allow for partial production, i.e., $x^1_i, x^2_i \in [0, X]$. We restrict attention to projects that cannot be scaled down. For example, a bridge can either be built or not built.
2.2 Late Project Approval Decision

In this subsection, we consider the possibility of deferring the project 1 approval decision until the beginning of period two. In this case, each project’s approval and funding decision can be conditioned on the agent’s reports of both projects’ costs. Hence, $x^1_i$ and $y^1_i$ are replaced by $x^1_{ij}$ and $y^1_{ij}$. The sequence of events is as follows.

The principal’s contracting problem is given in program (P2). The constraints in the program differ from the constraints in (P1) in the following respect. The $(SS^1)$ and $(SS^2)$ constraints in (P1) are replaced by a single self selection constraint (SS) in (P2). The $(SS^1)$ constraint in (P1) ensures the agent wants to submit a truthful report on project 1’s cost, given he does not know project 2’s cost. The (SS) constraint in (P2) ensures the agent wants...
to report truthfully, given he knows the costs of both projects at the time he is reporting.\footnote{4}

**Program P2. Project 1 approval decision made in period two**

$$\text{Max } \sum_{i=\text{L, H}} \sum_{j=\text{L, H}} p_i p_j \left[ c_0 x_{ij}^1 - y_{ij}^1 + c_0 x_{ij}^2 - y_{ij}^2 \right]$$

s.t.

$$y_{ij}^1 - c_i x_{ij}^1 + y_{ij}^2 - c_j x_{ij}^2 \geq y_{km}^1 - c_i x_{km}^1 + y_{km}^2 - c_j x_{km}^2 \quad \forall i, j, k, m \quad \text{(SS)}$$

$$y_{ij}^1 - c_i x_{ij}^1 \geq 0 \quad \forall i, j \quad \text{(RF}^1)$$

$$y_{ij}^2 - c_j x_{ij}^2 \geq 0 \quad \forall i, j \quad \text{(RF}^2)$$

$$x_{ij}^1, x_{ij}^2 \in \{0, X\} \quad \forall i, j \quad \text{(OF)}$$

### 3. Result

As a benchmark, suppose the principal also observes the cost of each project. In this case, the solution is first-best: all projects are approved and the principal transfers to the agent exactly the cost of production. In the first-best case, there is only a cost to waiting. By waiting, the principal delays obtaining the returns from a profitable project. Since $c_0 < 1$, waiting is strictly costly.

Observation. In the absence of the control problem, the principal strictly prefers not to defer the project 1 approval decision.

When the agent privately observes the cost of projects, the optimal contract involves a tradeoff between rationing projects and limiting the agent’s information rents. Consider a one-period, one-project setting as in Antle and Eppen [1985]. The optimal contract is either the Slack Contract or the Rationing Contract. Under the Slack Contract, the project is always approved and the agent is paid $c_{iH} X$. The principal’s payoff is $(c_0 - c_{iH}) X$. Under the Rationing Contract, the project is approved only if the agent reports the cost is $c_L$ and, in this event, the agent is paid $c_{iL} X$. The principal’s payoff is $p_L (c_0 - c_{iL}) X$. Since these are the only two contracts that provide incentives for the agent to report truthfully, by the Revelation Principle, one of

\footnote{4The (RF$^2$) constraints in (P1) are written assuming the agent can carry forward slack from the first period to the second period. In our model, the solution is unchanged even if the agent cannot carry slack forward because, with risk-neutral participants (with identical discount rates), it is costless for the principal to move any first period slack to the end of the second period. Hence, the resource feasibility constraints in (P1) and (P2) are equivalent. Also, as in (P1), the individual rationality constraints in (P2) are dominated by the resource feasibility constraints.}
them must be optimal. Comparing the two payoffs, Rationing is preferred to Slack if \( R \leq p_L \), where \( R = \frac{c_0 - c_H}{c_0 - c_L} \). (Note since \( 0 < c_L < c_H < c_0 \), \( R \) lies between 0 and 1.)

When the setting is extended to include a second period and a second project, and a project approval decision is made in each period (as in (P1)), the optimal contract is not necessarily a two-fold replication of the Slack or Rationing Contracts. By exploiting the agent’s uncertainty about the second-period cost when he is submitting his first-period report, the principal is sometimes (for some parameter values) better able to trade off production and rents.

The new contract exhibits memory in the sense that period-two allocations depend on both period-one and period-two cost reports. The contract is as follows. The agent is offered the Slack Contract in period one. If the period-one cost report is \( c_H \), the agent is offered the Rationing Contract in period two; if the period-one cost report is \( c_L \), the period-two project is always approved and the agent is paid the expected cost of the project. Using our notation, \( x_1^L = x_1^H = X, y_1^L = y_1^H = c_HX, x_2^L = x_2^H = X, y_2^L = y_2^H = c_LX, x_{HH}^2 = y_{HH}^2 = 0, x_{HL}^2 = x_{LH}^2 = X, y_{HL}^2 = y_{LH}^2 = (p_Lc_L + p_Hc_H)X \). (For a proof, see Antle and Fellingham [1990].)

When the Antle and Eppen setting is extended to include a second project and both project approval decisions are made at the same time (as in (P2)), the optimal contract is again not necessarily a two-fold replication of the Slack or Rationing Contracts. There is no uncertainty on the part of the agent to exploit. However, the principal’s information is improved. By basing the project 1 approval decision on the agent’s project 2 cost report, the principal can better trade off production and rents than under the Slack or Rationing Contracts.

There is a diversification effect introduced by multiple uncorrelated projects. As more projects are added, the probability distribution of NPVs becomes less dispersed. In response, the principal optimally chooses a cutoff such that production is lost only in the lower thin tail of the distribution and the agent earns significant rents only in the upper thin tail of the distribution.

In addition to a twofold replication of the Slack and Rationing contracts, the only other contract that can be a solution to (P2) is as follows. If the agent’s (project 1, project 2) cost reports is \((c_L, c_L), (c_L, c_H)\), or \((c_H, c_L)\), both projects are approved and the agent is paid \((c_L + c_H)X\). If the agent reports \((c_H, c_H)\), neither project is approved. Using our notation, \( x_{HH}^1 = x_{HH}^2 = y_{HH}^1 = y_{HH}^2 = 0 \); otherwise, both projects are approved and \( c_L + c_H \) is paid to the agent. We refer to this contract as the Late Approval Contract.

\(^5\) This contract is referred to as the Mem2 Contract in Antle and Fellingham. There is also a Mem1 Contract, which involves partial production. Since we consider only projects that can either be accepted or rejected in whole, Mem1 is not a feasible solution to (P1).
While in the first-best setting there is no advantage to waiting, the presence of a control problem sometimes creates value to the option of delaying the project 1 approval decision until period two. The following proposition provides conditions under which late project approval is strictly optimal.

**PROPOSITION.** In the presence of the control problem, the principal strictly prefers:

(i) not to defer the project 1 approval decision and offer the Rationing Contract if

\[ 0 < R < \frac{1 - c_0p_H}{c_0(1 + p_H)}. \]

(ii) to defer the project 1 approval decision and offer the Late Approval Contract if

\[ \frac{1 - c_0p_H}{c_0(1 + p_H)} < R < \frac{c_0p_Lp_H}{1 - c_0p_L}. \]

(iii) not to defer the project 1 approval decision and offer the Early Approval Contract if

\[ \frac{c_0p_Lp_H}{1 - c_0p_L} < R < \frac{p_L}{1 - p_Lp_H}. \]

(iv) not to defer the project 1 approval decision and offer the Slack Contract if

\[ \frac{p_L}{1 - p_Lp_H} < R < 1. \]

**PROOF.** From Antle and Fellingham [1990], the solution to (P1) is a twofold replication of the Slack Contract, a twofold replication of the Rationing Contract, or the Early Approval Contract. The principal’s expected residual corresponding to these contracts is \((c_0 - c_H)X + c_0(c_0 - c_L)X, p_L(c_0 - c_L)X + c_0p_L(c_0 - c_L)X,\) and \((c_0 - c_H)X + c_0p_Lp_L(c_0 - c_L)X + c_0p_L(c_0 - c_L)p_L(c_0 - c_H)X,\) respectively.

Replace the output feasibility constraints in (P2) with their continuous analogs, i.e., \(x_1^i, x_2^i \in [0, X].\) Doing so converts the integer program into a linear program and allows the use of standard duality arguments to characterize the solution of the revised program. In particular, duality arguments show that the solution to (P2) with the continuous output feasibility constraints is a twofold replication of the Slack Contract, a twofold replication of the Rationing Contract, or the Late Approval Contract. Since none of the three contracts involve partial production, these are also the solutions to (P2) with the integer output feasibility constraints. The principal’s expected residual corresponding to these contracts is \(2c_0(c_0 - c_H)X, 2c_0p_L(c_0 - c_L)X,\) and \(c_0(p_Lp_L + p_Lp_H + p_Hp_L)(2c_0 - c_L - c_H)X,\) respectively.

If the Slack (Rationing) Contract is optimal in (P2), it is strictly better for the principal not to defer the project 1 approval decision. Since \(c_0 < 1,\) the principal can obtain a higher payoff by not deferring the project 1 approval decision and offering the Slack (Rationing) Contract. The only time it is optimal to defer the project 1 approval decision is if (1) the Late Approval
### Table 1

<table>
<thead>
<tr>
<th>Timing and contract</th>
<th>Informational rents</th>
<th>Lost production</th>
<th>Delayed production</th>
</tr>
</thead>
<tbody>
<tr>
<td>do not defer &amp; Rationing</td>
<td>0</td>
<td>((1 + c_0)p_H(c_0 - c_H)X)</td>
<td>0</td>
</tr>
<tr>
<td>defer &amp; Late Approval</td>
<td>(c_0 p_L(c_H - c_L)X)</td>
<td>((1 + c_0)p_H(c_0 - c_H)X)</td>
<td>((1 - c_0)[p_L^2 (c_0 - c_L) + p_L p_H (c_0 - c_L)]X)</td>
</tr>
<tr>
<td>do not defer &amp; Early Approval</td>
<td>(p_L(c_H - c_L)X)</td>
<td>(c_0 p_H (c_0 - c_H)X)</td>
<td>0</td>
</tr>
<tr>
<td>do not defer &amp; Slack</td>
<td>((1 + c_0)p_L(c_H - c_L)X)</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Contract is optimal under (P2) and (2) the principal’s payoff under the Late Approval Contract is higher than the optimal contract in (P1). Note that (1) is automatically satisfied when condition (2) is satisfied. Comparing the principal’s payoffs under a twofold replication of the Slack Contract in (P1), a twofold replication of the Rationing Contract in (P1), the Early Approval Contract in (P1), and the Late Approval Contract in (P2) yields the conditions given in the proposition.

The allocations obtained under the Early Approval Contract cannot be achieved in the late project approval regime. Since the agent knows the project 2 cost prior to submitting his project 1 cost report in the late regime, he will not be willing to report the truth when the costs are \((c_L, c_H)\). Instead, he will report \((c_H, c_H)\). By reporting truthfully, the agent’s expected utility is \(c_0(c_H - c_L)X - c_0(c_H - p_L c_L - p_H c_H)X\). By reporting \((c_H, c_H)\), the agent’s expected utility is \(c_0(c_H - c_L)X\), which is greater. The agent’s uncertainty about project 2’s cost at the time of submitting the project 1 cost report is needed to make the Early Approval Contract allocations incentive compatible.

Also, the allocations obtained under the Late Approval Contract cannot be achieved in the early project approval regime because the approval decision for project 1 with a \(c_H\) cost report is conditioned on the project 2 cost report. If the project 2 cost report is low, the period 1 project is accepted; otherwise, it is rejected. Such conditioning is infeasible in the early project approval regime.

We conclude this section by highlighting the tradeoff between production and information rents that occur under the optimal contracts in our setting.\(^6\) In Table 1, we decompose the cost to the principal (relative to the first best solution) of using the contracts listed in the proposition.

As one moves down the contracts listed in Table 1, more (expected) information rents are paid but a positive NPV project is rejected less often. In the late approval regime, there is also the cost of delaying project 1 even when

\(^6\) We thank the referee for this suggestion.
project 1 is approved. An alternative way of obtaining the conditions presented in the proposition is to calculate the production-rents tradeoff under a given contract and compare it with its neighboring contracts. For example, the condition under which the Late Approval Contract is optimal can be found by calculating the total cost of information rents, lost production, and delayed production under it and comparing it to the total cost under the Rationing and the Early Approval contracts.

In particular, the Late Approval Contract is preferred to the Rationing Contract (Early Approval Contract) if and only if the first (second) inequality is satisfied.

\[
c_0 p_L^2 (c_H - c_L) X + (1 + c_0) p_H^2 (c_0 - c_H) X + (1 - c_0) [p_L^2 (c_0 - c_L) + p_L p_H (c_0 - c_H) + p_H p_L (c_0 - c_H)] X < 0 + (1 + c_0) p_H (c_0 - c_H) X + 0
\]

\[
c_0 p_L^2 (c_H - c_L) X + (1 + c_0) p_H^2 (c_0 - c_H) X + (1 - c_0) [p_L^2 (c_0 - c_L) + p_L p_H (c_0 - c_H) + p_H p_L (c_0 - c_H)] X < p_L (c_H - c_L) X + (1 + c_0) p_H (c_0 - c_H) X + 0
\]

The first inequality, when simplified, yields the condition \( R > \frac{1 - c_0 p_L}{c_0 (1 + p_H)} \). The second inequality, when simplified, yields the condition \( R < \frac{c_0 p_L p_H}{1 - c_0 p_L} \). These are the conditions listed in part (ii) of the proposition. The conditions under which each of the other three contracts are optimal can be derived similarly.

4. Conclusion

In this paper, we show that an option to wait can become valuable because of the introduction of a control problem. Waiting has both a cost and a benefit. The cost of waiting is that the agent’s uncertainty regarding future project cost realizations cannot be exploited. However, by delaying the project approval decision, the principal can condition the project 1 approval decision on the project 2 cost report. Such conditioning can be valuable because of a diversification effect.

In our paper, a control problem makes a real option valuable. Reversing this causality might be an interesting topic for future research. That is, are there settings in which real options create control problems? Presumably, a key feature of such a model would be participants who value the real option differently. What observed practices can be better understood by studying such models?

REFERENCES


