Capital Requirements in a Financially Driven Business Cycle Model

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We consider a simple overlapping generations economy where, because of asymmetric information and limited liability both in the loan and the deposits markets, firms have the incentive to undertake less efficient investment projects, while intermediaries have the incentive to monitor a smaller number of firms. Because of the positive relationship between the deposit interest rate and the level of monitoring, the lending activity of intermediaries may cause endogenous fluctuations in the level of economic activity.

In this economy, a higher capital requirement, introduced to render deposit contracts incentive compatible, implies a higher steady state stock of capital, fewer bankruptcies among intermediaries and smaller fluctuations in the level of economic activity.

(J.E.L. E32, D82, G28).

1. Introduction

Capital requirements are nowadays one of the most prominent tools of banking regulation and, given their importance and their widespread adoption in most countries, they have been the subject of intense theoretical and empirical scrutiny. Since the late 1970s, the effectiveness of capital requirements in reducing excessive risk taking by banks has been investigated utilizing a large variety of models which differ greatly in their underlying assumptions, such as the degree of market incompleteness, the degree of competition and the informational structure of the credit market.

Regardless of the differences in the approaches used to investigate the role and the effects of bank capital regulation, almost all the models use a partial equilibrium approach. Obviously, this approach is the right one if we want to understand, in depth, the impact of capital requirements on the behaviour of banks and on the complex interactions that arise, at the microeconomic level, in the credit market.

We must consider, however, that the ultimate goal of regulation is to reduce the dynamic instability of the aggregate economy that may be caused by malfunctioning financial markets and that instruments like capital requirements may be able to affect significantly the level of economic activity.
recent episodes of financial crisis, like the East Asian crisis of 1997 for example, the behaviour of financial intermediaries and the absence of an effective regulatory system has been blamed as a major determinant of the large output contraction that occurred in the region.¹

When these issues are addressed, partial equilibrium models are no longer adequate and it is necessary to use a dynamic general equilibrium analysis that allows a more complete study of the interaction between the financial sector and the real sector of the economy. To this effect, we propose in this paper a dynamic overlapping generations model, where the behaviour of banks is a source of aggregate instability for the economy, and we study the effects of capital requirements on the economy.

The model we consider here is similar to the model analysed in Mattesini (2001) which shows that the behaviour of intermediaries may be the cause of endogenous fluctuations in the level of economic activity.² The model is characterized by a double moral hazard problem. Because of asymmetric information and limited liability, borrowers may have the incentive to undertake riskier and less efficient investments and, therefore, banks must invest resources to monitor firms and make sure that they select the most efficient projects. At the same time, intermediaries, which are subject to stochastic shocks, may undertake actions to shift risk on to depositors, who are unable to observe the actions of lenders and are, therefore, protected by deposit insurance.

The moral hazard problem between intermediaries and firms generates dynamic instability in the economy. Since monitoring is costly, banks weigh costs and benefits and determine an optimal level of monitoring which, in turn, depends on the level of interest rates. When capital and wages increase, interest rates tend to decrease; and banks are pressured by competition to contain costs by reducing the level of monitoring. This has the effect of reducing the number of ‘good’ investment projects undertaken by capital producing firms and to increase the number of firms going bankrupt. Because of the larger number of inefficient projects, the capital stock in the following period will be lower. A lower capital stock, in turn, implies lower wages, higher interest rates, more monitoring; and the dynamic process starts again, leading to cycles in the level of economic activity.

Capital requirements in this model are essential because they guarantee that deposit contracts are incentive compatible. Moreover, they may represent an effective tool by regulators, because they influence the loan interest rate and, therefore, the number of firms that are monitored by intermediaries.

¹ See, among many others, Corsetti et al. (1998).
² In this respect, the model differs from the models proposed by Bernanke and Gertler (1989), Kyotaki and Moore (1997), Calmstrom and Fuerst (1997) and Bernanke et al. (1998) where financial intermediation amplifies the effects of exogenous productivity shocks. Endogenous cycles induced by credit market imperfections have been recently derived by Suarez and Sussman (1997) in a very different theoretical context.
affecting not only the probability of bank failure, but also the size and the depth of business fluctuations.

In section 2, we briefly describe the model; in section 3, we analyse the behaviour of consumers/depositors and that of capital producing firms; and, in section 4, we analyse the equilibrium in the credit market. The equilibrium of the whole model and its dynamics are studied in section 5 while, in section 6, we consider the effects of capital requirements.

2. The Model

We consider a discrete time economy populated by an infinite sequence of two-period lived overlapping generations. At each date, a continuum of young agents is born with a unit mass.

At each time $t$, a single final good is produced using capital $K_t$ and labour $L_t$. Output $Y_t$ is given by $Y_t = F(K_t, L_t)$ where $F$ is a standard constant returns to scale production function. We assume:

\begin{assumption}
\begin{align*}
F_k(K_t, L_t) &> 0 \\
F_L(K_t, L_t) &> 0 \quad \text{for all } (K_t, L_t) > 0 \\
F(0, L_t) &= F(K_t, 0) = 0
\end{align*}
\end{assumption}

Denoting by $k_t$ the capital–labour ratio and, by $y_t$, the output–labour ratio, the linear homogeneity of $F(\cdot)$ implies $y_t = f(k_t)$ where $f(0) = 0$, $f'(k_t) > 0$, $f''(k_t) < 0$. We also assume that capital depreciates completely in production in each period.

Perfect competition in the final good sector implies that the wage rate $w_t$ and the price of the input $k_t$, $\rho_t$ (both measured in terms of the good $y_t$), are given by

\begin{align*}
(1) & \quad w_t = f(k_t) - f'(k_t)k_t = w(k_t) \\
(2) & \quad \rho_{t+1} = f'(k_{t+1}) 
\end{align*}

Equations (1) and (2) imply zero profits for all firms producing $y_t$.

Young agents are divided into three types: ‘borrowers’, ‘uninformed lenders’ and ‘informed lenders’. Borrowers have access to a set of stochastic technologies for converting the time $t$ final good into time $t + 1$ capital, but are endowed with no labour. Uninformed lenders have no access to these technologies and cannot observe the actions undertaken by borrowers, but are endowed with a unit of labour which is supplied inelastically and use their wage income to fund young and old period consumption. Informed lenders also have no access to the borrowers’ technologies but can, instead, utilize a
monitoring technology that allows them to observe the actions of the borrowers at some cost.

3. Borrowers and Uninformed Lenders

Borrowers and informed lenders consume only in the second period of their lives, while uninformed lenders consume in both periods. We denote by $\alpha_1$, the fraction of borrowers in the economy, by $\alpha_2$, the fraction of informed lenders and, by $1 - \alpha_1 - \alpha_2$, the fraction of uninformed lenders.

Uninformed lenders at time $t$ choose the level of young age consumption $c_{1t}$ and old age consumption $c_{2t+1}$ to maximize a utility function $u(c_{1t}, c_{2t+1})$ which is strictly increasing, strictly concave, twice continuously differentiable and satisfies $\lim_{c \to 0} u'(c) = \infty$ and $\lim_{c \to \infty} u'(c) = 0$. At the end of the first period of their lives uninformed lenders can either store their savings, earning a gross return $r_{t+1} = 1$, or deposit their savings with informed lenders, earning a gross return $r_{t+1} \geq 1$ which they take as parametric. We assume that the government provides deposit insurance, so that the return on deposits is deterministic and independent of any shock to the profits informed lenders obtain from their intermediation/monitoring activity. Utility maximization subject to the constraints $c_{1t} = w_t - s_t$, $c_{2t+1} = r_{t+1}s_t$ gives us a savings function $s(w_t, r_{t+1})$. We assume that consumption in both periods is a normal good and that savings are not decreasing in the rate of return, i.e. we assume

**Assumption 2** $1 \geq s_w \geq 0, s_r \geq 0$.

Borrowers consume only in the second period of their lives. Since, as we will see in the next section, borrowers’ consumption $c_{t+1}^b$ is stochastic, we assume that borrowers are expected value maximizers, i.e.

**Assumption 3** $\mathbb{E}u(c_{t+1}^b) = \mathbb{E}(c_{t+1}^b)$.

Each borrower has access to two possible investment projects: a ‘good’ project G and a ‘bad’ project B. We assume

**Assumption 4** Project G transforms $q > 0$ units of the final good invested at time $t$ into $aq$ units of capital at time $t+1$. Project B transforms $q > 0$ units of the final good invested at $t$ into $bq$ units of capital at time $t+1$ with probability $p$ and 0 with probability $1 - p$.

**Assumption 5** $a > bp$.

To set production going, since capital is essential in the production
process, we assume that the first generation of old uninformed lenders are endowed with a level of capital $k_0 > 0$. We assume that the amount of the good needed to operate the investment project is not too large, i.e.

**Assumption 6**

$$\frac{(1 - \alpha)}{\alpha} w(k_0) \geq q$$

4. Informed Lenders

Being endowed with a monitoring technology, informed lenders set up at time $t$ a firm that we call ‘intermediary’ that borrows from uninformed lenders and specializes in lending to firms. At the end of time $t$, informed lenders receive an exogenous endowment $\omega$. Since such endowment is received at the end of the first period of the agents’ lives, it cannot be used by intermediaries for their lending activity. However, it can be used as collateral in the risky intermediation business. Intermediaries are required to pledge an amount $\mu \omega$ that will be transferred to the government (i.e. the deposit insurer) in case of default. We call $\mu$ a ‘capital requirement’. Informed lenders are also required to pay a tax $\tau$ in the second period of their lives. Denoting by $N$, the number of loans granted by each intermediary, we assume

**Assumption 7**

$$Nq \geq \mu \omega - (2\tau \mu \omega) \quad \omega \geq \frac{\tau}{1 - \mu}$$

implies that the capital requirement is always lower than the amount of loans granted to firms.

Since they are not endowed with any initial level of the final good, borrowers seek to finance investment projects by borrowing in the market. A loan contract will establish a repayment $x_{t+1}$ for the loan. If the firm undertakes the ‘bad’ investment and the project fails, the repayment will be zero. The credit market is clearly characterized by a moral hazard problem since the borrower, in the absence of monitoring, will not always choose the ‘good’ technology, but will choose it only if it gives higher expected profits, i.e. only if

$$\rho_{t+1}aq - x_{t+1}q \geq p(\rho_{t+1}bq - x_{t+1}q)$$

(3)

The market is characterized by asymmetric information and intermediated lending. We assume that individual lenders cannot observe borrowers but that there are some particular firms in the market, that we call intermediaries, that specialize in lending to capital producers and are endowed with a monitoring
technology. Intermediaries can observe a borrower’s choice at a cost $\gamma(n_t)/n_t$ where $1 > n_t > 1$ is the fraction of firms monitored at time $t$. We assume

**Assumption 8**

$$\gamma(n_t) = \gamma n_t^2$$

where

$$\frac{f'(a^q)pb}{2} \geq \gamma$$

Assumption 8 implies that the average cost function for an intermediary is quadratic and is therefore an increasing and convex function of $n_t$, which implies that monitoring is not subject to increasing returns. Since, to our knowledge, empirical evidence on this point is not generally available, we can rely on three types of arguments to justify this assumption. The first obvious one is that competition is hard to reconcile with increasing returns. The second is that monitoring implies mainly the acquisition of information; since this activity often requires the use of highly specialized personnel, it may imply an increase in average monitoring costs. An increase in the number of firms monitored, in fact, often implies the hiring of new officers which must be trained both formally and through a learning-by-doing process. The third argument is that information is often very specific to areas and sectors. Intermediaries that specialize in monitoring firms belonging to particular local productive systems or to particular sectors will not, in general, be able to reduce average costs by expanding the number of firms monitored. As a matter of fact, it is often argued that the comparative advantage of small local banks lies in the acquisition of information and in the ability to monitor efficiently.

Intermediaries collect deposits and lend to capital producing firms. If an intermediary decides to monitor, the borrower will be obliged to choose the ‘good’ project and will repay the amount $x_{t+1}$ with certainty, but the intermediary will face the monitoring costs $\gamma(n_t)/n_t$. If the intermediary does not monitor, the borrower might choose the ‘bad’ project. If he does, he will default on the loan with probability $p$. Given the large number of borrowers, $p$ not only represents the probability that a borrower that chooses a ‘bad’ technology defaults, but also the number of borrowers that go bankrupt.

In principle, intermediaries could try to induce firms to adopt the good technology by imposing an incentive compatibility constraint, i.e. by imposing a loan interest rate such that (3) is always satisfied. Since, however, we want to concentrate in this paper on the monitoring activity of intermediaries, we rule out this possibility by assuming that such an interest rate does not exist, i.e. that borrowers always choose the bad technology. A sufficient condition for this to occur is

$$f'(k_{t+1}) \geq (1 - p)/(a - bp)x_{t+1}. $$

Since we must always have $x_{t+1} > r_{t+1} > 1$, we immediately see that, if assumption 8 is satisfied, there is no loan interest rate $x_{t+1}$ that satisfies (3).
Assumption 9  For every $k_t > k_0$

$$f'(k_t) < \frac{1 - p}{a - bp}$$

Given this assumption, the only way intermediaries can induce firms to adopt the good technology is by monitoring them. In the real world, monitoring means a series of activities like the inspection of a firm potential cash flow, its balance sheet position, its management, etc. Often, monitoring means verifying that a firm respects the many covenants that are usually included in financial contracts to induce a diligent behaviour by borrowers. With this in mind, we assume that a monitor can always prevent a firm from using a ‘bad’ technology.

A crucial assumption of this model is that intermediaries’ revenues are subject to a stochastic shock $z$ which is distributed according to the function $G(z)$ with density $g(z)$ in the interval $[0, \hat{z}]$. We assume

Assumption 10

$$G(\hat{z}) = \frac{\hat{z}}{z} \quad g(z) = \frac{1}{z}$$

which implies that the shock is distributed uniformly in the interval $[0, \hat{z}]$. The shock $z$ is meant to represent the uncertainty which characterizes the intermediation business, such as sudden withdrawal of deposits, changes in macro-economic and financial market conditions, managerial and organizational failures. The intermediaries’ budget constraint implies that all the deposits collected at time $t$ are then lent to capital producing firms. Defining by

(4) \[ \hat{z}_t = Nq[x_{t+1}n_t(1 - p) + x_{t+1}p - \gamma n_t^2 - r_{t+1}] + \mu \omega \]

the level of $z$ above which an intermediary defaults, the consumption of an informed lender at time $t$ will be

(5) \[ c^i_{t+1} = \sigma^i_{t+1} + \omega - \tau \]

where

(6) \[ \sigma^i_{t+1} = Nq[x_{t+1}n_t - \gamma n_t^2 + px_{t+1}(1 - n_t)] - r_{t+1}d_t - z_t \text{ if } z_t \leq \hat{z}_t \]

$$\sigma^i_{t+1} = -\mu \omega \text{ if } z_t > \hat{z}_t$$

The expected profits of an intermediary from lending to a number $N$ of firms are therefore given by

Integrating by parts and equating to zero, we obtain

\[ E\pi^i_{t+1}(x_{t+1}, n_i) \equiv Nq[x_{t+1}n_i(1-p) + x_{t+1}p - \gamma n_i^2 - r_{t+1}]G(\hat{z}_i) \]

\[ - \int_{0}^{\hat{z}_i} zdG(z) - \mu \omega (1 - G(\hat{z}_i)) \]

Equation (8)

\[ E\pi^i_{t+1}(x_{t+1}, n_i) \equiv \int_{0}^{\hat{z}_i} G(z)dz - \mu \omega = 3D0 \]

Because of perfect competition in the intermediation business, we assume that (8) is always satisfied. It is important to notice that the existence of a capital requirement \( \mu \) in this model is necessary for deposit contracts to be incentive compatible. If \( \mu = 0 \), in fact, (8) could be satisfied by setting \( \hat{z}_i = 0 \), i.e. by choosing levels of \( n_i \) and \( x_{t+1} \) at which intermediaries would go bankrupt with probability one. If intermediaries revenues are stochastic, capital requirements play an important role in this economy since they prevent competitive banks from defaulting strategically on deposits.

Denoting by \( \theta_o \), the probability of being monitored, a firm will borrow if

\[ \rho_{t+1}[\theta_o a + (1-\theta_o) pb] - (1-p)x_{t+1} \geq 0 \]

i.e. if expected return from borrowing is always greater than the expected cost. As we will see later on (see footnote 7), given assumption 8 and the participation constraint, equation (9) is always satisfied.4

In the simple context we consider here, where the size of the loan is given and both borrowers’ and lenders’ profits are linear in the loan interest factor \( x_{t+1} \), intermediaries simply choose the optimal level of monitoring while the equilibrium loan interest rate is implied by the intermediaries’ zero profit constraint. Borrowers, given the loan interest rate, decide only whether to undertake the investment project and the type of project. We can therefore state

**Definition 1**

The optimal level of monitoring and the optimal loan interest rate are given by the levels of \( n_i \) and \( x_{t+1} \) that maximizes (7) subject to (8), (9), and the constraints: \( 1 \geq n_i, n_i \geq 0 \).

Let us ignore, for the moment, constraint (9). In this case, the first-order conditions are given by

\[ x_{t+1}(1-p) = 2\gamma n_i \iff 1 > n_i > 0 \]

\[ x_{t+1}(1-p) > 2\gamma n_i \iff n_i = 1 \]

\[ x_{t+1}(1-p) < 2\gamma n_i \iff n_i = 0 \]

4 Because of this, it is not necessary in this model to specify the process through which firms’ expectations are formed.

Equation (10) states that, for a profit maximizing intermediary, the cost of monitoring an extra firm must be equal to the expected marginal benefit, i.e. the expected interest on the loan. Given that
\[
\frac{d\pi_{t+1}}{dn_t} = \frac{2\gamma}{(1 - p)} > 0
\]
(13)
it establishes a crucial, direct relationship between the loan interest rate and the number of firms monitored. When the loan interest rate increases, the marginal benefit intermediaries receive from monitoring a large number of firms increases and, therefore, intermediaries increase monitoring up to the point at which the equality between marginal costs and marginal benefits is restored.

Given assumption 10, after defining \( \bar{\pi} \equiv Nq[x_{t+1} n_t (1 - p) + x_{t+1} p - \gamma n_t - r_{t+1} + \zeta(\mu)] \)
(14)
Substituting now (14), the first-order conditions (10), (11) and (12) can be rewritten as
\[
Nq \left[ \gamma n_t^2 + \frac{p^2 \gamma n_t}{1 - p} - r_{t+1} + \zeta(\mu) \right] = 3D0 \quad \Leftrightarrow \quad 1 > n_t > 0
\]
(15)
\[
Nq \left[ \gamma n_t^2 + \frac{p^2 \gamma n_t}{1 - p} - r_{t+1} + \zeta(\mu) \right] < 0 \quad \Leftrightarrow \quad n_t = 3D1
\]
(16)
\[
Nq \left[ \gamma n_t^2 + \frac{p^2 \gamma n_t}{1 - p} - r_{t+1} + \zeta(\mu) \right] > 0 \quad \Leftrightarrow \quad n_t = 3D0
\]
(17)
Given assumption 7, \( \zeta(\mu) < 1 \leq r_{t+1} \) and therefore (15) is a second-degree equation with the positive root
\[
\hat{n}_t = 3D \cdot \frac{p}{1 - p} \sqrt{1 + \left( \frac{1 - p}{p^2 \gamma} \right)^2 (r_{t+1} - \zeta(\mu))} - \frac{p}{1 - p} > 0
\]
(18)
The optimal level of monitoring, for our economy, is therefore given by the function \( \pi_t(r_{t+1}; \mu) \) where \( \bar{n}_t(r_{t+1}; \mu) = 1 \) if \( r_{t+1} > (\gamma(1 + p)/(1 - p) + \zeta(\mu) \) and \( \bar{n}_t(r_{t+1}; \mu) = \hat{n}_t \) if \( (\gamma(1 + p)/(1 - p) + \zeta(\mu) \geq r_{t+1} \).

Notice that
\[
\frac{d\hat{n}_t}{dr_{t+1}} = \frac{p}{2(1 - p)} \left[ 1 + \left( \frac{1 - p}{p \gamma} \right)^2 (r_{t+1} - \zeta(\mu)) \right]^{-1/2} > 0
\]
(19)
The positive effect of an increase in the deposit interest rate on monitoring is due to the fact that such an increase must be followed by an increase in the
loan interest rate. Given the positive relationship between the loan interest rate and the number of firms monitored implied by (13) this, in turn, implies an increase in the level of monitoring.

We can also show that

$$\frac{d\hat{n}_t}{d\mu} = \frac{p}{2(1-p)} \left[ \frac{1 + (1-p)^2}{p^\gamma} (r_{t+1} - \zeta(\mu)) \right]^{-1/2}$$

$$\times \frac{1}{Nq} \left[ -\omega + \frac{1}{2} (22\mu\omega)^{1/2} 22\omega \right]$$

$$> 0$$

i.e. an increase in the capital requirement $\mu$ has a positive effect on the level of monitoring. An increase in $\mu$, in fact, induces intermediaries to charge a higher loan interest rate and this, given (13) induces firms to monitor a larger number of firms.

5. General Equilibrium

Since the total number of firms in the market is $\alpha_1$ and the number of intermediaries is $\alpha_2$, the number of firms financed by each intermediary is $N = \alpha_1/\alpha_2$. The aggregate demand for loanable funds therefore is given by $\alpha_2 Nq = \alpha_1 q$ while the aggregate supply of loanable funds by uninformed agents is given by $(1 - \alpha_1 - \alpha_2) s_i(w_t, r_{t+1})$. In this case, equilibrium in the market for loanable funds implies

$$\frac{\alpha_1 q}{(1 - \alpha_1 - \alpha_2)} = s(w_t, r_{t+1})$$

(21)

Notice that, in this model, since the demand for capital is given, the level of savings does not play a major role in determining the economy’s capital stock. Given the capital stock in period $t$, the deposit interest rate simply moves to equate the supply of loanable funds to the given demand. Obviously, this is true if the level of savings is sufficient, i.e. the demand for loanable funds is never greater than $w(k)$. Since $w'(k) > 0$, this is guaranteed by assumption 6.

Given assumption 2 on the savings function, the implicit function theorem can be applied and (22) therefore defines an interest rate $r_{t+1}^*(w_t(k))$. Substituting $r_{t+1}^*(w_t(k))$ into $\hat{n}_t(r_{t+1}; \mu)$ we obtain a function

$$\hat{n}_t(r_{t+1}^*(w_t(k)); \mu)$$

which is decreasing in $k_t$, reaches the upper bound

5 Given (14) and the fact that $\zeta(\mu) = (\mu\omega - (22\mu\omega)^{1/2}) Nq/\hat{z}_t = (22\mu\omega)^{1/2}$, and therefore $\zeta(\mu) = \omega - \frac{1}{2} (22\mu\omega)^{-1/2} 22\omega = \omega - (\hat{z}/\hat{z}_t)\omega = \omega - (\zeta/G(\hat{z}))\omega < 0$. 

for $k_t = \kappa$ and the lower bound
\[ n_t(1; \mu) = n \]
for $k = \bar{k}$. The upper limit $\bar{k}$ is the level of $k_t$ at which
\[ r^*_{t+1}(w_t(k_t)) > \left( \gamma \frac{1 + p}{1 - p} \right) + \zeta(\mu) \]

A graphical representation of how the monitoring function
\[ \bar{n}_t(r^*_{t+1}(w_t(k_t)); \mu) = \bar{n}_t(k_t; \mu) \]
can be constructed from the main relationships of our model is illustrated in figure 1.

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The government in this model simply intervenes by offering deposit insurance and collecting taxes. We assume that the government always imposes a tax $\tau$ that balances the budget, i.e. such that

$$\alpha_2 \tau + \alpha_2 (1 - G(z_t)) \mu \omega = (1 - G(z_t))(1 - \alpha_1 - \alpha_2) s_t(w_t, r_{t+1})$$

$$\equiv (1 - G(z_t)) \alpha_1 q$$

(22)

The aggregate stock of capital is given by the output of the firms producing with the good technology and the output of those producing with the bad technology who did not default on their loans

$$k_{t+1} = \psi(k_t) \equiv [(a - pb) \pi(k_t; \mu) + pb] \frac{\alpha_1 q}{(1 - \alpha_1 - \alpha_2)}$$

(23)

The dynamic evolution of our economy depends crucially on the initial capital stock. If $k > k_0$, then $n_t = 1$ and $k_{t+1} = a a q$, i.e. the economy achieves immediately a constant stock of capital. If, instead, $k_0 > k$ then the capital stock in period $t+1$, given the function $\pi(k_t; \mu)$, depends on the previous period’s stock of capital. Notice that

$$\frac{dk_{t+1}}{dk_t} = \psi'(k_t)$$

$$\equiv \frac{\alpha_1 q p}{(1 - \alpha_1 - \alpha_2) 2(1 - p)}$$

$$\times \left[ 1 + \left( \frac{1 - p}{p^2 \gamma} \left(r^*_t(k_t) - \zeta(\mu) \right) \right)^{-1/2} \frac{(1 - p)^2 dr^*_{t+1} dw_t}{p^2 \gamma dw_t dk_t} \right]$$

< 0

(24)

The downward sloping difference equation reflects the endogenous cyclical-ity of our economy. An increase in the capital stock increases output and therefore the income of uninformed lenders which, given the demand for loanable funds, implies a fall in the deposit interest rate. A lower interest rate induces intermediaries to decrease the number of firms monitored; and this induces firms to adopt the ‘bad’ technology. Because of this, the stock of capital in period $t+1$ is lower than in period $t$ and the whole process leading

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6 Given assumption 7, this implies a level of $\omega$ such that

$$\omega \geq ((1 - G(z_t)) Nq/1 - \mu - (1 - G(z_t)) \mu$$

7 Given assumption 1, (13) and the concavity of the production function,

$$\rho_{t+1} [(a + (1 - \theta) pb)] = f'(k_{t+1}) [(a + (1 - \theta) pb)]$$

$$f'(a a q) pb \geq 2 \gamma x_{t+1}$$

which implies that the participation constraint (9) is always satisfied.

to an increase in the interest rate and to an increase in monitoring starts again. Equation (23) is represented in figure 2.

We immediately see that, provided that \[ ((a - pb)\bar{n} + pb) \] is sufficiently low, (23) has a unique fixed point, i.e. a unique steady state that we denote as \( \hat{k} \). The dynamics of our economy crucially depends on the slope of the function \( \psi(k_i) \) around \( \hat{k} \). If \( \psi'(\hat{k}) \in (-1, 0) \) and \( k_0 \) is sufficiently close to \( \hat{k} \), the cycles will eventually die out as the economy converges to \( \hat{k} \). If \( \psi'(\hat{k}) < -1 \), instead, the economy might generate limit cycles, i.e. cycles that perfectly replicate themselves over time. These cycles will be of order 2 since, as it is well known (Grandmont, 1986), a monotonic first-order difference equation cannot produce higher-order cycles. As in Mattesini (2001), we can therefore prove

**Proposition 1**

Suppose \( \hat{k} \) is a fixed point of \( k_{i+1} = \psi(k_i) \) and \( k_0 > \hat{k} \). Defining

\[
\Theta \equiv [(a - pb)\bar{n}(k_0) + pb]a_q
\]

and

\[
\Omega \equiv [(a - pb)\bar{n} + pb](a_1 q/(1 - a_1 - a_2))
\]

Figure 2:
a set of sufficient conditions for the existence of a two-period cycle \((k_1^*, k_2^*)\) where \(\bar{k} \equiv k_2^* > \hat{k} > k_1^* > k_0 > \underline{k}\) is given by

\[
\psi'(\hat{k}) < -1
\]

\[
[((a - pb)\bar{\mu}(\Theta) + pb)\frac{\alpha_1 q}{1 - \alpha_1 - \alpha_2}] < k_0
\]

\[
[((a - pb)\bar{\mu}(\Omega) + pb)\frac{\alpha_1 q}{1 - \alpha_1 - \alpha_2}] < \bar{k}
\]

**Proof**

To prove the existence of a two-period cycle, it is necessary to prove that the second iterate of \(\psi(k_t)\), \(\psi^2(k_t)\), cuts the 45° line at \((k_1^*, k_2^*)\) where \(\bar{k} \equiv k_2^* > \hat{k} > k_1^* > k_0 > \underline{k}\).

Notice that if \(\bar{k}\) is a fixed point of \(k_{t+1} = \psi(k_t)\), then \(\psi(\bar{k}) = \hat{k}\), which implies

\[
\psi^2(\bar{k}) = \psi(\psi(\bar{k})) = \psi(\hat{k}) = \bar{k}
\]

Moreover, notice that

\[
\frac{\partial \psi^2(\bar{k})}{\partial \bar{k}} = \frac{\partial \psi(\psi(\bar{k}))}{\partial \bar{k}} = \psi'(\psi(\bar{k}))\psi'(\hat{k}) = (\psi'(\hat{k}))^2
\]

If \(\psi'(\hat{k}) < 1\), then

\[
\frac{\partial \psi^2(\bar{k})}{\partial \bar{k}} = (\psi'(\hat{k}))^2 > 1
\]

Hence, since

\[
\psi^2(\bar{k}) = \left((a - pb)\bar{\mu}(\Theta) + pb\right)\left(\frac{\alpha_1 q}{1 - \alpha_1 - \alpha_2}\right) < k_0
\]

\(\psi^2(\hat{k})\) cuts the 45° line from below. If

\[
\psi^2(\hat{k}) \equiv \left((a - pb)\bar{\mu}(\Omega) + pb\right)\left(\frac{\alpha_1 q}{1 - \alpha_1 - \alpha_2}\right) < \bar{k}
\]

then \(\psi^2(\hat{k})\) crosses over again in the interval \((\hat{k}, \bar{k})\) and, therefore, we have a two-period cycle.

6. Capital Requirements

Capital requirements in this economy have the purpose of reducing the moral hazard problem between intermediaries and depositors. As we saw in section 3, in fact, a positive $\mu$ makes it unprofitable to undertake excessive risks, and an increase in $\mu$ increases the number of firms monitored. Differentiating (23) evaluated at the steady state $k$ and given (20), we obtain

$$\frac{\partial \hat{k}}{\partial \mu} = \frac{\partial \hat{n}_t}{\partial \mu} > 0$$

(25)

In this model, where the whole stock of capital depends on the number of firms which, having been monitored, undertake ‘good’ investments, capital requirements have a positive effect on the steady state stock of capital by inducing banks to increase monitoring.

Capital requirements also achieve the objective of reducing the number of bankruptcies among intermediaries. Substituting (10) into (4) and given assumption 10, we obtain in fact

$$G(\hat{z}_t) = \frac{\hat{z}_t}{z} = \frac{Nq}{z} \left[ \gamma \frac{\hat{n}_t^2(k_i; \mu) + p^2 \hat{n}_t(k_i; \mu)}{1 - p} - r_{t+1}^\ast(k_i; \mu) \right] + \mu \omega$$

which represents not only the probability that an intermediary does not default, but also the number of intermediaries that do not default, and we can immediately verify that

$$\frac{\partial \hat{z}_t}{\partial \mu} = \left[ \gamma \frac{\hat{n}_t^2(k_i; \mu) + p^2 \hat{n}_t(k_i; \mu)}{1 - p} \right] \frac{d\hat{n}_t}{dk_i} + \mu \omega > 0$$

(26)

An interesting question that arises at this point is whether capital requirements can also influence the equilibrium dynamics of our economy. Differentiating (24), we immediately see that

$$\frac{d|\psi'(k_i)|}{dm} < 0$$

(27)

Equation (27) tells us that the higher are capital requirements, the smaller is the slope of the function $\psi(k_i)$. This means that when the capital stock in period $t$ increases, the subsequent decrease in the stock of capital that occurs in period $t + 1$ is smaller the higher is $\mu$. In other words, higher capital requirements tend to decrease the size of cyclical fluctuations in the level of economic activity. When the stock of capital increases, wages increase and the deposit interest rate decreases which, in turn, pushes down the loan interest
rate and the level of monitoring. The higher the capital requirement, the smaller is the decrease that competition among intermediaries will induce on the loan interest rate and on the level of monitoring and, therefore, the smaller is the decrease in the stock of capital.

7. Conclusions

We have analysed a model where the financial intermediation system may be responsible for aggregate instability, an idea that was common among pre-Keynesian business cycle theorists but that has received less attention in recent times. The model is characterized by a double moral hazard problem. The first problem is the one that arises between intermediaries and borrowers. Being able to observe the choices of capital producing firms only at a cost, intermediaries invest resources in monitoring and the monitoring activity of intermediaries is crucial in determining the number of efficient investment projects and the level of economic activity. Since monitoring depends, in turn, on aggregate economic conditions and ultimately on the stock of capital, the economy may experience economic fluctuations.

The second problem arises between intermediaries and depositors and is given by the fact that, operating in an uncertain environment, banks may be led by competition to save on monitoring costs and to shift risks on to depositors. This problem, at least in part, can be avoided by imposing capital requirements. This form of government intervention is effective in increasing the stock of capital, reducing bankruptcies among intermediaries and in reducing the size of the endogenous cycles experienced by our model economy.
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