Government spending, interest rates, and capital accumulation in a two-sector model

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Abstract. This paper investigates dynamic impacts of a temporary fiscal expansion in a two-sector growth model. If the expansion falls on consumption-investment commodities, capital accumulation can be either promoted or reduced and the short-term interest rate unambiguously rises. If the expansion falls on consumption commodities, capital accumulation is crowded out and the short-term interest rate declines during the period of the fiscal expansion. It is also shown that fiscal spending on the consumption commodity can move the short- and long-term interest rates in opposite directions. JEL Classification: E43, E62, O41

Dépenses gouvernementales, taux d’intérêt, et accumulation de capital dans le cadre d’un modèle à deux secteurs. Ce mémoire examine les impacts dynamiques d’une politique de relance fiscale temporaire dans le cadre d’un modèle à deux secteurs. Si la relance passe par les biens de consommation et d’investissement, cela peut promouvoir ou non l’accumulation de capital, et les taux d’intérêt à court terme s’accroissent. Si la relance passe par les biens de consommation, il y a effet d’éviction sur l’accumulation de capital, et les taux d’intérêt chutent durant la période de relance fiscale. On montre aussi que des dépenses publiques portant sur les biens de consommation peuvent entrainer les taux d’intérêt à court et à long terme dans des directions opposées.

1. Introduction

Recently, one-sector growth models with perfect foresight and intertemporal utility maximization have been frequently adopted as a benchmark for studying dynamic effects of fiscal policy (e.g., Barro 1981, 1989; Abel and Blanchard 1983; Judd 1987; and Aschauer 1988). These authors derive several stylized predictions. One
of the most important ones is that a temporary fiscal expansion raises the interest rate. However, Barro (1997, 454–9) obtains an opposite empirical result: a temporary fiscal expansion lowers the average interest rate in the United States. Moreover, Denslow and Rush (1989) find a negative correlation between a temporary fiscal expansion and the long-term interest rate for the United States but a positive correlation for France.

Several authors attempt to construct models that can consistently explain these mixed findings. In order to do so, they introduce some additional factors into the models, such as external effects of public spending on household utility or private production (Djajić 1987; Ihori 1990; and Palivos and Yip 1996), interchangeability between productive capital and durable consumption goods (Mankiw 1987), endogenous time preference (Devereux 1991) and a combination of these factors (Chang, Tsai, and Lai 1998).

In this paper we show that a straightforward extension of a standard two-sector static model into a dynamic optimization setting naturally provides us with a consistent result with Barro’s finding on the effect of temporary fiscal spending on the real interest rates.¹

Specifically, we consider an economy where one of the two commodities is used for both consumption and investment, while the other is used only for consumption. In this setting we examine the dynamic effect of a temporary increase in fiscal spending on interest rates and capital accumulation. Since we use a two-sector model, we can investigate how different the effect of temporary fiscal spending is if it falls on consumption commodities or on consumption-investment ones. Furthermore, while in the standard one-sector model the short-term interest rate and capital accumulation are always negatively correlated, in the two-sector model they can be correlated either positively or negatively. Thus, in our model it can occur that temporary fiscal spending raises the short-term interest rate and yet stimulates capital accumulation.

We also analyse the effects of such fiscal policy on the term structure of interest rates.² It is shown that the effects are heavily dependent on whether a temporary fiscal expansion falls on the consumption commodity or on the consumption-investment commodity. When the increase falls on the consumption-investment commodity, the pattern of the response of the long-term interest rate is almost the same as that of the short-term interest rate although the magnitude of the response of the long-term rate is smaller. On the other hand, if it falls on the consumption commodity, the pattern of the response of the long-term interest rate may be different from that of the short-term rate.

¹ Effects of fiscal policy in a growing two-sector economy are analysed by Hoon (1992), Mino (1996), and Ono and Shibata (2000). They examine the effect of a ‘permanent’ increase in fiscal spending or capital income taxes. In contrast, we focus on the effect of a ‘temporary’ increase in fiscal spending on capital accumulation and the term structure of interest rates.

In the next section we construct a two-sector growth model with optimizing agents, and derive the equilibrium dynamics. The relationship between the short- and long-term interest rates is also derived. In section 3 we analyse effects on capital accumulation and interest rates of a temporary increase in government expenditure. The paper is concluded in section 4.

2. The model

Let us first present the basic structure of the model. There are two commodities, commodity 1, which is used for both consumption and investment, and commodity 2, which is used only for consumption. For simplicity, the population is assumed to be 1.

2.1. The production sector

Sector \( j \) has the following constant-returns-to-scale production function:

\[
f_j(k_j)L_j, \quad j = 1, 2,
\]

where \( k_j \) and \( L_j \) are the capital-labour ratio and labour input of sector \( j \), respectively. We assume that \( f_j(k_j) \) satisfies the following standard neoclassical properties and the Inada conditions:

\[
f_j''(k_j) > 0, \quad f_j''(0) = \infty, \quad f_j'(\infty) = 0, \quad j = 1, 2.
\]

Profit maximization of competitive firms yields the following set of first-order conditions:

\[
\begin{align*}
    r &= f_1'(k_1) = pf_2'(k_2) \\
    w &= f_1(k_1) - f_1'(k_1)k_1 = p[f_2(k_2) - f_2'(k_2)k_2],
\end{align*}
\]

where commodity 1 is taken as numeraire, and \( p, r, \) and \( w \), respectively, denote the price of commodity 2, the instantaneous (short-term) interest rate, and the wage.

2.2. The household sector

We consider a representative household with perfect foresight. Since the population is normalized to unity, the per capita value of a variable also represents its aggregate value. The household has the following utility function:

\[
U = \int_0^\infty [u(C_1) + v(C_2)]e^{-\rho t}dt,
\]

where \( C_j \) is consumption of commodity \( j \) (\( j = 1, 2 \)) and \( \rho \) is the constant subjective discount rate. Functions \( u \) and \( v \) satisfy \( u' > 0, \ u'' < 0, \ v' > 0 \) and \( v'' < 0 \). The flow budget equation is

\[
\dot{B} = rB + w - C_1 - pC_2 - \tau,
\]

where \( B \) denotes non-human wealth, and \( \tau \) a lump-sum tax.
Given the time paths of \( p(t) \), \( r(t) \), \( \tau(t) \), and \( w(t) \), the representative household maximizes (5) subject to (6). The optimal conditions are

\[
\begin{align*}
    u'(C_1) &= q \\ 
    u'(C_2) &= pq \\ 
    \frac{\dot{q}}{q} &= \rho - r \\ 
    \lim_{t \to \infty} q(t)B(t)e^{(-\rho t)} &= 0,
\end{align*}
\]

where \( q \) represents the co-state variable of \( B \). Note that if a jump in a policy parameter at time \( T \) is preannounced, \( q \) cannot jump at time \( T \); that is, \( \lim_{t \uparrow T^-} q(t) = \lim_{t \downarrow T^+} q(t) \). Intuitively speaking, since \( q \) is a kind of asset price, that is, the shadow price of \( B \), any anticipated jump in \( q \) is inconsistent with no arbitrage. Thus, from (7), any anticipated jump in \( C_1 \) cannot occur. Moreover, (8) implies that \( \nu'(C_2)/p \) must not jump at time \( T \). However, this does not necessarily mean that each of \( p \) and \( C_2 \) continuously moves then. In fact, we shall later show that \( p \) and \( C_2 \) exhibit discontinuous changes at time \( T \) so as to avoid an anticipated jump in \( q \).

By rearranging conditions (7)–(10) we obtain

\[
\begin{align*}
    -\frac{u''(C_1)}{u'(C_1)} \dot{C}_1 &= r - \rho \\
    \frac{v'(C_2)}{u'(C_1)} &= p \\
    \lim_{t \to \infty} B(t)e^{\int_t^\infty r(s)ds} &= 0.
\end{align*}
\]

2.3. The government

For simplicity, it is assumed that the government imposes only a lump-sum tax and keeps a balanced budget. Thus, its budget constraint is

\[ G_1 + pG_2 = \tau. \]

Note that even if the government issues bonds, none of the following arguments are modified, since the Ricardian equivalence proposition always holds in the present setting. It implies that a reduction in the lump-sum tax together with an increase in government bonds exercises no effect. Therefore, we shall focus on the case where the lump-sum tax and fiscal spending are increased by the same amount.

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3 This property is proved in a similar way to Kemp and Long (1977). See also Fisher and Turnovsky (1992) for this point.
4 Formally, from the envelop theorem, \( q = \partial V/\partial B \).
5 In the case of a one-good economy, consumption \( C \) cannot jump, so that co-state variable \( q \) does not jump.
2.4. Market equilibrium and dynamics

We first consider the factor market equilibrium conditions. From (3) and (4), we obtain \(k_1, r\) and \(w\) as functions of only \(p\):

\[ k_1 = k_1(p), k_1'(p) = \frac{f_2(k_2)}{f_1''(k_1)(k_2 - k_1)} \]

\[ k_2 = k_2(p), k_2'(p) = \frac{f_1(k_1)}{p^2f_2''(k_2)(k_2 - k_1)} \]

\[ r = r(p), r'(p) = \frac{f_2(k_2)}{k_2 - k_1}, \]

\[ w = w(p). \]

To avoid analytical complexity, we assume the no-factor-intensity-reversal condition to be globally satisfied. Specifically, it is assumed that commodity 1 (the consumption-investment commodity) is more capital intensive than commodity 2 (the consumption commodity):\(^6\)

\[ k_1(p) > k_2(p) \quad \text{for all } p. \]  

(16)

Using (15) we have the factor market equilibrium conditions:

\[ K = k_1(p)L_1 + k_2(p)L_2, \quad L_1 + L_2 = 1, \]  

(17)

where \(K\) stands for the aggregate capital stock. From (17) we obtain \(L_j (j = 1, 2)\) as a function of \(p\) and \(K\):

\[ L_1 = \frac{K - k_2(p)}{k_1(p) - k_2(p)}, \quad L_2 = \frac{k_1(p) - K}{k_1(p) - k_2(p)}. \]  

(18)

Let us next consider the equilibrium conditions of the two commodity markets. Since commodity 2 cannot be accumulated, its equilibrium condition is given by

\[ C_2 + G_2 = f_2(k_2)L_2. \]  

(19)

From (12) \(C_2\) is solved as \(C_2 = v^{-1}(pu'(C_1))\). Substituting this and (18) into (19) gives

\[ v^{-1}(pu'(C_1)) + G_2 = f_2(k_2(p)) \left( \frac{K - k_1(p)}{k_2(p) - k_1(p)} \right). \]  

(20)

\(^6\) Even if we assume the opposite, all the mathematical expressions obtained in this paper remain intact as long as the saddle-point stability holds. It is more natural, however, to assume that the consumption-investment commodity is more capital intensive than the other commodity, as is assumed in this paper.
From (20) \( p \) is obtained as a function of \( C_1, K, \) and \( G_2 \):
\[
p = p(C_1, K, G_2), p_{C_1} > 0, p_K > 0, p_{G_2} > 0,
\]
where the signs of the partial derivatives above are derived in Appendix A.1.

The market equilibrium condition of commodity 1 is given by
\[
C_1 + \dot{K} + G_1 = f_1(k_1(p))L_1,
\]
since it is used for both consumption and investment. Substituting (18) and (21) into (22), we obtain the dynamic equation of \( K \):
\[
\dot{K} = f_1[k_1(p(C_1, K, G_2))]
\left(\frac{K - k_2[p(C_1, K, G_2)]}{k_1[p(C_1, K, G_2)] - k_2[p(C_1, K, G_2)]}\right)
- C_1 - G_1.
\]

Substituting \( r(p) \) in (15) and \( p(C_1, K, G_2) \) in (21) into (11) yields the dynamic equation of \( C_1 \):
\[
- \frac{u''(C_1)}{u'(C_1)} \dot{C}_1 = r(p(C_1, K, G_2)) - \rho.
\]

(23) and (24) describe the complete dynamics of \( C_1 \) and \( K \) when initial capital \( K(0) \) is given. Hence, by taking account of the transversality condition, we determine the two-dimensional equilibrium dynamics on the \( C_1 - K \) plane.

In appendix A.2 we show that the \( \dot{C}_1 = 0 \) locus is downward sloping and the \( \dot{K} = 0 \) locus is upward sloping. The phase diagram of the dynamics is illustrated in figure 1. From the phase diagram it is easily seen that the system is saddle-point stable. The saddle-point stable equilibrium path is depicted as \( AE_0 \) (or \( A'E_0 \)).

2.5. The term structure of interest rates

Following Blanchard (1984) and Fisher and Turnovsky (1992), we define the long-term interest rate \( R \) as the yield on consols paying a constant real coupon flow of unity.\(^7\) Then, the consol price is \( 1/R. \) Since the consol price should equal the bond price that eternally yields a unit value,
\[
\int_t^\infty e^{-\int_t^s r(u)du} ds = 1/R(t).
\]

Also, the bond price paying \( \{r(t)\}_{t=0}^\infty \) is 1:
\[
1 = \int_t^\infty r(s)e^{-\int_t^s r(u)du} ds.
\]

\(^7\) For simplicity, we assume that the net supply of consols is zero. Then, all of the equilibrium conditions obtained earlier remain intact.
From these two equations,

\[ R(t) = \frac{\int_t^\infty r(s)e^{-\int_t^r r(u) \, du} \, ds}{\int_t^\infty e^{-\int_t^r r(u) \, du} \, dv} \]

\[ = \int_t^\infty \left\{ r(s) \frac{e^{-\int_t^r r(u) \, du}}{\int_t^\infty e^{-\int_t^r r(u) \, du} \, dv} \right\} ds, \]

which shows that \( R(t) \) is a weighted average of future short-term interest rates.
3. Temporary fiscal expansions and interest rates

In this section we investigate the effects of unanticipated temporary fiscal spending on each commodity. Throughout the analysis we assume the economy to be initially in the steady state.

3.1. A temporary fiscal expansion falling on commodity 1

Let us first analyse the effect of a ‘permanent’ increase in $G_1$. As is demonstrated in appendix A.2, an increase in $G_1$ moves the $\dot{K} = 0$ locus downward, with the $\dot{C}_1 = 0$ locus unchanged, as shown in figure 2. Consequently, the economy moves along the path given by $AE_1$, and hence capital accumulation is stimulated and the long-run capital stock rises. This result makes a sharp contrast with the prediction of the standard one-sector model, where fiscal expansion does not affect capital accumu-
lation in the long run. In our model, the steady-state rate of interest must equal the exogenously given subjective discount rate, $r$, and hence the permanent increase in $G_1$ has no effect on the steady-state interest rate, $r$, causing no effects on the steady-state relative price, $p$. With factor prices constant, the increase in the relative output of commodity 1 can be achieved only by an increase in the economy’s overall capital-labour ratio, which means an increase in the capital stock. This mechanism does not work in the standard one-sector model.

Using these observations, we examine the effects of a ‘temporary’ increase in $G_1$ on capital accumulation, interest rates, and the relative price. Suppose that the government announces at time $t_1$ that fiscal spending is raised at time $t_1$ and will be reduced to the original level at time $t_2$. As noted in subsection 2.2, consumption of commodity 1 does not jump at time $t_2$, since it was announced. Thus, in figure 2 there are two possible patterns for the optimal response to the policy: (i) if $t_2$ is sufficiently large (long fiscal expansion), then path $BCE_0$ is realized; and (ii) if $t_2$ is small (short fiscal expansion), then path $FDE_0$ is realized. In case (i), capital initially increases and then decreases until $t_2$. After time $t_2$, capital again begins to increase, approaching the original level. In case (ii), capital continues to decrease until $t_2$ and thereafter increasingly approaches the original level. In both cases $C_1$ initially jumps downward and then increases gradually towards the original level. These movements are summarized in figure 3.

Let us next consider the effects on the interest rates and the relative price. Since the steady-state interest rate equals $r$ and Euler equation (24) holds, the movement of $C_1$ in figure 3 implies that the short-term interest rate jumps upward at time $t_1$ and $r(t) > r$ for $t \in (t_1, \infty)$. From (2), (15), (16), and (21), it follows that the movement of $p$ is opposite to that of the short-term interest rate and is continuous over time, implying that $r$ is continuous as well.8

Since from (25) $R$ is a weighted discounted sum of future short-term interest rates, it stays lower than the short-term rate at any point in time if the short-term rate monotonically decreases. Figure 4 illustrates typical time paths of the short- and long-term interest rates.

These are rather interesting results. The higher $t_2$ is, the more likely capital initially accumulates, while both the short- and long-term interest rates unambiguously rise. Thus, a rise in the short-term interest rate and capital accumulation simultaneously occur. This result is in sharp contrast to that of the standard one-sector model, where the short-term interest rate is always negatively correlated to the capital level.

The mechanism that generates this result is quite simple. An increase in $G_1$ raises demand for commodity 1 and thus increases its price $(1/p)$; that is, it reduces $p$. Since commodity 1 is capital intensive, this decline in $p$ will lead to an increase in the interest rate, $r$. This is a common mechanism in two-sector models, such as the Heckscher-Ohlin model.

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8 We can easily show that $r'(p) < 0$. 

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3.2. A temporary fiscal expansion falling on commodity 2
As obtained in appendix A.2, a 'permanent' increase in $G_2$ shifts the $\dot{C}_1 = 0$ and $\dot{K} = 0$ loci downward, causing the steady-state capital stock to decline; that is, $dK(\infty)/dG_2 < 0$. Note that in figure 5 the initial jump from $E_0$, caused by
an increase in $G_2$, can be either upward or downward, as demonstrated by points $A$ and $B$.

Now we examine the effects of an unanticipated ‘temporary’ increase in $G_2$. As illustrated in figure 5, there are two possible patterns for the optimal response of the economy: (i) $C_1(t_1)$ jumps upward ($FGE_0$), and (ii) $C_1(t_1)$ jumps downward ($CDE_0$). In either case, after the initial jump both $C_1$ and $K$ decrease until $t_2$ and thereafter increasingly return towards each original level. Figure 6 illustrates these movements.

We next obtain the movements of $p$ and $r$. Note that $p$ is a continuous function of $C_1$, $K$, and $G_2$, given by $(21)$. Since after the initial jump $C_1$ and $K$ continuously move as illustrated in figure 6, whereas $G_2$ jumps only at $t_1$ and $t_2$, $p$ continuously decreases for $t \in (t_1, t_2)$, jumps downward at $t_2$, and thereafter continuously increases towards the original level.

Since in figure 6 $\dot{C}_1 < 0$ for $t \in (t_1, t_2)$ and $\dot{C}_1 > 0$ for $t \in (t_2, \infty)$, (11) implies that $r(t) < \rho$ for $t \in (t_1, t_2)$ and that $r(t) > \rho$ for $t \in (t_2, \infty)$. From (15) and (16) $r$ is a decreasing function of only $p$. Using these two properties and the above-mentioned movement of $p$, we find that $r$ jumps downward at time $t_1$, continuously increases for $t \in (t_1, t_2)$, jumps upward from a lower level than $\rho$ to a higher level than $\rho$ at $t_2$, and thereafter continuously decreases toward $\rho$. This is a natural result; when the government increases its demand for commodity 2 over some interval, the price of commodity 2, $p$, rises and hence the interest rate, $r$, falls.

Typical time paths of $r$ and $R$ are depicted in figure 7. Since from (25) $R$ is the weighted average of $r$ over time, it cannot jump at time $t_2$. Since $r$ is below $\rho$ for $t \in (t_1, t_2)$, and above $\rho$ for $t \in (t_2, \infty)$, the time path of $R$ would be like that represented in figure 7. It should be noted that the long-term interest rate stays


10 If $t_2 - t_1$ is almost infinite, the transition path becomes close to $AE_1$ or $BE_1$ in figure 5. Thus, both capital and consumption decrease monotonically throughout the transition, and from (15), (16), and (21), $r$ jumps downward at time $t_1$ and then increasingly approaches $\rho$. Since $R$ is a weighted average of $r$ over time, the direction of a change in the time path of $R$ is basically the same as that of $r$. 

\[FIGURE 4\]
higher than \( r \) during some interval before the policy reversal, whereas the short-term interest rate remains lower than \( r \) at any time before the reversal. That is, the average long-term interest rates may be higher than the usual level during fiscal expansion, while the average short-term interest rate is definitely lower than the usual level.

It should be emphasized that we obtain an opposite result to that of the standard one-sector model; that is, a temporary fiscal expansion on commodity 2 lowers \( r \) during the expansion period. Moreover, the response of \( R \) can significantly differ from that of \( r \). This implies that we have to pay attention to the composition of fiscal spending, instead of its aggregate level, in analysing the effects of fiscal expansion on interest rates.

3.3. Fiscal expansion falling on both commodities
Let us briefly mention the case where the government increases fiscal spending on both commodities at the same time. The effects of such a policy on the interest rates

![Figure 5](image_url)
are obtained by combining the two opposite effects analysed above. If fiscal spend-
ing is allocated to the two commodities so as to keep the marginal rate of substitu-
tion between the two commodities unchanged, our dynamic works as if there were only one commodity; that is, the effects on the interest rates are the same as those
in the standard one-sector model. If fiscal spending affects the marginal rate of substitution, the steady-state level of capital changes. As is shown in property (iv) of appendix A.2, the capital stock increases when $p_{G_1}dG_1 > p_{G_2}dG_2$, while it decreases when the opposite inequality holds. In a special case where $u(C_1) + v(C_2) = \alpha \log C_1 + (1 - \alpha) \log C_2$, we have a clearer result: if the ratio of commodity $i$ ($i = 1, 2$) in fiscal spending is higher than that in consumption, its effect works in the same direction as in the case of fiscal spending on commodity $i$.

4. Concluding remarks

Using a simple two-sector growth model, we have shown that an unanticipated temporary increase in fiscal spending may either crowd in or crowd out capital accumulation, depending on whether the government spends on the consumption commodity or the consumption-investment commodity.

If a temporary increase in fiscal spending on the consumption-investment commodity occurs, the short-term interest rate initially jumps upward and thereafter declines towards the original level. Although the short-term interest rate is kept higher than the steady-state level, capital accumulation is initially accelerated if the period of fiscal expansion is relatively long. On the other hand, if the fiscal expansion is towards the consumption commodity, the short-term interest rate initially falls and gradually increases. At the time of the policy reversal it jumps upward beyond, and then decreases towards, the original level. Capital decumulates during the period of the expansion and thereafter accumulates towards the original steady-state level. Effects of such policies on the term structure of interest rates also heavily depend on whether the fiscal expansion falls on the consumption-investment commodity or on the consumption commodity.
We conclude this paper by suggesting directions for further research. First, it is necessary to investigate empirically the predictions of our model. It is especially important to direct our attention to the composition of fiscal spending in testing the predictions. Second, it would be interesting to endogenize labour supply in a way similar to that of Palivos and Yip (1996). We guess that, since the interest rate dynamics can have various patterns even under our specification, and since the mechanism behind our results lies in the standard two-sector structure, various patterns of the interest rate dynamics similar to those obtained in this paper can occur under a more general utility function. Endogenous labour supply makes the dynamics rather complicated, however, and thus there may appear dynamics richer than ours. Finally, as in Djajić (1987), Ihori (1990), and Palivos and Yip (1996), incorporating benefits from government spending into our model is also an important issue. We will leave these issues for our future research.

Appendix

A.1. The partial derivatives of the $p$ function

Totally differentiating (20) and applying (3), (4), and (15) to the result yields

$$dG_2 + \frac{pu''}{u''} dC_1 - \frac{f_2}{k_2 - k_1} dK = Adp,$$

where

$$A = -\frac{u'}{u''} + \frac{1}{(k_2 - k_1)^2} \left( (f'_2 k_2 - f_2 k'_1) + \frac{f_1 L_2}{p^2 f''_2} - \frac{f_2^2 L_1}{f'_1} \right) > 0.$$  

Therefore, from (16) and (A1),

$$pc_1 = \frac{pu''}{u''}A > 0, pK = \frac{f_2}{A(k_1 - k_2)} > 0, pG_2 = \frac{1}{A} > 0,$$

which implies (21).

A.2. The properties of the $\dot{C}_1 = 0$ and $\dot{K} = 0$ loci

Setting $K = 0$ in (23) and $\dot{C}_i = 0$ in (24) yields

$$r(p(C_1, K, G_2)) = \rho,$$

$$C_1 + G_1 = f_1[k_1(p(C_1, K, G_2))] \left( \frac{K - k_2[p(C_1, K, G_2)]}{k_1[p(C_1, K, G_2)] - k_2[p(C_1, K, G_2)]} \right).$$

Differentiating (A3) and (A4) and applying (3), (4), and (15) to the result, we obtain

$$\dot{C}_1 = 0 :$$

$$pc_1 dC_1 + pK dK + pG_2 dG_2 = 0,$$
\[ K = 0: \]
\[
(\Phi p_{c_1} - 1)dC_1 + \left( \frac{f_1}{k_1 - k_2} + \Phi p_K \right) dK - dG_1 + \Phi p_{g_2} dG_2 = 0, \tag{A6}
\]
where
\[
\Phi = \left( \frac{f_1 - f'_1 k_1 + f'_2 k_2}{f_1''} \left( \frac{K - k_2}{k_1 - k_2} \right) + \frac{f_1^2}{p^2 f_2''} \left( \frac{k_1 - K}{k_1 - k_2} \right) \right)^2.
\]
Substituting (18) into the above equation, we have
\[
\Phi = \left( \frac{f_1 - f'_1 k_1 + f'_2 k_2}{f_1''} \frac{f_2 L_1}{f_1''} + \frac{f_1^2}{p^2 f_2''} L_2 \right) \left( \frac{1}{k_1 - k_2} \right)^2 < 0. \tag{A7}
\]
From (4), (A2), and (A7),
\[
\frac{f_1}{k_1 - k_2} + \Phi p_K = -\frac{f_2}{A(k_1 - k_2)^2} \times \left( \frac{f_2 f_1'}{f_1''} L_1 + \frac{f_1^2 f_2'}{p^2 f_2''} L_2 + \frac{f_1 u'}{f_2 v''} (k_1 - k_2) \right) > 0. \tag{A8}
\]
Using (17), (22), (A3), and (A5)–(A8), we find the following properties:

(i) The \( \dot{K} = 0 \) locus has a positive slope, whereas the \( \dot{C}_1 = 0 \) locus has a negative slope.

(ii) An increase in \( G_1 \) shifts only the \( \dot{K} = 0 \) locus downward in figure 2. Thus, unambiguously,
\[
\frac{dC_1(\infty)}{dG_1} < 0, \quad \frac{dK(\infty)}{dG_1} > 0.
\]

(iii) An increase in \( G_2 \) shifts both loci downward in figure 5. Also,
\[
\frac{dC_1(\infty)}{dG_2} = -\frac{f_1 p_{g_2}}{p_K (k_1 - k_2) + f_1 p_{c_1}} < 0,
\]
\[
\frac{dK(\infty)}{dG_2} = -\frac{p_{g_2} (k_1 - k_2)}{p_K (k_1 - k_2) + f_1 p_{c_1}} < 0.
\]

(iv) When both \( G_1 \) and \( G_2 \) increase, \( C_1(\infty) \) decreases because both \( dG_1 \) and \( dG_2 \) have negative effects on \( C_1(\infty) \), as stated in properties (ii) and (iii) above. However,
the effect on $K(\infty)$ depends on the relative size between $dG_1$ and $dG_2$, since they affect $K(\infty)$ in opposite directions. Calculating the total effect gives

$$
dK(\infty) = \frac{1}{B} \left( dG_1 - \frac{p_{G_2}}{p_{C_1}} dG_2 \right),$$

where

$$B = \frac{p_k}{p_{C_1}} + \frac{f_1}{k_1 - k_2} > 0.$$

From (A9), the steady-state capital stock does not change if $p_{C_1} dG_1 = p_{G_2} dG_2$. From (A2) this condition is equivalent to $pu''dG_1 = v''dG_2$. The implication of this property is the following. From (12) the marginal rate of substitution between the two commodities is given by $v'/u'(= p)$. Therefore, if $C_1$ and $C_2$ change so as to satisfy $pu''dC_1 = v''dC_2$, the marginal rate of substitution remains unchanged, that is, the relative demand for the two commodities remains intact. If fiscal spending is exercised so as to keep this property valid, capital accumulation is not affected in the long run.

References


Aschauer, David Alan (1988) ‘The equilibrium approach to fiscal policy,’ *Journal of Money, Credit, and Banking* 20, 41–62


