Speculative attacks with unpredictable or unknown foreign exchange reserves

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Abstract. During a currency crisis, speculators usually do not know the value of a central bank's foreign exchange reserves. In this paper I show that modelling speculators as having imperfect knowledge of reserves enriches the predictions of the classical model of speculative attacks. With realistic lags in reserve reporting and costs to unsuccessful speculation, successful speculative attacks will involve a jump depreciation, unsuccessful attacks may occur, attacks may occur when fundamentals are improving, attacks may not be preceded by large increases in interest rates, and fixed exchange rates may be abandoned with no attack and no decline in the money supply. JEL Classification: F31

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1. Introduction

In the standard or classical model of a speculative attack on a fixed exchange rate, an attack occurs because of an inconsistency in policy. A central bank tries to maintain the fixed rate at the same time that it steadily expands domestic credit. These goals become incompatible when reserves are exhausted, and the peg comes to an end. Krugman (1979) and Flood and Garber (1984) showed that the end takes the form of a sudden loss of central bank reserves, before the bank would have exhausted reserves in the absence of speculation. Krugman (1996) and Garber (1996) have defended the idea that an inconsistency in policy drives most currency crises.

The standard model predicts that there is no jump depreciation in an attacked currency and that the money supply jumps down (due to the reserve loss) at the time of the attack. In reality, attacks typically are accompanied by the opposite pattern, a sudden depreciation (rather than a smooth transition to a float) and no drop in the money supply. Flood, Garber, and Kramer (1996) described this pattern for the 1994 Mexican crisis. In a study of currency crises between 1975 and 1997, the International Monetary Fund (1998a) found little evidence of monetary contractions coinciding with attacks.

Historical experiences with fixed exchange rates also feature some unsuccessful speculative attacks, pegs that are abandoned without an attack occurring, attacks that are not preceded by increases in interest rates, and attacks that occur when fundamentals did not seem to be deteriorating. These observations also are inconsistent with the standard model.

In this paper I explore the implications for the standard model of speculative attacks of adding two realistic features. First, the central bank’s foreign exchange reserves cannot be observed continuously and are reported with a lag. This lag played a role in the Mexican attack, where foreign exchange reserves were not reported during the second half of 1994. Before the attack on the Thai baht in 1997 the Bank of Thailand had invested many of its reserves in forward contracts to deliver dollars for baht. Uncertainty about the value of those contracts contributed to the market’s perception that the central bank’s defence of the baht was untenable (see International Monetary Fund 1998b, box 2.11). During 1996 the Russian Federation misreported its reserve data by $1.2 billion by booking some transactions through FIMACO, an offshore subsidiary of its central bank (see International Monetary Fund 1999b). In the autumn of 1997 the Bank of Korea held many of its foreign exchange reserves at overseas branches of domestic banks, which turned out to be illiquid. In the Korean case, the central bank itself was unsure of the value of its reserves (see International Monetary Fund 1998b, box 2.5). In response to criticisms of this lack of transparency, the group of twenty-two developed and developing countries assigned a working group to examine reserve reporting, including details of net positions and the frequency of reports. This group reported in October 1998, making recommendations on the frequency and timeliness with which central banks publish data. Similarly, the IMF is currently seeking to enforce reserve reporting requirements in a number of countries to which it lends. Its Special Data Dis-
Seminario Standard was strengthened in 1999 to require more detail on the composition of reserves (see International Monetary Fund 1999a). Even if a country’s reporting is up to the IMF’s standards, however, only monthly reports are required. Finally, structural applications of speculative attack models by Blanco and Garber (1986), Cumby and van Wijnbergen (1989), Goldberg (1994), Jeanne (1997), and Melick (1996) are limited to low-frequency data because measurements of fundamentals are not available at high frequency. It seems reasonable to model market participants as facing the same limited information.

This unobservability of current fundamentals seems even clearer if the fundamental is interpreted broadly to include potential borrowing or readiness to tolerate high interest rates or unemployment. A good example is found in uncertainty about the Hong Kong government’s preparedness to defend its dollar during 1997–98. A broad interpretation of the fundamental also seems necessary for European countries, where reserves are not a constraint, as Obstfeld (1994) has argued. Burnside, Eichenbaum, and Rebelo (1998) similarly suggested that reserve inadequacy cannot explain the Asian currency crisis, while Eichengreen, Rose, and Wyplosz (1995) found in a cross-crisis empirical analysis that reserve behaviour alone cannot account for attacks. Carrera (1999) reached the same conclusion in a study of daily data from Mexico during 1992–94.

The analysis in this paper is consistent with a definition of fundamentals that is broader than reserves, provided that they are exogenous. I retain the key features of the classical model – atomistic speculators, exogenous policy, and trending fundamentals – which lead to a unique equilibrium. Thus, there are no self-fulfilling attacks which bring about a change in policy that validates the attack. Obstfeld (1986, 1994) and Morris and Shin (1998, 2000) provided important contributions in models with self-fulfilling attacks, while Flood and Marion (1999) reviewed both types of attack models. I aim to show that the predictions of the classical model are richer and more realistic with unobservable reserves, but not to argue that no attacks contain self-fulfilling components. For example, the exogenous-policy setting may be inappropriate for the Asian crisis, where governments were not running growing fiscal deficits prior to floating.

The second feature added to the standard model is a cost to unsuccessful speculation against a fixed exchange rate. The cost may take the form of a large interest-rate differential if the central bank mounts an interest rate defence. It may also arise from capital controls (such as those adopted in Malaysia in September 1998), market intervention (as in Hong Kong in August 1998), or wide bid-ask spreads in money markets as liquidity falls during a currency crisis. The IMF’s International Capital Markets (1993) provides a catalogue of these costs during the 1992–93 European attacks. With such costs, speculators do not face a pure one-way bet against the pegged currency.

With these two modifications, the model is consistent with some speculation being unsuccessful. In successful attacks the currency typically will jump depreciate. The money supply is predicted to fall by less than in the standard model, and in some cases not to fall at all. Prior to a successful attack, domestic interest rates are
predicted to rise by less than in the standard model. Finally, an attack may occur when, in retrospect, current fundamentals were improving.

In section 2 I outline the standard, deterministic model as a benchmark and to establish notation. In section 3 unpredictability in domestic credit expansion and hence in the duration of the peg is introduced. Domestic credit now follows a Wiener process, which is the natural extension of the continuous-time, deterministic model. The paper’s main contribution is in section 4, in which the case in which current reserves are unknown is examined. Section 5 studies speculators who are uncertain about the floor on reserves that will lead the central bank to abandon the fixed exchange rate. It shows that this uncertainty is equivalent to uncertainty about current reserves. In section 6 the paper is concluded. For ease of reading, sources for the distribution theory are collected in a brief appendix.

2. Classical attack model

In the classical, log-linear model of a speculative attack, a central bank maintains a fixed exchange rate but also continuously buys bonds from the government. These two policies are incompatible, and eventually the central bank’s foreign exchange reserves are exhausted. At that time there is a speculative attack, and the currency floats. The model was developed by Krugman (1979) and Flood and Garber (1984).

Consider a small open economy, whose log nominal exchange rate $e$ satisfies the asset-pricing equation:

$$ e(t) = m(t) + \alpha \frac{de}{dt}, \quad \alpha > 0, \tag{1} $$

where $m$ is the log money supply. This relationship may be derived, for example, from the monetary model of the exchange rate. The central bank’s balance sheet identity is log-linearized as

$$ m = b + r, \tag{2} $$

where $b$ denotes bonds (domestic credit) and $r$ denotes foreign exchange reserves. Domestic credit expands steadily and deterministically:

$$ db = \mu dt, \tag{3} $$

which eventually leads to the demise of the fixed exchange rate. Initially, that rate is fixed at $\bar{e}$. For simplicity, the model abstracts from the narrow target zone around $\bar{e}$ and assumes that the exchange rate is fixed precisely. A steady decline in reserves offsets (sterilizes) the growth in central bank holdings of government bonds. While the peg lasts, $de/dt = 0$, and so

$$ \bar{e} = m = b + r. \tag{4} $$
Denote by $j$ the duration of the fixed exchange rate after time $t$. At that point, domestic credit will be given by

$$b(t + j) = b(t) + \mu j.$$  

(5)

Flood and Garber (1984) introduced the notion of the shadow exchange rate, $\tilde{e}$, which is the floating rate that would prevail if reserves were exhausted. Using equation (1), the shadow rate is

$$\tilde{e}(t) = b(t) + \alpha \mu,$$  

(6)

because domestic credit continues to grow at rate $\mu$ during a float. Speculators attack as soon as they can make profits through resale of the currency, so the exchange rate is given by

$$e = \max[\tilde{e}, \tilde{\tilde{e}}].$$  

(7)

Thus, as the float begins, there is no predictable jump in the exchange rate, so the fixed and floating rates coincide:

$$\tilde{e} = \tilde{e}(t + j) = b(t + j) + \alpha \mu.$$  

(8)

The attack occurs when $\tilde{e}$ hits $\tilde{\tilde{e}}$ from below. Equivalently, define

$$\tilde{\tilde{b}} = b(t + j) - \tilde{e} - \alpha \mu,$$  

(9)

so the attack occurs when $b$ reaches $\tilde{\tilde{b}}$.

To solve for the duration of the peg, combine the forecast for domestic credit (5) with the no-arbitrage condition (8) to give

$$\tilde{e} = b(t) + \mu j + \alpha \mu.$$  

(10)

Since $\tilde{e} = b(t) + r(t)$, however, the lifespan of the fixed exchange rate after time $t$ is

$$j = \frac{r(t) - \alpha \mu}{\mu}$$  

(11)

for $\mu > 0$. This duration is increasing in initial reserves $r(t)$ and decreasing in the drift in domestic credit $\mu$.

To see that the peg ends with a discrete loss of reserves, note that the exchange rate $e$ does not jump at $t + j$ but that $de/dt$ jumps from zero to $\mu$. From the exchange-rate model (1), then, $m$ must jump down by $\alpha \mu$, in the form of an instantaneous loss of the central bank’s remaining reserves.

Figure 1 illustrates the classical model. The dark lines show $\tilde{e}$ rising to the barrier at $\tilde{e}$, while the lighter lines show the underlying behaviour of domestic credit and reserves. The profit in an attack is $\tilde{e} - \tilde{\tilde{e}}$ or equivalently $b(t) - \tilde{\tilde{b}}$. An attack occurs as soon as the profit is greater than zero, so the fixed exchange rate ends when $b$ hits $\tilde{\tilde{b}}$ and $\tilde{e}$ hits $\tilde{\tilde{e}}$. Reserves drop to zero at that instant. The timing of the
attack could be described using the behaviour of reserves, $r$, the shadow exchange rate, $\bar{e}$, or the fundamental, $\bar{b}$, each reaching a certain threshold. This paper is focused on the fundamental, $\bar{b}$, relative to its implicit threshold $\bar{b}$, because that is the exogenous variable.

3. Unpredictable reserves

Uncertainty about the central bank’s foreign exchange reserves may be introduced in several ways. In this section I focus on uncertainty about the future evolution of $\bar{b}$ and hence of $r$, an uncertainty that may be shared by the central bank. Flood and Garber (1984) made this extension in a model with discrete time and increments to $\bar{b}$ distributed exponentially. Blanco and Garber (1986), Dornbusch (1987), Cumby and van Wijnbergen (1989), Goldberg (1994), Melick (1996), and Flood and Marion (2000) also studied discrete-time models with stochastic policies or other fundamentals. Here, I adopt a continuous-time model with normal increments to $\bar{b}$, which is a standard format in target-zone models, for example, and elsewhere in financial theory. This format also includes the classical model of section 2 as a special case and serves as a precursor to the case with unknown reserves.

Now suppose that $\bar{b}$ solves the stochastic differential equation

$$db = \mu dt + \sigma dz, \quad \mu \geq 0,$$

(12)
where $z$ is a standard Brownian motion. Thus, $db$ is a Wiener process with instantaneous mean $\mu$ and variance $\sigma^2$. The restriction on $\mu$ ensures that reserves will be exhausted. The asset-pricing equation becomes

$$e(t) = m(t) + \alpha E_t \frac{de}{dt}, \quad \alpha > 0,$$

(13)

where $E_t$ denotes an expectation at time $t$ conditional on the information set generated by past values of $b$, knowledge of the law of motion (12), and observation that a float has not yet begun. Writing $E_t$ assumes that for all $b \in \mathbb{R}$ there exists a unique probability measure $P_b$ such that $b$ is a $(\mu, \sigma)$ Brownian motion on a probability space $(\Omega, \mathcal{F}, P_b)$ with starting state $b(0)$ under $P_b$.

Formally, the duration of the fixed exchange rate is

$$j = \inf \{s : b(t + s) = \bar{b} \},$$

(14)

which is now a random variable. This is the first time (counting from the current time, $t$) that $b$ reaches $\bar{b}$, also called the first passage time. Assume that $b(t) < \bar{b}$ so that $j > 0$.

The probability density function of the first passage time $j$ of $b$ to $\bar{b}$ is inverse Gaussian:

$$f(j;b(t), \bar{b}) = \frac{\bar{b} - b(t)}{\sigma \sqrt{2\pi j^3}} \exp \left( -\frac{(\bar{b} - b(t) - \mu j)^2}{2\sigma^2 j} \right)$$

$$= \frac{\bar{b} - b(t)}{\sigma \sqrt{j^3}} \phi \left( -\frac{(\bar{b} - b(t) - \mu j)^2}{\sigma^2 j} \right)$$

(15)

for $j > 0$, where $\phi$ denotes the standard normal density. The corresponding cumulative distribution function is

$$F(j;b(t), \bar{b}) = \int_0^j f(s;b(t), \bar{b}) \, ds$$

$$= 1 - \Phi \left( \frac{\bar{b} - b(t) - \mu j}{\alpha j^{1/2}} \right) + \exp \left( \frac{2\mu (\bar{b} - b(t))}{\sigma^2} \right)$$

$$\times \Phi \left( \frac{-2\bar{b} + b(t) - \mu j}{\alpha j^{1/2}} \right),$$

(16)

where $\Phi(x)$ is the cumulative standard normal integral from $-\infty$ to $x$. $F(j;b(t), \bar{b})$ gives the probability that the first passage time from $b(t)$ to $\bar{b}$ is less than or equal to $j$. The cumbersome notation, with $f$ and $F$ taking $b(t)$ and $\bar{b}$ (in addition to $j$) as arguments, is adopted to clarify section 4, which involves a different initial condition and barrier.
Figure 2 illustrates two examples of the first passage time density $f(j; b(t), \bar{b})$. In each case $\bar{b} - b(t) = 5$ and $\sigma = 1$. The dark curve represents the case $\mu = 0.6$, while the light curve represents the case $\mu = 0.3$. Notice that the high value of drift is associated with the left-hand density and shorter durations. Also, the probability of the barrier’s being reached in a neighbourhood around $j = 0$ is zero, so there are no surprises. This property stems simply from the continuity of the path of a Wiener process. Speculators continuously observe the fundamental, $b$, and so if it is not at the barrier $\bar{b}$, then there is no attack imminent, because there are no jumps in $b$.

Moments of $j$ exist only if $\mu > 0$. In that case, the average duration of the peg is

$$E_j = \frac{\bar{b} - b(t)}{\mu},$$  \hspace{1cm} (17)

which is the same as the duration in the deterministic case. The variance of the duration is

$$\text{Var}(j) = \frac{[\bar{b} - b(t)]\sigma^2}{2\mu^3}. \hspace{1cm} (18)$$

The forecast uncertainty of the attack’s timing depends on two factors, in addition to domestic credit’s intrinsic variability, $\sigma^2$. First, the time until an attack is less predictable the further is the level of domestic credit $b(t)$ from the value that will trigger an attack, $\bar{b}$. As $b(t)$ approaches $\bar{b}$ over time, the duration of the peg
becomes more predictable, even though the variance of \( b \) itself does not change. Second, the forecast variance is decreasing in \( \mu \), the average growth rate of domestic credit. The more rapid is the growth of \( b \), the less uncertainty there is about the timing of the peg’s demise.

Analysing the first passage time density allows one to construct forecasts of the exchange rate at some future date, say, \( t + \tau \). If the attack occurs after the horizon \( \tau \), then \( e = \bar{e} \) until \( t + \tau \). The probability of this event is

\[
\text{Prob}(j > \tau) = 1 - F(\tau; b(t), \tilde{b})
\]

\[
= \tilde{F}(\tau; b(t), \tilde{b})
\]

\[
= \int_{\tau}^{\infty} f(j; b(t), \tilde{b})dj,
\]

where \( \tilde{F} \) is the standard notation for the survivor function, the probability that the barrier has not yet been reached. The other possibility is that the duration of the peg is less than \( \tau \). If \( j \leq \tau \), then the exchange rate floats for a period of length \( \tau - j \), starting from a value of \( \bar{e} \), so its expected value is \( \bar{e} + \mu(\tau - j) \). Combining these two possibilities gives

\[
E[e(t + \tau)] = \bar{e} \int_{\tau}^{\infty} f(j; b(t), \tilde{b})dj + \int_{0}^{\tau} f(j; b(t), \tilde{b})[\bar{e} + \mu(\tau - j)]dj
\]

\[
= \bar{e} + \int_{0}^{\tau} f(j; b(t), \tilde{b})\mu(\tau - j)dj,
\]

which is greater than \( \bar{e} \) when \( \tau > 0 \), to reflect the possibility of an innovation in \( b \) that prompts an attack and float.

The integral in equation (20) may be solved numerically, given parameter values. Alternatively, this calculation may be simplified by writing the forecast in terms of normal integrals. Conditional on an attack’s having occurred, the forecast of the exchange rate is

\[
\bar{e} + \mu(\tau - j) = b(t + j) + \alpha\mu + \mu(\tau - j)
\]

\[
= b(t) + \alpha\mu + \mu(\tau - j)
\]

\[
= b(t) + \alpha\mu + \mu\tau
\]

for \( \tau > j \). Domestic credit \( b \) follows a martingale, so the event of its having passed \( \tilde{b} \) does not affect its forecasts. Thus, equation (20) becomes

\[
E[e(t + \tau)] = \text{Prob}(j > \tau)\bar{e} + \text{Prob}(j \leq \tau)(b(t) + \mu\tau + \alpha\mu)
\]

\[
= [1 - F(\tau; b(t), \tilde{b})]\bar{e} + F(\tau; b(t), \tilde{b})[b(t) + \mu\tau + \alpha\mu],
\]
where $F(\tau; b(t), \tilde{b})$ can be calculated readily using the standard normal cumulative distribution function and equation (16). As $\tau \rightarrow \infty$ $F(\tau; b(t), \tilde{b}) \rightarrow 1$, the forecast of the future spot rate becomes $b(t) + \mu \tau + \alpha \mu$, the floating value.

These forecasts of future spot rates may be embodied in forward rates and interest differentials. For example, let $i(t, \tau)$ denote the yield on a pure discount bond at time $t$ with term (time to maturity) $\tau$, and let $i^*(t, \tau)$ denote the corresponding interest rate on a foreign-currency bond. Under approximate, uncovered, interest rate parity, the differential prior to an attack is given by

$$i(t, \tau) = i^*(t, \tau) + \frac{E[e(t + \tau)] - \tilde{c}}{\tau}.$$  \hspace{1cm} (23)

Prior to an attack, this differential is zero for $\tau = 0$, because of the zero-instantaneous-hazard property of $f$. At longer maturities, there is a forward discount on the currency prior to the attack, a feature also present in Flood and Garber's (1984) discrete-time model. The forward discount rises over time. After an attack, the differential is $\mu$ at all maturities, its value under a float (see Svensson 1992). Thus, the term structure slopes upward, and its slope rises as an attack approaches. Then the term structure becomes flat after an attack. Ozkan and Sutherland (1998) reach the same conclusions about the term structure prior to a speculative attack, using different mathematical methods, for the case of driftless fundamentals.

The classical model includes the instantaneous rate of change of the exchange rate in the asset-pricing equation (1) or, equivalently, includes the instantaneous interest rate in the demand for money in the underlying monetary model. If, instead, an exchange-rate change over some positive horizon enters equation (1) or money-demand depends on a finite-maturity interest rate, then, as the interest rate gradually rises over time, reserves will decline continuously to zero without a discrete attack. The model including an instantaneous rate of change nevertheless illustrates that sudden reserve losses can be an equilibrium phenomenon while being consistent with rising international interest differentials (at all positive maturities) as an attack approaches.

In this section I have added a stochastic fundamental to the standard model of speculative attacks. The difference from the deterministic model of section 2 is that the timing of the attack is now uncertain. That means that the variance of the duration of the fixed exchange rate can be found explicitly (in equation (18), for example). The difference from Flood and Garber (1984), who also considered random fundamentals, is that the Wiener process here gives rise to a different, simple expression for the expected, future exchange rate (22). They modelled the fundamental in discrete time and with exponentially distributed increments.

Next, in section 4, I modify the environment to feature common but incomplete information about reserves. In that case, successful speculative attacks will be preceded by smaller interest-rate increases than are worked out here. That incomplete information also leads to a richer range of outcomes in the model, including unsuccessful attacks and fixed exchange rates sometimes ending without attacks.
4. Unknown reserves

In section 3 the paths of \( b \), and hence \( r \), cannot be predicted exactly, but the current values are known. As argued in the introduction, however, the assumption that market participants know the current value of the central bank’s foreign exchange reserves is not realistic. This difficulty in accurately measuring reserves has been a problem for formal, empirical work with models of speculative attacks, even after the fact. It seems reasonable to model speculators as having no better data while attacks are occurring than econometricians have later.

Again, I model the monetary policy as I did in section 3. The central bank continues to expand domestic credit, on average, as in equation (12). Its behaviour is simple, as it is in the classical model. When it runs out of reserves, the peg is replaced by a float. Although speculators can deduce the value of \( m \) from \( \bar{e} \), the central bank conceals the underlying mixture of \( b \) and \( r \); so not knowing \( r \) is equivalent to not knowing \( b \).

There is a large number of atomistic, risk-neutral speculators with common information. This situation contrasts with the set-up of Morris and Shin (1998), where speculators receive idiosyncratic, noisy signals of \( b \) or other fundamentals. Morris and Shin (2000) have independently proposed a model in which fundamentals are reported to speculators with a delay, again with differences in information across speculators. They carefully describe the effects of the reserve lag on the evolution of beliefs in a model with driftless fundamentals and with strategic interaction among speculators.

In the present case, each speculator knows a value \( b(0) \), the value of the fundamental at time 0. The idea is that reserves are reported only periodically and cannot be observed continuously. A second, realistic feature of reserve reporting is that it occurs with a lag. In other words, the value of \( b(0) \) applies to time 0, but it may not have been announced until later. However long the lag, the key feature is that the current fundamental, \( b(t) \) is not known. In this section I describe how speculators can forecast it and how differences between their forecast and the actual fundamental can enrich the range of outcomes in the model.

To construct these forecasts, remember that speculators also know the policy parameters \( \mu \) and \( \sigma \), and that a float has not yet begun. Because the central bank runs out of reserves at the latest when \( b = \bar{e} = \bar{b} + \alpha \mu \), speculators at time \( t \) know that \( b \) has not yet reached \( \bar{e} \). Their forecast of the current value of the fundamental can be modelled using a first passage time problem. This problem differs from the one in section 3. There, we imagined that speculators know the current value of the fundamental, \( b(t) \). Given its random walk behaviour, speculators can forecast \( j \), the time until a future attack, which will occur when \( b \) reaches the trigger point \( \bar{b} \).

In contrast, in this section the speculators do not know the current value of the fundamental and so must estimate it given a report of some lagged value \( b(0) \). They do not know whether or not \( b(t) \) has reached the threshold \( \bar{b} \) that would make a speculative attack profitable. However, they do have one added piece of information
beyond the lagged reserve report that tells them $b(0)$. Recall from the model’s structure (4) that

$$\tilde{e} = b + r,$$

so that if $b$ rises to $\tilde{e}$, then reserves fall to zero and a float begins. (Of course, a float may begin even earlier because of a speculative attack, as was shown in sections 2 and 3.) Thus, if speculators observe that a float has not begun, they deduce that all previous values of $b$ lie below $\tilde{e}$. This information affects their estimate of the current value. This estimate is denoted with shorthand notation:

$$E_0 b(t) = E[b(t)|b(0), b(s) \leq \tilde{e}; 0 \leq s \leq t].$$

This is the forecast of the current value of $b$, given its last-reported value $b(0)$ and knowledge that reserves have not yet been exhausted.

The rest of the model’s structure is unchanged. The asset-pricing equation is

$$e(t) = m(t) + \alpha E_0 \frac{de}{dt},$$

with $\alpha > 0$. Now $\tilde{e}$, the shadow floating exchange rate, is based on an estimate of current domestic credit:

$$\tilde{e}(t) = E_0 b(t) + \alpha \mu.$$  

The term $\alpha \mu$ is unchanged, because the drift in $b$ is known, and that is the expected change in the exchange rate in the event of a float.

If speculators did not know that $b$ has not yet reached $\tilde{e}$, then $b(t) \sim N(b(0) + \mu t, \sigma^2 t)$ and the forecast of $b(t)$ would simply be $b(0) + \mu t$. With this information, however, the conditional density of $b$ is somewhat different. It can be written in terms of the standard normal probability density function, $\phi$, and the corresponding cumulative distribution function, $\Phi$. Below the barrier at $\tilde{e}$ the density of $b$ and $t$ is given by

$$g(b, t) = \phi \left( \frac{b - b(0) - \mu t}{\sigma \sqrt{t}} \right) \cdot \exp \left( -\frac{2\mu (\tilde{e} - b(0))}{\sigma^2} \right)$$

$$\times \Phi \left( \frac{b - \tilde{e} - b(0) - \mu t}{\sigma \sqrt{t}} \right) \left( \frac{1}{\Phi(t; b(0), \tilde{e})} \right).$$

where

$$\Phi(t; b(0), \tilde{e}) = \Phi \left( \frac{\tilde{e} - b(0) - \mu t}{\sigma \sqrt{t}} \right) - \exp \left( -\frac{2\mu (\tilde{e} - b(0))}{\sigma^2} \right)$$

$$\times \Phi \left( \frac{-\tilde{e} - b(0) - \mu t}{\sigma \sqrt{t}} \right).$$

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This density $g$ is not simply truncated Gaussian, because previous, as well as current, values of $b$ must lie below $\tilde{e}$.

Figure 3 graphs some examples of the density $g(b, t)$. Speculators know that $b(0) = 0$, from the most recent report on reserves. Monetary policy is known to have parameters $\sigma = 1$ and $\mu = 0.3$ (the light curves) or $\mu = 0.6$ (the dark curves). The value of $\tilde{e}$ is 8, so that is the maximum possible value for $b$. The left-hand curves apply at $t = 3$, and the densities are virtually normal. The density with higher drift is to the right, as one would expect. The right-hand curves take a later snapshot, at $t = 10$. Note that $g(\tilde{e}, t) = 0$. At $t = 10$ the densities are clearly non-normal, owing to conditioning on the observation that $b$ has not yet reached $\tilde{e}$. The idea is that speculators reason that the drifting fundamental must eventually be arbitrarily close to $\tilde{e}$, even when they observe that it has not yet reached that value.

The forecast of the current value of the fundamental, $b(t)$ is

$$E_0 b(t) = \int_{-\infty}^{\tilde{e}} b(t) g(b, t) db$$

$$= b(0) + \mu t - 2(\tilde{e} - b(0)) \exp \left( \frac{2\mu(\tilde{e} - b(0))}{\sigma^2} \right)$$

$$\times \Phi \left( \frac{-\tilde{e} + b(0) - \mu t}{\sigma \sqrt{t}} \right) (\tilde{F}(t; b(0), \tilde{e}))^{-1}. \quad (29)$$
The behaviour of this conditional forecast over time is just what one would expect, even if the actual expression (29) is unfamiliar. When \( b(0) \) is far from \( \bar{e} \), this forecast is close to the unconditional forecast \( b(0) + \mu t \). The conditional forecast rises over time and bends, approaching \( \bar{e} \) asymptotically. It lies strictly below \( b(0) + \mu t \) after time 0, because of speculators’ knowledge that \( b(t) \) has not yet passed \( \bar{e} \).

To see when a speculative attack will occur, consider the possible outcomes when speculators buy the central bank’s foreign exchange reserves. If they do this and \( b \) lies above \( \bar{b} \), then the shadow floating exchange rate \( \bar{e} \) lies above \( \bar{e} \) (see figure 1). That means that a float will lead to a depreciation of the domestic currency, and profits for speculators who have acquired foreign currency. If, instead, \( b \) lies below \( \bar{b} \) and speculators still acquire the central bank’s reserves, then \( \bar{e} \) lies below \( \bar{e} \) so that the domestic currency would appreciate – and speculators make losses on their foreign currency holdings – if a float were to begin. Given these possible outcomes, speculators would follow this trigger rule: attack as soon as \( E_0 b(t) \) is greater than \( \bar{b} \).

A problem with this explanation for the trigger rule is that speculative attacks that lead to appreciations are rare. A central bank under attack yet having an undervalued currency (with \( \bar{e} < \bar{e} \)) is more likely to defend its peg by imposing costs on speculators, such as temporarily large onshore interest rates or capital controls. To allow for this more realistic description of central bank behaviour, I assume that the losses from early attacks are a proportion \( \omega \) of the undervaluation of the currency. Thus, the loss from buying reserves when \( b \) is less than \( \bar{b} \) is \( \omega [\bar{e} - \bar{e}(t)] = \omega [\bar{b} - b(t)] \).

Risk-neutral speculators will purchase reserves when their expected profit passes zero:

\[
\int_{-\infty}^{\bar{b}} g(b, t) \omega \left[ b(t) - \bar{b} \right] db + \int_{\bar{b}}^{\bar{e}} g(b, t) \left[ b(t) - \bar{b} \right] db = 0. \tag{30}
\]

The left-hand side of this equation is negative at time 0, but rises over time until a purchase occurs when it passes 0. At that time, the probability that a purchase loses money, multiplied by the loss, equals the probability that it makes money, multiplied by the gain. If the loss in an unsuccessful attack is much less than the profit from a successful one, so that \( \omega \) is small, then attacks will occur relatively early and often and with a low probability of success. If the loss in an unsuccessful attack is just as large as the profit from a successful one, so that \( \omega = 1 \), then the behaviour of the central bank again supports the following plan of attack: speculators attack when their forecast of the current value \( b(t) \) reaches the trigger point \( \bar{b} \). For simplicity, I focus on this case, though the qualitative results do not depend on this choice of \( \omega \).

In summary, when \( \omega = 1 \), the condition that describes profits passing zero (30) is simply \( E_0 b(t) = \bar{b} \) or, equivalently, \( \bar{e} = \bar{e} \). Thus, the attack involves the same trigger point given in section 2 or 3, but now the attack occurs when the estimate of the current value of the fundamental reaches \( \bar{b} \) and thus when the estimate of the cur-
rent shadow exchange rate reaches the fixed exchange rate, \( \tilde{e} \). The attack may occur before or after \( b(t) \) reaches \( \tilde{b} \), however, because the actual \( b(t) \) may be less or greater than the estimate.

Now four outcomes are possible. They are illustrated in figure 4, which graphs some simulations of \( b(t) \) from a process with \( \mu = 0.3 \) and \( \sigma = 1 \). The initial, reported value for \( b \) is \( b(0) = 0 \), while \( \tilde{b} = 5 \). I set \( \alpha = 10 \) so that \( \tilde{e} = \tilde{b} + \alpha \mu = 8 \), as in figure 3. The solid, bending curve in the figure is \( E_0 b(t) \), which is the same across simulations because it does not make use of information after time 0. Realizations \( \{b(t)\} \) are shown as dashed lines.

First, the lowest realization in figure 4 illustrates a case in which the attack is early. An attack is triggered when \( E_0 b(t) = \tilde{b} \), but at that time the value \( b(t) \) is well below the forecast, so that speculators lose money and the attack ceases. As dis-
cussed above, the central bank maintains the peg and its reserves. One can imagine the speculators learning the current value of reserves, or equivalently \( b(t) \), during an unsuccessful attack. In that case, the process begins again with this as the initial condition. Alternatively, they may learn from their losses only that policy was less expansionary than they had thought: \( b(t) < E_0 b(t) = \bar{b} \). If that is all they learn, then they revise their forecast, shifting down the dark line in figure 4, and attack again later when their forecast rises to \( \bar{b} \). It cannot be profitable to attack simply to acquire information, however, given the costs associated with an unsuccessful attack.

This unsuccessful attack may appear to be probing the resolve of the central bank. Krugman (1996, sect. 5) showed that such apparent probing also can occur when there is uncertainty about the central bank’s loss function, even though speculators are atomistic. Earlier, Krugman (1979) argued that uncertainty about the reserve floor can lead to an unsuccessful attack, eventually followed by a successful one. Here, speculators are atomistic, and the central bank is known to follow the traditional trend and zero reserve floor, but with lagged reporting.

Second, if \( E_0 b(t) = b(t) \), then the attack unfolds as in the standard model. Reserves jump down and the money supply drops by \( \alpha \mu \). There is no jump in the exchange rate. This outcome has a probability of zero and so is not shown in figure 4.

Third, if \( E_0 b(t) < b(t) \), then the attack yields profits; for, as the central bank’s reserves are exhausted, the exchange rate will jump up. Recall that \( \bar{b} \) is defined as the domestic credit level at which, if the peg ends, there is no jump in the exchange rate. So, if domestic credit has expanded beyond that level, then there will be a jump up in \( e \) and also a smaller jump down in \( r \) and in the money supply than in the case with known reserves. The middle realization in figure 4 (labelled ‘late’) shows this outcome, in which \( b \) has already reached \( \bar{b} \) when the attack occurs. In figure 4 the fundamental \( b \) happens to be rising at the time of the attack, but it could also be falling before and during a successful attack.

Fourth, the top realization in figure 4 (labelled ‘very late’) shows a case in which \( b \) not only passes \( \bar{b} \) earlier than expected by speculators, but in fact reaches \( \bar{e} \) before an attack occurs. In other words, \( b \) may reach \( \bar{e} \) before \( E_0 b(t) \) reaches \( \bar{b} \). In this case, the exchange rate jumps up by \( \alpha \mu \) and there is no jump in the money supply, which begins growing at average rate \( \mu \). The float begins by surprise, before an attack occurs. Here, the realization of domestic credit is so much more expansionary than is forecast that reserves fall to zero and the fixed exchange rate is abandoned without an attack. This outcome – in which reserves are much less than speculators expect – yields predictions opposite to those of the standard model but more closely matching some historical episodes.

In these last two cases, the interest differential (23) reflects the forecast of current and future values of domestic credit rather than the more expansionary actual path. Thus, interest rates will not rise before the attack by as much as the standard model with complete information predicts (in section 3). The last two cases also show that successful attacks feature jump depreciations, since only if \( E_0 b(t) = b(t) \) is there no jump, and this occurrence has zero probability because the conditional density of \( b \) has no atom at \( \bar{b} \). Also, the fixed exchange rate may give way to a float
without a jump down in the money supply. The money supply does not jump if \( r \) hits zero (so \( b \) hits \( \bar{e} \)) before an attack occurs. The probability of no jump in the money supply is simply the probability that \( b(t) \) hits \( \bar{e} \) before \( E_o b(t) \) hits \( \bar{b} \). An analytical expression for this probability does not exist, because \( E_o b(t) \) does not follow a time-homogeneous Wiener process. However, this probability clearly is increasing in \( t \), the time since the last report on reserves, and in \( \sigma^2 \), the underlying volatility in monetary policy.

As noted earlier, these examples assume that losses from early speculation balance gains from late speculation. If, instead, the potential gains exceed the potential losses, then there will be many speculative purchases that yield small losses and fewer that yield large gains. Attacks will occur earlier, on average, than in the example worked out above, which means that jumps in the exchange rate will be less likely.

5. Unknown reserve floor

In section 4 I modelled speculators’ uncertainty about the current value of reserves. Another way to introduce uncertainty into the classical model is to attach it to the floor for reserves that prompts the central bank to abandon the fixed exchange rate. In this section I will show that not knowing the reserve floor is formally equivalent to not knowing the current value of reserves, because it is the difference between current reserves and the floor that drives attacks.

Suppose that the central bank’s policy is to abandon the fixed exchange rate as soon as reserves, \( r \), fall to some floor \( r_f \), which now need not be zero. The idea is that the central bank needs to maintain some reserves for a purpose other than maintaining the fixed exchange rate, and this need continues under floating. Alternatively, this floor could be less than zero, because the central bank may borrow reserves. Then, in the classical, deterministic model the trigger value for the fundamental is

\[
\bar{b} = \bar{e} - a \mu - r_f. \tag{31}
\]

The duration of the peg is

\[
j = \frac{r(t) - r - a \mu}{\mu}. \tag{32}
\]

The higher the floor on reserves (the foreign exchange the central bank will maintain after floating) the lower the value of the fundamental that triggers an attack and the sooner the attack occurs. With this simple change, the standard model then is consistent with an attack leading to a float, even though reserves have not been exhausted. Taiwan’s experience during the Asian crisis fits this pattern. Of course, a second-generation model in which a central bank loss function replaces this mechanical policy rule might be a better way to think of the decision to abandon a peg.
Krugman (1979, sect. 5) first considered the possibility that speculators are uncertain about the floor for reserves. Cumby and van Wijnbergen (1989) and Willman (1989) modelled the reserve floor as a fixed but unknown number. In a discrete-time version of the classical model, they used numerical methods to derive probabilities of attacks. Krugman (1979) and Willman (1989) considered the case without a cost to unsuccessful speculative attacks. They showed that in this case there will be a series of unsuccessful attacks preceding a successful one. Krugman showed that an attack occurs as soon as it might succeed in depleting reserves. That outcome occurs because speculators have a one-way bet.

Outcomes with an unknown reserve floor also can be derived if, instead, there is a cost to unsuccessful attacks, as shown in section 4. To see how these attacks might occur, next suppose that the value of \( r \) is unknown to speculators and that it can be modelled as a random variable. Then the trigger rule for a speculative attack is

\[
b(t) = E_i(b) = \tilde{c} - \alpha \mu - E_i(r),
\]

or, equivalently, because \( \tilde{c} = b + r \):

\[
r(t) = E_i(r) + \alpha \mu.
\]

From the trigger rule (33) it is easy to see that this alternative way to add uncertainty to the standard model leads to the same possible outcomes as in section 4. First, if \( b(t) \) is less than \( \tilde{b} \), then the attack is early. The floor on reserves is lower than speculators forecast. The central bank replenishes its reserves, while speculators revise their forecasts and then attack again later. Second, if \( b(t) \) is greater than \( \tilde{b} \), then the attack is late, and speculators make profits. Equivalently, reserves have fallen closer to the floor than speculators expected. Third, the floor on reserves, \( r \) may be so much higher than expected that a float begins without an attack. This outcome occurs if \( r \) falls to \( \tilde{r} \) before it falls to \( E_i(r) + \alpha \mu \). Equivalently, \( b(t) \) may rise to \( \tilde{c} - \tilde{r} \) before it reaches \( \tilde{c} - \alpha \mu - E_i(r) \), the threshold that would trigger an attack.

Speculators may use their knowledge that a float has not yet begun to revise their forecast of the reserve floor, \( E_i(r) \) over time. As \( r(t) \) drifts down, the forecast \( E_i(r) \) will also decline; for if a float has not begun, then any probability mass around \( r(t) \) in the density function for \( r \) can be shifted down. How these revisions are modelled depends on the probability density function chosen for \( r \). If that density function were unconditionally Gaussian but then conditioned on the observation that \( \tilde{r} < r \) because a float has not begun, then the same probability tools used in section 4 would apply.

6. Conclusion

In this paper a classical model of speculative attacks is studied, in which a fixed exchange rate gives way to a float because of an underlying conflict between two policy goals. The two key additions to the model are the assumption that reserves
are not continuously observable and a cost of speculation. Given a policy $\mu$, $\sigma$, $b(0)$, the model yields several different possible outcomes. Speculative purchases may yield profits or losses. There can be unsuccessful attacks (like the one on the French franc in September 1992), which appear to be probing the resolve of a central bank. The fixed exchange rate also may be abandoned with a jump up in the exchange rate and a jump down in the money supply, though the jump in $e$ is larger than it is in the deterministic model (where it is zero) and the jump in $m$ is smaller. The rise in interest rates prior to a successful attack is less than that predicted by the traditional model with complete information. Finally, the peg may end with a depreciation and no movement in the money supply (no attack) if reserves are far enough below the value forecast by speculators. These predictions arise without interaction between speculators, contingent policy rules, or other extensions.

When $E_0b(t)$ hits $\bar{b}$ and speculators buy reserves, there may be no shock in $b$ itself, or there may be a favourable shock, so that an attack may appear to occur for no reason, even though it is related to forecasts of fundamentals. A feature of attacks with unknown reserves is that they may come as a surprise to an outside investigator who does not know $\mu$, $\sigma$, and $\omega$, and a macroeconomist who observes historical data will generally find that attacks do not occur when $b(t)$ hits $\bar{b}$. The timing of an attack does not come as a surprise to insiders, however, since, given $\mu$, $\sigma$, $\omega$, $b(0)$ and $\bar{b}$, the attack timing is completely predictable. Yet insiders may still be surprised by the actual value of $b$ relative to their forecast, so that the exchange rate may jump. Insiders also may be surprised by the end of the fixed exchange rate if $b$ hits $\bar{e}$ before $E_0b(t)$ hits $\bar{b}$.

As figure 4 illustrates, the curved path of $E_0b(t)$ lies below the linear average path of the fundamental, given by $b(0) + \mu t$. Thus, if $\omega$ is large enough, then lagged reserve reporting may delay the attack on average. This effect might create a motive for the central bank’s delay in revealing reserves. This finding contrasts with a result of Morris and Shin (1998) that transparency may make a fixed exchange rate last longer by preventing self-fulfilling attacks. The analysis here is consistent with the reluctant transparency often observed, along with the central bank’s desire that the fixed exchange rate continue. This conclusion, of course, takes the reporting frequency as given. If central banks that chose frequent reporting were those with strong fundamentals, then the forecast $E_0b(t)$ for remaining central banks would be higher than that modelled here.

In this paper I have considered a fixed exchange rate, but the same method applies directly to a crawling peg. The first passage time of $b(t)$ with drift $\mu$ to a linear barrier $\bar{b} + \beta t$ has the same distribution as that of a Wiener process with drift $\mu - \beta$ to a barrier at $\bar{b}$. Sections 3 and 4 could be revised accordingly.

The first passage time density has been the building block in this paper. As Abrahams (1986) notes, a complete analytical solution to the one-sided first passage time problem is available only for the Wiener process with drift used here. Barndorff-Neilsen, Blaesild, and Halgreen (1978) link the parameters of the generalized inverse Gaussian density to the parameters of time-homogeneous diffusions. More generally, approximations or numerical methods would be necessary to examine models in which domestic credit follows a different process.
Numerical methods also could be used to consider forecasts of the current value of fundamentals, $b(t)$, that depend on additional information. For example, suppose that the density $g$ describing the possible current values of $b$ depends on some information $\theta(t)$, in addition to $b(0)$, $t$, $\mu$, $\sigma$, and $\bar{c}$. Then news in $\theta$ that shifts up the density may trigger a speculative attack. Similarly, a new report on reserves – even though it is made with a lag – may lead to an upward revision in the forecast of the current fundamental and so bring about an attack.

Appendix: Distribution theory

The first-passage time density (15) was derived by Karlin and Taylor (1975, 363) or Cox and Miller (1965, 221). For the corresponding cumulative distribution function (16), see Harrison (1985, 14) or Cox and Miller (1965, 221). The moments of $j$, (17) and (18), were calculated by Cox and Miller (1965, 222). The distribution of a Wiener process with an absorbing barrier (27), also was derived by Cox and Miller (1965, 221). The corresponding mean (29) was given by Lancaster (1990, 120). Lancaster (1990, sect. 5.7) provided a readable summary of distribution theory for the Wiener process with a single barrier. Calculating the mean (29) (the solid, curved path shown in figure 4) requires very precise evaluations of the normal cumulative distribution function. I used MAPLE$^\text{TM}$ for these evaluations.

References

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