In a recent article, MacDonald and Ollinger (MO) estimate cost functions of hog slaughtering plants. While the estimates they obtain provide useful information about scale economies and consolidation in hog slaughter, incorrect inferences are drawn about substitutability (and lack thereof) between hogs and other inputs in processing pork. The purpose of this comment is to derive the correct measures of input substitutability and to show that the error in using the Allen elasticities of substitution to draw inferences about substitutability among inputs can be substantial.

MO in discussing Allen elasticities of substitution (AES) between pairs of inputs (hogs, labor; hogs, other materials; hogs, capital) report that the magnitudes of the parameter estimates indicate that there “. . . is essentially no substitution between hogs and labor or between hogs and other materials. There does appear to be some degree of substitution between hogs and capital, reflecting perhaps opportunities to use capital equipment to increase yields of meat from hog carcasses” (p. 340). While MO are not denying that *any* substitutability between hogs and other inputs exists, their inference based on the AES estimates is that substitutability, if present, is quite limited. This inference is erroneous as the following discussion shows.

In an important article, Blackorby and Russell (BR) show that the Morishima elasticity of substitution (MES) and not the AES preserves the essential characteristics of the usual Hicksian notion of substitutability. Specifically, BR show that the MES is the relevant measure of curvature or ease of substitution along an isoquant in the case of more than two inputs. In contrast, they demonstrate that for more than two factors, the AES cannot be regarded as a valid measure of the ease of substitution or curvature of the isoquant.

In its simplest form, the MES is calculated as the derivative of the logarithm of the optimal quantity ratio $x_i/x_j$ with respect to the logarithm of the price ratio $p_i/p_j$, with variation in the price ratio in the $j$th coordinate direction. The formula for the MES is (BR, p. 885)

$$ M_{ji} = \epsilon_{ij} - \epsilon_{jj} $$

where $\epsilon_{ij}$ is the price elasticity of demand for the $i$th factor with respect to the $j$th factor price holding output constant. This elasticity is related to the AES as

$$ \epsilon_{ij} = \frac{\text{AES}_{ij}}{S_j} $$

where $S_j$ is the share of the $j$th input in total cost. The interpretation of the MES in (1) is the effect on the quantity ratio $x_i/x_j$ from approximately a 1% change in the price ratio $p_i/p_j$, allowing only the $j$th price to vary.

Using the data on AESs and $S_j$'s from MO's table 2 (p. 340), table 1 reports the MESs for variation in the prices of labor, animals (hogs), materials, and capital on the respective quantity ratio. As the table reveals, most of the relationships indicate substitutability and not complementarity. The effects on the quantity of hogs relative to the quantities of labor, materials, and capital from changes in the prices of labor, materials, and capital are shown in the second row. A 10% increase in the price of labor causes the ratio of hogs to labor to increase 3.34%, a 10% increase in the price of materials causes the ratio of hogs to materials to increase 1.77%, and a 10% increase in the price of capital causes the ratio of hogs to capital to increase 14.93%. In contrast, the effects of a given change in the price of hogs on the quantity ratios are shown in the second column where a 10% increase in the price of hogs causes the ratio of labor to hogs to
Table 1. Morishma Elasticities of Substitution in Hog Slaughter

<table>
<thead>
<tr>
<th>Quantity Ratio (Numerator)</th>
<th>Price Change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Labor</td>
</tr>
<tr>
<td>Labor</td>
<td></td>
</tr>
<tr>
<td>Animals</td>
<td>0.334</td>
</tr>
<tr>
<td>Materials</td>
<td>0.737</td>
</tr>
<tr>
<td>Capital</td>
<td>0.621</td>
</tr>
</tbody>
</table>

The fact that the elasticities of substitution are not symmetric (or even of the same sign in some instances) should not be surprising in view of the fact that the MES is naturally asymmetric (BR, p. 885). What is important to recognize is that substitutability is not negligible when looking at the effects of changing the price of labor, materials, or capital. Most significantly, whether the price of capital or price of hogs is changed, the elasticities indicate a large degree of substitutability between hogs and capital. Indeed, the degree of substitutability between hogs and capital is one of the largest among the different pairs of factors! Therefore, the suggestion by MO that substitutability between hogs and other inputs in pork production is quite limited is not correct.

As previous authors have argued (Wohlgenant 1989; Goodwin and Brester), substitutability between the agricultural raw product and other factors in processing/marketing is an important feature of the food industry. To ignore this feature—or to relegate it as insubstantial—could lead to serious misconceptions about the nature of the food processing sector (i.e., whether it can be characterized as a value-added process), the nature of the relationship between retail and farm prices, and the degree of market power (Wohlgenant 1999). In the case of more than two factors—as is the case in MO’s study—Allen elasticities of substitution provide no information about pair-wise substitutability. The relevant elasticities are Morishma elasticities, which in the case of the hog processing industry indicate a much greater degree of substitutability than the Allen elasticities.

References


