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Housing Return and Construction Cycles

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This paper presents a general equilibrium model of the residential housing market. Within the model housing returns, housing construction, mortgage loan terms, and household maintenance behavior are all endogenous. These interacting elements tie expected housing returns to expected changes in family wealth. As a result: (1) families are credit constrained; (2) mortgage loan-to-value ratios can be used to forecast future housing returns; (3) developers acquire land when expected housing returns lie above the rate of interest and then develop when housing returns lie below. Thus, their holdings and construction decisions also forecast housing returns.

This paper presents a general equilibrium model in which interactions between homeowners, banks, and developers lead to fluctuations in housing prices, mortgage loan terms, and both the deterioration and redevelopment of the housing stock. In contrast, most models take the actions of one or more of the parties as exogenous. For example, real options papers treat housing prices as an exogenous process generally unaffected by the development being modeled. Other papers take the down payment required by purchasers as exogenous and then analyze the resulting impact on housing prices. Here every party is modeled as an optimizing agent. Of course, the cost of this general equilibrium approach is additional modeling complexity. Nevertheless, it remains possible to obtain a great many results which have received both empirical verification and are new to the theoretical literature.

Williams (1993b) and Shiller and Weiss (2000) present evidence that when people do not have a stake in their home's value, they do not expend much effort on care, which then leads to increased degradation of the physical structure. This naturally creates a moral hazard problem between homeowners and their creditors. In the model presented here, this conflict results in credit rationing, a phenomenon which has ben observed in the empirical work of Duca and Rosenthal (1991), Rosenthal, Duca and Gabriel (1991), Ambrose, Pennington-Cross and Yezer (1998), and Haurin, Hendershott and Wachter (1997). All of these papers show that at the interest rates charged by banks, there exist

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families that would like to borrow money in order to purchase a house, but are nevertheless turned away.

While credit rationing helps solve the moral hazard problem, it also leads to periods in which housing returns have positive expected abnormal returns as families occasionally find themselves unable to bid away all of the future rents. Of course, in such cases one might expect potential landlords to step in and compete away any excess returns. Otherwise, these entities should be able to profit by purchasing, renting, and then selling homes after the excess returns come to fruition. However, as the paper shows, this strategy is not economically profitable, even in the face of high forecasted housing returns, so long as landlords are less efficient at maintenance than homeowners. In pricing terms, a home is always worth more to a family that wishes to live in it than to an institution that wishes to purchase and rent it out. For potential landlords predictable excess housing returns simply never exist. In part, this may help to explain why few single-family homes are rented out even in countries like Canada that do not have mortgage interest deductions.

While moral hazard issues may lead housing in good condition to exhibit time-varying excess returns, the same cannot be said for dilapidated buildings. Properties in already poor condition are obviously somewhat immune to such issues. As a result, the usual arguments lead the model to conclude that in equilibrium these lots will always have expected returns that equal the current discount rate. Empirically, this implies that the price ratio for homes in good condition versus those in dilapidated condition should narrow during periods of high expected growth. Rational developers therefore acquire land when it is cheap (has a high expected return) relative to homes in good condition, and develop and sell their land when it becomes relatively expensive (has a low expected return). In other words, developer holdings should forecast future housing returns. This same pattern implies that unusually large positive housing price shocks will tend to lead abnormally high levels of construction.

Many of the links between land prices, housing prices, and construction activity described above have recently been documented in the data. Coulson (1999) finds that price increases induce builders to finish accumulated inventory rather than start new units. Bulan, Mayer and Somerville (2000) also examine builder reactions to housing prices and find that a price increase leads to a significant increase in the probability that new construction will take place. These empirical findings are consistent with the model's prediction that builders will accumulate buildings in anticipation of a price rise, then sell after the rise is over. Thereafter, they bide their time until a new price shock can be anticipated. Evidence in this area can be found in Rosenthal (1999), who finds that when land prices become relatively high, current construction goes down.

According to the model, expected housing returns should be tied to expected changes in local incomes. Since there is no reason to believe such changes should follow a random walk, this may help to explain a number of empirical findings. Case and Shiller (1989) and Meese and Wallace (1994) conclude that housing prices do not follow a random walk. However, even more to the point is Capozza and Seguin's (1996) finding that housing prices "over-react" to income growth, the very mechanism that the model posits as the driving force behind the time-varying expected returns.

In comparing the results in this paper with those from the "stochastic cities" literature founded by Capozza and Helsley (1990), it may be useful to separately analyze data from large mature cities and their less fully developed counterparts. In fact, many accepted "truths" regarding the general housing stock (and especially new construction) come from national data which for the most part represents areas of the country other than the very largest cities.[1] For example, the model predicts that in a mature city, housing in good condition will display time-varying expected returns while dilapidated homes and vacant lots will not. In contrast, outside of such cities, most models conclude that both developed land and land in need of development will have constant expected returns. Roughly, in developing areas, the difference between new housing prices and vacant lots should simply be the cost of construction. However, in developed cities this need not be so. According to the model, during economic booms the price difference should narrow, and at the peak they should spread apart enough that a housing construction boom ensues.

The model also produces predictions regarding cycles in the observed set of mortgage contracts. Credit rationing causes higher rates of expected economic growth to produce higher expected housing returns. Higher economic growth rates thus reduce the moral hazard problem between banks and homeowners which allows for higher equilibrium LTV ratios. Empirically this should imply that LTV ratios can be used to forecast housing returns, a prediction that has been verified by Lamont and Stein (1999).

The paper has four sections and an Appendix. The first section presents the theoretical model. This section lays out the model's representation of the housing stock and the level of economic activity. It also describes the agents within the model and presents the problems faced by a family that moves into a city. It then combines all of these elements to calculate the equilibrium actions taken by the model's agents and the resulting supply and price of housing. Finally, the first

[1] Topel and Rosen's (1988) paper on the subject is typical in its use of nationwide data.

section lays out the model's equilibrium properties, describes the return process, mortgage rates, and equilibrium LTVs and describes the housing supply. The second section discusses a number of extensions to the model including more general utility functions, heterogenous wealth levels, and heterogenous housing quality. The third section relates this model to previous studies. The final section contains the paper's conclusions and the Appendix provides all of the proofs.

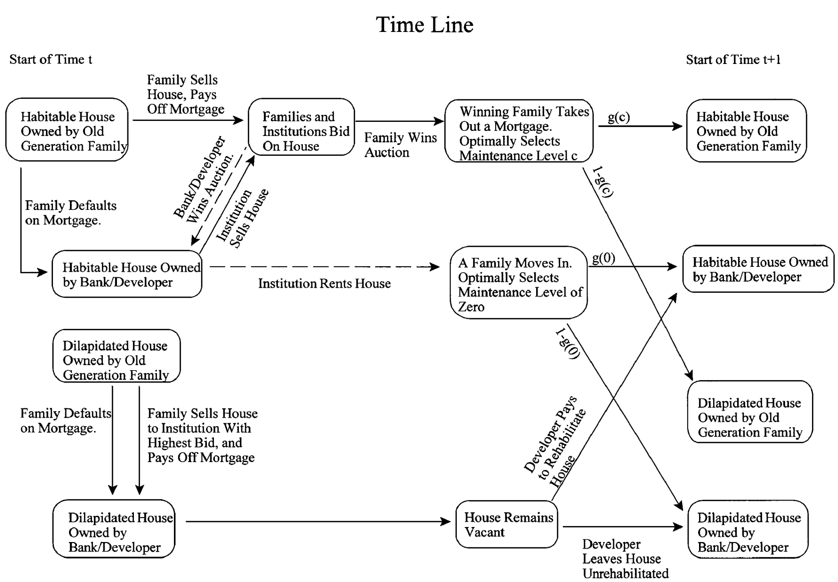
The Model

Figure 1 presents an outline of events within the model.

Geography and the Level of Economic Activity

Consider a city with a central business district (like New York) for which land within commuting distance of the business district exists in finite supply. There exist N lots within this area upon which single-family homes can be built. Homes are either dilapidated and must be rebuilt or are in good "move-in"

Figure 1 ■ Homes start each period in one of four states. Habitable, or dilapidated, and either owned by a family, or an institution. Habitable homes can either be sold to incoming families, or institutions. If owned by an institution it can either be resold or rented out. Dilapidated homes can only be transferred to an institution. Family occupied homes require care, which they set to c and this determines the probability a home will remain habitable $g(c)$ or become dilapidated $1-g(c)$. Developers can rehabilitate dilapidated homes. Dotted lines represent out of equilibrium moves.



condition. Dilapidated homes provide so little utility that potential residents would rather move into an apartment, live in another city, or fix up the house before living in it. Anyone that cannot afford a single-family house is accommodated elsewhere, perhaps in a rental apartment or another city.

The model has neither a beginning nor ending date. What differentiates one period from the next are two economic state variables: w_t and s_t. The variable w_t captures the general level of economic activity in the economy with higher values representing a more prosperous society. The variable s_t captures the rate at which the economy is currently growing and interacts with w_t via the following dynamic equation:

$$w_{t+1} = s_t w_t \delta_{t+1}, \tag{1}$$

where δ_t represents a forecast error. Throughout, the paper assumes that the δ_t and the s_t are independent across time periods with time independent distribution functions of $f(\delta)$ and $q(s)$, respectively. Since every variable in the model will be tied to the economic activity variable, w_t, it is natural to restrict both s and δ to values between zero and plus infinity. Finally, to cut down on the notational burden, the expected values of both random variables are normalized to one.

Agents

At the start of period t, the generation of inhabitants that moved in the previous period moves out and a new generation moves in. In real life, people move in and out of a city for any number of reasons that have nothing to do with the housing market. For example, they may get married, change jobs, or even die. Thus, the model takes the decision to move as an exogenous event.

Families possess a utility function over housing and wealth of the following form:

$$U(\theta, W_{t+1}) = \theta W_{t+1} - c_t. \tag{2}$$

In the equation, θ represents an indicator variable that equals one if the family lives in a house that is not dilapidated and zero if it lives elsewhere. The variable W_{t+1} equals the family's total wealth when it moves out of town. This functional form has the advantage of implying that above all else, families prefer to live in single-family housing rather than apartments or another city. However, given the family's residence, it prefers more wealth to less. The variable c_t equals the utility spent by the family to care for their housing purchase. A family can vary its level as they see fit in response to their economic environment. Here, care should not be construed simply as roof repairs and the like, but also the updating of the home to current standards of technology and taste.[2]

When members of the old generation leave the city, they put their homes on the market. The new generation then bids on these homes, with each house going to the highest bidder. In case of a tie among the bidders, the winner is selected via a random draw that gives each of the high bidders an equal chance to win. Each incoming family arrives with a wealth level of $W_t = w_t$. (Recall w_t is just a state variable; thus this definition just normalizes it to equal the current wealth of the incoming generation.) In an attempt to purchase a home they can combine their wealth with a bank loan. Banks offer mortgages in which they lend an amount ℓ in exchange for a commitment from the borrower to repay m the next period when the family moves out of the city. The model assumes that mortgages are nonrecourse loans. Thus if the borrower defaults the bank can take possession of the house, but cannot otherwise punish the borrower.[3]

In the event of default, the bank takes possession of the house and can either sell it immediately, rent it out, or leave it vacant. If either of the latter two options are employed, then it can then try to sell the house at some later date. Since casual observation indicates that empty homes tend to deteriorate rather quickly, the model assumes that an unoccupied home receives an amount of care equal to zero.[4]

Banks are profit-maximizing, competitive, risk-neutral institutions. They will issue mortgages so long as they believe that the expected return on the loan

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[2] Since c_t does not come out of the budget constraint, it literally represents the personal time and effort that people put into keeping a home in shape in order to prevent it from deteriorating. The model pursues this route for simplicity. Adding c_t to the budget constraint does not alter the qualitative conclusions. Quantitatively, however, changing c_t from effort to cash strengthens the model's predictions regarding the existence and influence of credit constraints. If c_t represents cash instead of effort, then the banks will further restrict their loans to ensure that the owners of the house have both the incentive and funds to care for it. Under the model's current assumptions, the banks do not need to worry about the latter issue.

[3] Empirically, borrowers do default on their loans and frequently because the house is worth less than the mortgage. See Lekkas, Quigley and Van Order (1993) and Deng, Quigley and Van Order (1996) for evidence on this issue. Also, while the paper only examines traditional loans, it can be easily generalized to include others types such as equity participation loans. The model only requires that larger mortgage repayment provisions reduce the owner's payoff in some states of nature. So long as this holds, owner-provided care will fall as the repayment provisions increase, and the model's qualitative results will remain unchanged.

[4] The assumption that only occupants can care for a residence is made primarily to help simplify the analysis. However, one can easily modify the model to allow for care by the lender as well. Also, see Shiller and Weiss (2000) for a detailed discussion regarding both why banks cannot contract around the moral hazard issue, and why maintenance is important for preserving a home's value.

equals or exceeds the current interest rate, r. In addition to banks, the city also contains developers. Developers can restore dilapidated homes at a cost of $hw_t s_t$. The assumption that housing construction costs depend upon the state variables captures the idea that in a wealthy or fast-growing economy, local resource demands go up and this in turn adds to building costs (a phenomenon empirically verified by Somerville 1999). For simplicity, also assume that a payment of $hw_t s_t$ dollars in period t allows the developer to produce a home in that same period.

While, in principle, the banks and developers are separate entities, to reduce the level of notation (and without loss of generality) it is easier to treat them as one and the same. This essentially implies that when a bank forecloses on a dilapidated house, it turns it over to its development office instead of selling it to a separate corporation. Thus, the discussion will use the terms bank and developer interchangeably as appropriate.

The Incoming Family's Problem

When families move into the city, they attempt to purchase a home. The successful ones then decide how much care to put into their residence. Higher maintenance levels cost more in lost utility, but increase the future value of the house. Since the model assumes that the housing stock is of homogenous quality, care can only influence the probability that any one house will remain in good condition next period.

Based upon the above discussion, a homeowner's optimization problem can be written as

$$\max_c g(c/s_t w_t) \int_{\psi} [P_{t+1} - m] f(\delta_{t+1}) q(s_{t+1})\, d\delta_{t+1}\, ds_{t+1}$$

$$+ [1 - g(c/s_t w_t)] \int_{\varphi} [V_{t+1} - m] f(\delta_{t+1}) q(s_{t+1})\, d\delta_{t+1}\, ds_{t+1} - c. \tag{3}$$

In Equation (3) ψ represents the set of states such that $P_{t+1} \geq m$ when the home remains in good condition, and φ represents the set of states where the homeowner remains solvent even though the home is dilapidated. The function V represents the value of the dilapidated house. Naturally, both P and V are functions of the state variables s and w. The function $g(c/s_t w_t)$ yields the probability that the home will remain in good condition. The division of c by $s_t w_t$ implies that homeowners face the same type of state-dependent costs as builders. To see this, note that if $s_t w_t$ doubles, then the family must spend twice as much to produce the same probability that the home will remain in good condition. This is similar to the paper's earlier assumption that builders must spend twice as much to rehabilitate a dilapidated house when $s_t w_t$ doubles.[5]

For care to matter, it is necessary that the level of care produced by a family have a meaningful impact upon their home's expected value. To guarantee that this is true, the paper imposes a number of technical restrictions on g, which can be found in the Appendix.

At this point some may argue that care, unlike its portrayal in the model, is self-enforcing. Presumably residential care always takes place since a deteriorating home is unpleasant to live in. After all, nobody wants to live in a house where rainwater comes in through the living room. However, this argument misses the fact that many forms of maintenance only impact the long-term value of the structure and not the immediate quality of life provided to its residents. A home requires updating as tastes and technologies change to preserve its value, not its livability. More traditional examples exist as well. Consider a home supplied by well water. Typically, good maintenance practice requires the addition of chemicals designed to balance the water's pH. If this is not done the acidity level will cause the home's plumbing to corrode, which will eventually require major expenditures to correct. Nevertheless, the water will often remain palatable and thus not impact the quality of life experienced by the home's residents. Thus, while lenders can count on residents to perform some types of home maintenance, there are other types that may not be preformed if the residents do not have the necessary financial incentives.

Equilibrium

The paper assumes that the level of care set by the family cannot be imbedded in the mortgage loan agreement. Thus, mortgage loans contain simply an amount the bank will lend and a balloon payment that the borrower will owe the following period. Under these circumstances, the homeowner's first order condition can be written as

$$g' \frac{1}{s_t w_t} \int_{\psi} [P_{t+1} - m] f(\delta_{t+1}) q(s_{t+1})\, d\delta_{t+1}\, ds_{t+1} \;-\; g' \frac{1}{s_t w_t} \int_{\varphi} [V_{t+1} - m] \times f(\delta_{t+1}) q(s_{t+1})\, d\delta_{t+1}\, ds_{t+1} - 1 = 0. \tag{4}$$

Due to the moral hazard problem posed by the homeowner's care decision, banks may choose to restrict the size of the loan they will extend to any one

[5] Equivalently, one can assume g is a function of c alone and that the family's utility cost of care equals $cs_t w_t$. The paper's results are essentially identical in either case. Under both formulations the family must double its utility costs of care when $s_t w_t$ double in order to keep the probability that the home will remain in good shape constant.

family. This in turn restricts that family's maximum bid for a house. However, prior to formulating the bank's problem one must address the issue of whether or not a home that has been repossessed will always be sold to a family that wishes to occupy it. This not only affects the bank's problem but also the distribution of the price process. If, in some states of nature, speculators wish to buy and hold homes either unoccupied or for rent, then they can outbid families that wish to occupy them, and this must be accounted for. Fortunately, none of these concerns arise in equilibrium since one can prove that if a home is in good condition, the highest bidder will always be a family that wishes to take possession. Or equivalently, if a bank forecloses on a house, it immediately resells it to a family.

Proposition 1 *Developers and banks never rent or leave vacant any home in good condition.*

Proposition 1 shows that the paper's results hold even though there exist wealthy individuals or institutions that can potentially purchase and hold housing in order to speculate on its return. The difficulty speculators face is that while owner-occupied housing may have an above-market return, in any particular period, unoccupied and rental housing does not. Thus, speculators cannot profitably bid away homes from potential residents. As a result, banks and developers do not influence the equilibrium price process despite the fact that they may have sufficient funds to purchase large quantities of the housing stock. Proposition 1 may therefore help to explain why almost all single-family homes are owner-occupied in both the United States (85.5% according to the U.S. Census Bureau, 1990) and Canada (90.3% according to the Canadian Census, 1996).[6]

Proposition 1 also shows that without a moral hazard component other models will have difficulty replicating the array of results produced later in this paper. For example, consider a model in which bankruptcy induces transactions costs on the part of a lender. Lenders will then restrict the size of their loans, leading to credit constraints on individual families. Thus, it appears a pure bankruptcy cost model can reproduce the results presented here. But this is not the case. Absent moral hazard, bankruptcy costs will not stop wealthy institutions from purchasing housing and then renting it out when the expected appreciation rate makes it profitable to do so. In fact, such costs will encourage them to do so since they have a competitive advantage vis-à-vis borrowers. Thus, expected housing

[6] Naturally, tax arguments (which are not covered here) may also explain the high owner occupancy rates within the United States. However, in Canada, mortgage interest payments are not deductible, making the tax argument a somewhat less compelling explanation for their high owner occupancy rate.

returns must always remain equal to or below the cost of capital. Conversely, Proposition 1 shows that moral hazard issues can prevent institutions from capping returns since rental units produce lower returns than owner-occupied dwellings. It is this wedge that endows the model with a number of unique and interrelated features.

The equilibrium mortgage contract must provide the highest possible loan amount, which is equivalent to setting m in order to maximize ℓ. This requirement, combined with the results from Proposition 1, implies that the mortgage contract must solve

$$\max_m g(c/s_t w_t)\left\{\int_{\neg\psi} P_{t+1} f(\delta_{t+1}) q(s_{t+1})\, d\delta_{t+1}\, ds_{t+1} + m\int_{\psi} f(\delta_{t+1}) q(s_{t+1})\, d\delta_{t+1}\, ds_{t+1}\right\} + [1 - g(c/s_t w_t)]\left\{\int_{\neg\varphi} V(s_{t+1}, w_{t+1}) \times f(\delta_{t+1}) q(s_{t+1})\, d\delta_{t+1}\, ds_{t+1} + m\int_{\varphi} f(\delta_{t+1}) q(s_{t+1})\, d\delta_{t+1}\, ds_{t+1}\right\}. \tag{5}$$

The region of integration, $\neg\psi$, represents the set of states where bankruptcy occurs and the home remains in good condition ($P_{t+1} < m$). If the price of the house falls below the mortgage payment, the family defaults, the bank takes possession, and then sells the house. This is represented by the first integral. The second integral covers those states of nature where the value of the home exceeds the mortgage payment due the bank. Whenever this happens the bank expects the family to repay the loan and keep the house. The second term in braces represents the value to the bank when a house has deteriorated to the point of being dilapidated. In the region of integration $\neg\varphi$ the bank obtains a dilapidated house during foreclosure, while in the region φ the homeowner prefers to pay off the loan.

With the assumptions behind the formulation of Equation (5) now verified, one can differentiate it with respect to m. Setting the result equal to zero produces the following first order condition,

$$g'\frac{dc}{dm}\frac{1}{s_t w_t}\left\{\int_{\neg\psi} P_{t+1} f(\delta_{t+1}) q(s_{t+1})\, d\delta_{t+1}\, ds_{t+1} + m\int_{\psi} f(\delta_{t+1}) q(s_{t+1})\, d\delta\, ds\right\} + g\int_{\psi} f(\delta_{t+1}) q(s_{t+1})\, d\delta_{t+1} ds_{t+1} - g'\frac{dc}{dm}\frac{1}{s_t w_t} \times \left\{\int_{\neg\varphi} V_{t+1} f(\delta_{t+1}) q(s_{t+1})\, d\delta_{t+1}\, ds_{t+1} + m\int_{\varphi} f(\delta_{t+1}) q(s_{t+1})\, d\delta\, ds\right\} + [1-g]\int_{\varphi} f(\delta_{t+1}) q(s_{t+1})\, d\delta_{t+1}\, ds_{t+1} = 0 \tag{6}$$

that must characterize the mortgage that yields the maximum loan amount. To help keep the notation compact, g' represents the derivative of g. The dc/dm term represents the fact that as the bank demands an ever higher mortgage payment, the level of care taken by the homeowner will change. The next proposition indicates that c weakly decreases in m.

Proposition 2 *The family's optimal level of care weakly decreases in the required mortgage payment. Formally,* $dc/dm \leq 0$.

While the model predicts that the level of expected care declines with the size of the mortgage, this should not be construed to imply that the model also predicts that homeowners will not (in equilibrium) care for their homes. In equilibrium, banks adjusts their lending terms so that homeowners will in fact wish to engage in maintenance.

While the model assumes that each family inhabits its house for only one period, one can still draw some conclusions about continuous time data. Based upon Proposition 2 homeowners should adjust their care in response to their current loan-to-value ratio. If the value of their home falls, they should then reduce their expenditures on care, which should lead to a lower expected value of the home in the future. Lekkas, Quigley and Van Order (1993) provide some empirical evidence consistent with this hypothesis. They find that homes with high loan-to-value ratios suffer greater value declines in default than homes with initially lower loan-to-value ratios. This is consistent with the idea that care matters. High loan-to-value homes initially receive less care, and if prices in general turn down, then these homes fall both faster and by more than their better cared for brethren.

Additional evidence on the importance of care and its relationship to a homeowner's loan-to-value ratio comes from Case, Shiller, and Weiss, Inc.'s appraisal model. Their model reduces the expected value of a property by about 20% if it is in default (see the Case Shiller Weiss, Inc. web site). That is, their model predicts that if you compare two homes in the same area, and one is in default and the other is not, then the "distressed" property will typically be worth 20% less. Since both homes are impacted by the same neighborhood factors, presumably the difference lies in the fact that distressed homes tend to be in less desirable physical condition than their neighbors. Within the model this occurs when families deep in debt reduce their maintenance activities in response to the fact that the bank will reap most of the benefits.

In equilibrium, the conjectures held by both the bank and the city's families regarding the distribution of future housing prices must be accurate. Following standard practice, the solution to the model is found by initially conjecturing a solution to the price process. This conjecture is then used to solve for the actions of each player and in turn their actions are used to derive the dynamics of the price process. If the conjecture holds true, in that the prices produced by the players matches the conjectured price process, then an equilibrium has been found.

Within the current setup, the appropriate conjectures are that the banks will lend an amount $\ell_t = w_t s_t k_\ell$, and in return they will require a mortgage repayment of $m_t = w_t s_t k_m$. Should the bank find itself in possession of a dilapidated house, then its expected profit from building and selling a new house at the optimal date will equal $V_t = w_t s_t k_V$. Finally, in response to both the bank's lending practices and the resulting housing price process, homeowners will set their care level to $c = w_t s_t k_c$, where k_ℓ, k_m, and k_c are constants.

Since families will spend whatever is necessary to move into a house, they will bid up housing prices until they equal the sum of the current loan amount and the family's wealth level. Thus, $P_t = \ell_t + w_t$, and given the conjectured solution to ℓ_t, one can rewrite this as $P_t = w_t(s_t k_\ell + 1)$. Increasing the index one period yields $P_{t+1} = w_{t+1}(s_{t+1} k_\ell + 1)$ and since $w_{t+1} = w_t s_t \delta_{t+1}$, this implies $P_{t+1} = w_t s_t \delta_{t+1}(s_{t+1} k_\ell + 1)$.

Substituting the conjectured solution into the homeowner's first order condition, Equation (4) becomes

$$g' \int_{\psi} (k_\ell s_{t+1} + 1)\delta_{t+1} - k_m) f(\delta_{t+1}) q(s_{t+1})\, d\delta_{t+1}\, ds_{t+1}$$

$$- g' \int_{\varphi} (k_v s_{t+1} \delta_{t+1} - k_m) f(\delta_{t+1}) q(s_{t+1})\, d\delta_{t+1}\, ds_{t+1} - 1 = 0, \qquad (7)$$

where the $s_t w_t$ terms in the integral have been canceled out with those arising from the differentiation of g. Similarly, the first order condition characterizing the equilibrium mortgage payment (6) now equals

$$g' \frac{dc}{dm} \left[\int_{\neg\psi} (k_l s_{t+1} + 1)\delta_{t+1} f(\delta_{t+1}) q(s_{t+1})\, d\delta_{t+1}\, ds_{t+1} \right.$$

$$\left. + k_m \int_{\psi} f(\delta_{t+1}) q(s_{t+1})\, d\delta_{t+1}\, ds_{t+1} \right] + g \int_{\psi} f(\delta_{t+1}) q(s_{t+1})\, d\delta_{t+1}\, ds_{t+1}$$

$$- g' \frac{dc}{dm} \left[\int_{\neg\varphi} k_V s_{t+1} \delta_{t+1} f(\delta_{t+1}) q(s_{t+1})\, d\delta_{t+1}\, ds_{t+1} + k_m \int_{\varphi} f(\delta_{t+1}) \right.$$

$$\left. \times\, q(s_{t+1})\, d\delta_{t+1}\, ds_{t+1} \right] + (1 - g) \int_{\varphi} f(\delta_{t+1}) q(s_{t+1})\, d\delta_{t+1}\, ds_{t+1} = 0. \qquad (8)$$

In addition to Equations (7) and (8) in equilibrium banks must earn a zero expected profit on their loans. Thus, the condition

$$k_\ell = \frac{1}{1+r}\left\{g\left[\int_{\neg\psi}(k_\ell s_{t+1}+1)\delta_{t+1}f(\delta_{t+1})q(s_{t+1})\,d\delta\,ds_{t+1}\right.\right.$$
$$+k_m\int_{\psi}f(\delta_{t+1})q(s_{t+1})\,d\delta_{t+1}\,ds_{t+1}\Bigg] + (1-g)\left[\int_{\neg\psi}k_V s_{t+1}\delta_{t+1}f(\delta_{t+1})\right.$$
$$\left.\left.\times\, q(s_{t+1})\,d\delta_{t+1}\,ds_{t+1} + k_m\int_{\varphi}f(\delta_{t+1})q(s_{t+1})\,d\delta_{t+1}ds_{t+1}\right]\right\} \tag{9}$$

must also be satisfied, where r is the appropriate discount rate. This leaves the valuation function for a dilapidated house k_V which must satisfy

$$k_V = \frac{1}{1+r}\left\{\int_{\xi}[(k_\ell s_{t+1}+1)\delta_{t+1} - hs_{t+1}\delta_{t+1}]f(\delta_{t+1})q(s_{t+1})\,d\delta_{t+1}\,ds_{t+1}\right.$$
$$\left.+\int_{\neg\xi}k_V s_{t+1}\delta_{t+1}f(\delta_{t+1})q(s_{t+1})\,d\delta_{t+1}\,ds_{t+1}\right\}, \tag{10}$$

where ξ represents the set of future states under which the bank builds and then sells a house on the lot, and $\neg\xi$ represents the set of states where the bank postpones construction until at least the next period. Based upon the above set of equilibrium equations one can now solve for k_c, k_ℓ, k_m, and k_V. Furthermore, the conjectures will be self-fulfilling since the level of care, the size of the mortgage loans, the required balloon payment, and the implicit value function (V) can all be obtained independently of w_t or s_t's current value.

Properties of the Equilibrium

Expected housing returns, mortgage rates, and LTVs. Given solutions to k_c, k_ℓ, k_m, and k_V one can now calculate the expected return to a house (r_{ht}) over the next period as

$$r_{ht} = \frac{E(P_{t+1})}{P_t} = \frac{E[(k_\ell s_{t+1}+1)s_t\delta_{t+1}]}{k_\ell s_t+1} = \frac{(k_\ell+1)s_t}{k_\ell s_t+1}, \tag{11}$$

where the last equality follows from the assumption that $E(s_{t+1})$ and $E(\delta_{t+1})$ equal 1. This equation represents the typical definition used by people that construct real estate price indices (for example, repeat sales indices such as those found in Case and Shiller 1989 or hedonic models such as Meese and

Wallace 1994). Since it does not include either the cost of maintenance or the probability that the house will become dilapidated next period, the equation does not represent the actual expected return to a homeowner with a 100% equity position in the house. Nevertheless, this is the proper definition for somebody holding a lot without a home in good condition on it that wishes to know if construction should take place this period or next, which is the issue under study in this paper. However, with relatively little difficulty, the results regarding expected returns can be modified to account for other perspectives.

Equation (11) implies that the return to housing over the next period lies between 0 and $(k_\ell + 1)/k_\ell$ as s_t varies between zero and plus infinity (assuming k_ℓ is nonnegative). The next proposition refines this range by finding restrictions on the value of k_ℓ.

Proposition 3 *The value of k_ℓ is strictly greater than* 0 *and less than* $1/r$*. It equals* $1/r$ *only when there does not exist a moral hazard problem.*

Based upon Equation (11), the variation in expected housing returns depends critically upon the equilibrium value of k_ℓ. The value of k_ℓ in turn depends upon the importance of the moral hazard problem. Since (11) is strictly increasing in s_t, the expected return to housing must range from 0, when s_t equals 0, to a high of $(k_\ell + 1)/k_\ell$ as s_t goes to infinity. Thus, based upon Proposition 3, when care is irrelevant, housing returns vary from 0 to r, while in an environment where care is important, expected returns need not have any upper bound. These results are summarized in the next proposition.

Proposition 4 *Expected housing returns are strictly increasing in the local rate of economic growth, $d\{E(P_{t+1})/P_t)\}/ds_t > 0$. At s_t goes to* 0 *the expected return to housing goes to zero; as s_t goes to infinity the expected return to housing approaches* $(k_\ell + 1)/k_\ell$.

Spiegel and Strange (1992) also study how moral hazard influences expected housing returns. They conclude that the need for maintenance by a home's occupants can produce housing prices that exhibit predictable supernormal returns. The current paper extends this result to a multiperiod setting. However, by deriving this result within a dynamic model, the current analysis goes further by showing that predictable excess housing returns can occur in *any* economy where care matters if s_t happens to be large enough in that period. Conversely, the dynamic setting also shows that for s_t small enough, housing prices will exhibit predictable *below*-market returns. All of this implies that expected housing returns will depend upon both the current rate of growth in the local economy (s_t) as well as the current interest rate.

Since housing prices within the model are determined by the equilibrium mortgage contract, the model also provides predictions regarding how this contract varies with expected economic growth.

Proposition 5 *In equilibrium mortgage interest rates are invariant over time since* $m_t/\ell_t = k_m/k_\ell$*, while LTVs vary procyclically with expected rates of economic growth since* $\ell_t/P_t = s_t k_\ell/(s_t k_\ell + 1)$.

This proposition indicates that mortgage contracts will adjust to changing economic conditions in the local economy through the LTV demanded by banks rather than through the mortgage interest rate. Within the model, this occurs because the LTV acts as a mechanism through which the bank can induce the homeowner to care for the house. As shown in Proposition 4, high rates of expected economic growth lead to high expected housing returns. But, higher housing returns allow the bank to relax the LTV constraint.[7] For a given LTV, the higher the expected returns in the housing market, the greater the homeowner's incentive to care for the house. The mortgage interest rate stays constant within the model since the bank's required return on its loans always equals r and this number does not vary with local economic conditions.

Housing supply and construction. New housing can only be built on lots where good homes have become dilapidated. The next proposition examines whether this will ever happen in equilibrium.

Proposition 6 *In equilibrium, homeowners set* c *such that* $g < 1$*. Thus, homes become dilapidated with some positive probability.*

The model thus produces the realistic result that over time the general housing stock will deteriorate and require replacement.

Typically, models predict that expected returns in a competitive market will always equal the interest rate, in which case developers never have any reason to accumulate inventory. Instead they can buy lots at the competitive price just prior to construction. However, this model predicts that expected returns will be time-varying and thus developers do have an incentive to accumulate and draw down inventory.

[7] As noted in the introduction, the model's prediction that LTV ratios forecast expected housing returns is consistent with the empirical findings in Lamont and Stein (1999).

Proposition 7 *Developers accumulate dilapidated homes when expected income growth,* s_t*, exceeds some critical bound* s^**, and they construct new housing when it is below* s^**.*

Proposition 7 predicts that developer inventories can be used to forecast future changes in housing prices, with inventories leading large price increases.

While Proposition 7 indicates that developer inventories will vary with expected housing returns, this does not imply that they expect to earn above-market returns on their inventory. The expected return to holding a lot with a dilapidated home cannot exceed the interest rate. If it did, those with the available funds would simply bid up its price until the rents were eliminated since maintenance is not an issue for such homes. Thus, as s_t increases, the price ratio of lots with and without a home in good condition decreases.

Proposition 8 *The price ratio for homes in good condition to those in dilapidated condition declines with* s_t*. Mathematically,*

$$\frac{d\left[\frac{P_t}{V_t}\right]}{ds_t} = -\frac{k_v}{s_t^2 k_\ell^2} < 0. \tag{12}$$

Intuitively, Proposition 8 follows from the fact that developers are bidding up the price of lots with dilapidated homes in order to speculate on their future value, while at the same time, credit-constrained homebuyers are unable to drive up the price of homes in good condition by a similar amount. Observationally then, developers can follow a simple rule: When the relative value between lots with and without salable homes is narrow, hold on to the land; when it widens, build. Within the data, this should show up as a positive correlation between housing construction and the spread between dilapidated homes and those in good condition. This phenomenon also creates an empirical link between current housing construction and past changes in housing prices.

Proposition 9 *The expected volume of current housing construction increases with the previous period's return to housing. Formally, let* n_t *equal the number of homes built in period t; then* $dE[n_t|r_{ht-1}]/d(r_{ht-1}) > 0$*.*

Despite the assumption of proportional construction costs, changes in the price level for housing still remain linked to the amount of new construction. This occurs because builders are rational profit-maximizing agents in the model. When expected housing returns are unusually high, they increase their inventories and when low, they reduce them. Since unusually high housing returns tend to be followed by more normal expected returns, this produces an empirical link in which high returns to housing lead to a higher than average number of units under construction.

While Propositions 7 and 9 have not been tested explicitly, Coulson (1999) provides some indirect evidence in support of their predictions. He writes, "This is puzzling. A rise in price is evidently causing builders to finish accumulated inventory . . . rather than commence new projects" (Coulson 1999, p. 100). These duel results are consistent with Propositions 7 and 9's predictions. According to Proposition 7, inventories should forecast price increases. Once the price increase takes place, building then commences (Proposition 9). After that, further abnormal returns should no longer exist, on average, and development should then return to more normal levels.

Extensions

Alternative Utility Functions

Under the paper's current assumptions, each family inelastically demands one house. Then, given it obtains one, the family acts as a risk-neutral wealth-maximizing agent with respect to care and default. To the degree that alternative utility functions change these properties, the model's quantitative properties will change. However, since any reasonable utility function will require families to prefer better housing to inferior housing and more money to less, they will produce essentially the same qualitative results.

For example, suppose families possess an additive utility function of the form $U_1(\theta, W_{t+1}) = k_{u1}\theta + (k_{u2} + k_{u3}\theta)W_{t+1} - k_{u4}c_t$, where k_{u1}, though k_{u4} are constants. Setting k_{u2} to zero will produce results that are identical to those generated with the utility function in Equation (2). This generalization simply allows the house to pay a utility dividend that is independent of wealth. Since families already inelastically demand one unit of single-family housing, this additional incentive will not alter their decision rules.

With k_{u2} greater than zero in U_1, families no longer inelastically demand one unit of single-family housing. Nevertheless, the qualitative results remain essentially unchanged. This is easily seen by noting that once a family moves into a house, its care decision is still governed by Equation (7), and thus the bank's loan decisions must still satisfy Equations (8) and (9). What changes are the reservation values for the would-be homeowner. For s_t large nothing changes. Families correctly believe that on average the next generation will bid up housing prices. Thus, while the current generation is willing to bid away any rents (and then some), credit constraints will stop them from completely doing so. As with the utility function in (2), this leads to expected housing returns above r. For small values of s_t, housing prices will be lower and exhibit higher expected future returns under U_1 than (2), but again they will be qualitatively similar. When s_t is small, families correctly believe that future incoming families will not have the income to support high price levels. Under U_1 families will then trade off the current value of housing consumption with future wealth. This causes them to bid down the price of housing in the current period relative to families with the utility function in (2). Still, in both cases, the "housing dividend" allows expected housing prices to appreciate at less than the interest rate over the next period. Of course, for k_{u1} large enough or k_{u2} small enough, both U_1 and (2) will produce identical results even for small s_t as the "housing dividend" comes to dominate the family's decision.

Lender and Third Party Care

The model's qualitative results are robust to extensions that allow for lender or third party care, so long as these outside parties are less efficient than occupants at home maintenance. This seems like the natural assumption, since care by the lender is likely to be less timely and involve other inefficiencies. Furthermore, contractual difficulties may arise leading to costly litigation regarding what care the lender is responsible for and what care is the resident's responsibility.[8] One can model the relative inefficiency of lender maintenance by assuming that lender care involves a fixed cost and that, at the margin, the benefits of a dollar of lender care are less than the benefits from a dollar of residential care. In this case, if the fixed costs are high enough, then in equilibrium the banks will not chose to care for the house, and the model's results will remain unchanged. For lower fixed costs, the bank will expend some resources on care, but it will also restrict the amount it lends to induce care by the residents, too. The important point is that even in this case, credit constraints for the purpose of encouraging residential care will arise.

Alternative "Mortgage" and "Rental" Contracts

The first section of the paper restricts lenders and borrowers to a simple mortgage contract with a repayment amount that does not depend upon the home's eventual value or condition. This naturally raises the question as to whether or not more general financial arrangements may change the results. Qualitatively, the answer is no, and, in fact, the optimal contract is very close to a standard mortgage agreement.

[8] Given the extensive litigation over this issue in condominiums and co-operatives it is apparently very difficult to write precise contracts regarding the definition and limits of home maintenance.

Instead of restricting families and banks to a standard loan agreement, allow them to write whatever contract they wish. In this setting, the bank offers to lend ℓ_t in exchange for a "mortgage" repayment of $m(P_{t+1}, V_{t+1}, \theta_h)$, where m is now an arbitrary function, and θ_h is an indicator variable describing the state of the house (0 for dilapidated and 1 for good) when the contract comes due. The basic equilibrium restrictions remain unchanged. First, given the loan agreement, the family's level of care will maximize its expected wealth. Second, the bank must earn an expected return of r. Third, the function m must maximize the loan amount ℓ_t.

Proposition 10 *The optimal mortgage contract sets* $m(P_{t+1}, V_{t+1}, 0) = V_{t+1}$ *if the house becomes dilapidated. If the house remains in good condition, then setting* $m(P_{t+1}, V_{t+1}, 1) = w_t s_t k_m$ *is weakly optimal.* (*Note: here the equilibrium value of* k_m *will lie below the value derived in the state independent setting of Section* 1.)

Unlike the state independent contract, the optimal contract forces the family into default whenever the house becomes dilapidated irrespective of its actual value. Thus, the difference between the optimal state-dependent contract and a standard mortgage contract is, as a practical matter, trivial. In practice, one expects a dilapidated house to sell for less than the current amount owed on the mortgage in just about every state of nature that one is likely to see. Thus, a simple mortgage contract should have approximately the same value as the optimal state-dependent contract. Furthermore, a state-independent mortgage contract is much less likely to result in costly litigation. While it may be easy to describe a home's condition within a model, it is not clear that it can be done within a courtroom. If the home's condition is not legally verifiable, then a standard mortgage contract is a globally optimal contract. Since the introduction of a state-dependent loan agreement does not alter the functional form of the solution, all of the paper's qualitative results remain unchanged.

Alternative Relationships Between Construction Costs and the State of Nature

The model assumes that construction costs vary one to one with the state of nature. In contrast, models deriving like Capozza and Helsley (1990), Capozza and Sick (1991), and Bar-Ilan and Strange (1996b) assume that construction costs are invariant to the state of nature. The truth probably lies between these two extremes. While construction costs no doubt increase with local economic activity, it is probably less than proportional. However, even with this change most of the paper's primary qualitative conclusions will remain unchanged. What will change is the link between current housing prices and construction levels. If housing construction costs do not increase proportionally with the state of nature, then as the state of nature increases, housing becomes relatively cheap to build. As a result, developers will construct housing in wealthy areas during periods of time when they would not in less well-to-do communities.

Heterogenous Wealth Levels

Allowing for heterogenous wealth levels does not, in principle, pose any difficulties. As in almost any model with market clearing, the marginal purchaser sets the price, which in this case will be the richest family that cannot buy a house. Unfortunately, incorporating heterogenous wealth into a multiperiod setting makes it impossible to obtain a closed-form solution. With heterogenous wealth levels, the current supply of housing (a random variable) now impacts the equilibrium since it impacts the wealth of the marginal family. This adds another state variable and one whose distribution depends on the recent history of the economy. Thus, the wealth of the marginal family will now depend upon both the current state of the economy and its recent history as s_t impacts development. Nevertheless, since the marginal family is always credit constrained (or they would bid more in order to move into a house), they still set the equilibrium price, and therefore the paper's general qualitative conclusions must hold.

Heterogenous Housing Quality

Heterogenous housing quality in the absence of heterogenous wealth does not alter the model in a meaningful way. Suppose that the variable θ in the family's utility function (Equation (2)) is now continuous and represents the quality of the house owned by the family. Since the marginal family will still bid up to the limit of their credit constraint, the housing price process will not change. In order for heterogenous housing quality to have a significant impact on the model, it must be accompanied by heterogenous wealth. While a closed-form solution is now unavailable, one can still characterize the equilibrium bids within a particular period.

Clearly, with heterogenous housing quality and wealth, the equilibrium bids will sort wealthier families into better homes, since they can outbid their poorer neighbors. Sort the homes by increasing quality $1, \ldots, k$ and the people by decreasing wealth $1, \ldots, n$ with $n > k$. (In fact, the model requires that n exceed the number of lots zoned for single-family housing.) Family $k + 1$ represents the wealthiest family that cannot afford to purchase a house, and thus family k need only outbid them to obtain residence k. Since family $k + 1$ just misses out on a house, they must bid to the point where their credit constraint binds, and thus the price of the kth home equals the wealth of the kth family plus the maximal amount they can borrow. Now consider the price of home $k - 1$. In order to move into this house, family $k - 1$ must outbid family k. This means that the price of the $k - 1$ home will depend both upon the quality difference between it and house k and the wealth of families k and $k - 1$. Suppose the quality difference is large. Then the price of house k will equal the most family $k + 1$ can bid, while the price of house $k - 1$ will equal the most family k can bid. In this case, both prices look like those found within any one period in a model of homogenous housing quality and family wealth. On the other hand, suppose the quality difference is relatively small. Then family k will only bid to the point where it is indifferent between house k and $k - 1$. Thus, family $k - 1$ need only bid enough to make family k just barely worse off in house $k - 1$. How do you know that family $k - 1$ will do this? Since family $k - 1$ has a larger wealth endowment, it can borrow less than family k and still win the house. Because family $k - 1$ can buy the house while borrowing less, that family will take better care of the house (a straightforward conclusion that derives from Equation (4)), and thus it will provide them with a higher expected utility. This implies that family $k - 1$ will always outbid family k for home $k - 1$. However, in terms of the housing price process, the important thing to note is that the price of the kth home plays a pivotal role in determining the price of house $k - 1$. Since the price of house k depends upon the credit limit faced by family $k + 1$, the price of the higher quality homes is also influenced by it, and therefore one expects the general qualitative conclusions of this paper to hold under these conditions also.

Relationship to the Current Literature

Generally, papers on housing construction treat the problem as a subset of the real options literature and seek to determine the optimal stopping time. Papers within this literature include Titman (1985), Majd and Pindyck (1987), Williams (1991), Capozza and Sick (1991, 1994), Capozza and Li (1994), Grenadier (1995), and Bar-Ilan and Strange (1996a, 1998). These models are in a sense purely financial in that they can be applied to any asset that can be described as an option to invest within an environment where the developer is a price taker. Noncompetitive counterparts to this literature can be found in models by Williams (1993a), Grenadier (1996), and Williams (1997). Other papers by Capozza and Helsley (1990), Capozza and Sick (1991), and Bar-Illan and Strange (1996b) modify the real options problem to allow for the possibility that a home's distance from a city's center will influence its desirability. What distinguishes the current paper is the introduction of a moral hazard problem between homeowners and their lenders, which then induces a price process with time-varying returns that may include periods with above-market expected returns. In contrast, housing prices in the above models can never experience expected above- or below-market returns since prices always adjust to eliminate such rents.

This paper builds upon Spiegel and Strange (1992), and to some extent Williams (1993b). Both of these articles presume that housing maintenance plays an important role. However, they differ from the current paper in that they do not focus on price and construction dynamics. Instead, Williams (1993b) seeks to explain why single-family housing tends to be owner occupied while larger complexes are run by a landlord who rents out the units. In his model, people either rent units or own them outright. Large complexes are then rented out because landlords have a maintenance technology that is relatively inefficient when applied to small complexes and relatively efficient when applied to large complexes. Note that, in contrast to the current paper, the moral hazard problem in Williams' (1993b) does not impact owner-occupied housing. Spiegel and Strange (1992) examine a single-period model in which the moral hazard issue between mortgage lenders and their borrowers leads to credit rationing and potentially above-market expected returns. In that model, the home's terminal value derives from an exogenously specified price process, which then drives the initial transaction price. The current paper extends the analysis in a number of directions by allowing for construction, a dynamic infinite horizon setting, and the endogenous derivation of all prices.

Another set of related articles examines the relationship between housing price volatility and credit constraints. In papers by Stein (1995) and Ortalo-Magné and Rady (1998, 1999) potential homeowners are faced with an exogenously specified down payment requirement when bidding on a house. Thus, when a home increases in value, the owner has additional equity that can be used to bid for additional housing, which cycles into yet further increases in housing prices. Conversely, a decline in prices tightens the down payment constraint and thus stifles demand. These duel impacts lead to a very volatile housing price series in which trading volume is positively correlated with price increases. The Ortalo-Magné and Rady (1998, 1999) papers also examine how life cycle issues may further impact the price process. In contrast, the current paper uses an endogenously derived credit contract and then focuses more on the development of the housing stock and the correlation between the equilibrium mortgage contract and the local economic environment.

Conclusion

This paper presents a model of the housing development process within a mature city. The model allows for the endogenous construction decisions by homebuilders, the mortgage contracts offered by banks, and the bidding and care decisions by families wishing to move into a neighborhood. When these elements are combined, housing prices become linked with the rate of economic growth in the local economy. This linkage implies that developers will look to the current state of the economy before they decide to build new housing.

The model provides a number of empirical predictions, many of which have already been shown to hold in the data. In equilibrium, the model predicts that LTV ratios will forecast future housing returns, with high ratios preceding high housing returns—a prediction that was recently confirmed in Lamont and Stein's (1999) empirical study. Another prediction that appears to hold in the data (for example, Case and Shiller 1989) is that housing returns will experience periods in which they predictably yield above- or below-market returns. However, the model also makes a number of as yet untested predictions. First, the model predicts that while housing in good condition may experience predictable time-varying excess returns, dilapidated homes will not. This leads to a second prediction, which is that during economic growth cycles, the price ratio between homes in good condition and those in dilapidated condition will narrow. Related to these untested results there exists some indirect evidence from Coulson (1999) that developer holdings can be used to forecast future expected housing returns and that above-normal housing returns should forecast above-normal construction levels. Similarly, Rosenthal (1999) provides some evidence for the model's prediction that when land prices go up builders will slow down construction until they once again fall to more normal levels. Finally, the model also helps to explain why relatively few individuals and institutions rent out single-family homes. As the model shows, potential buyers can make offers that are sufficiently attractive to potential lessors that it pays potential landlords to sell the house rather than rent it out.

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Appendix

Restrictions on g

The first restriction on g is that it must be sensitive enough to the level of care that agents believe their actions will materially impact its value. For example, suppose that a home's probability of remaining in good condition does not in any way depend upon the level of care expended by the homeowner. Then obviously homeowners will not expend utility resources on care and lenders will not restrict their lending practices in order to encourage homeowners to engage in care either. A second restriction on g is that it must allow care to be sufficiently cost effective that there exist some conditions under which a family will not set it equal to zero. Otherwise one is once again faced with a situation where the bank's lending practices cannot influence the behavior of its borrowers. Third, g must not make care "too cheap." If care is too cost effective, then homeowners will simply set it to the point where the probability that the house becomes dilapidated equals zero and the moral hazard problem will once again fade away.

Based upon the above three criteria, the moral hazard issue will matter so long as one assumes that g has the following properties:

1. $g(1) - g(0) > 0$,
2. $g'(0) > 1$, and
3. for c such that $g = 1$, $g' = 0$.

The first condition states that if the homeowner does not care for the house it becomes dilapidated with some probability, and this probability is strictly higher than for a well-maintained home. The second condition states that at the point where care equals zero, a small amount of care will return more in expected housing value than it costs. The third condition states that at the margin, if the probability that the house will remain in good condition equals one, the marginal benefit from additional care equals zero. Obviously this must be true for the right derivative, but the condition also imposes it for the left derivative. Thus, an alternative (but slightly more restrictive) assumption would be that g is twice continuously differentiable for all levels of care.

Proposition 1 *Developers and banks never rent or leave vacant any home in saleable condition.*

Proof To prove the proposition, first note that if a family moves into a rental house they expend zero effort on care since their return for doing so equals zero. Thus, vacant and rental homes receive the same level of care: zero. This implies that the expected value of the house in the following period is the same whether the owner rents it out or leaves it vacant. Thus, renting must dominate (since the owner earns a fee in the meantime), and no homes are allowed to stand vacant between sales.

To show that banks always sell rather than rent saleable homes in their possession, it is only necessary to show that there exists some mortgage contract that allows a family to pay more for the house then the bank can get in expected value from its rental. As noted above, a rented house receives no care, and thus, with probability one, will become dilapidated next period. Thus, the value to either a bank or developer of renting a currently saleable home for one period is the present value of owning a dilapidated home one period hence plus the rental fee. Clearly the rental fee must be less than or equal to the current wealth level (w_t) possessed by a family, which implies the rental value is strictly less than what a family can offer to buy the house. To see why, consider a suboptimal mortgage with m set to infinity. From Equation (4), a family with this mortgage will set c to zero and thus, with probability one, the bank will take possession of a dilapidated house one period hence. Thus, this mortgage contract has the same value as a currently saleable home that is rented for w_t. However, a family can make at least a weakly higher bid since, due to the moral hazard problem, a mortgage contract with m equal to infinity will not necessarily maximize the available loan size. Q.E.D.

Proposition 2 *The family's optimal level of care weakly decreases in the required mortgage payment. Formally,* $dc/dm \leq 0$.

Proof Differentiate Equation (4) with respect to m and then rearrange to show that $dc/dm \leq 0$ after recalling that since c maximizes (4), the second derivative with respect to c must be negative. Q.E.D.

Proposition 3 *The value of* k_ℓ *is strictly greater than* 0 *and less than* $1/r$. *It equals* $1/r$ *only when there does not exist a moral hazard problem.*

Proof The proof begins by showing that k_ℓ is strictly positive. Rearrange Equation (9) to solve for k_ℓ producing

$$\begin{aligned} k_\ell &\left[1 - \frac{1}{1+r} g \int_{\neg\psi} s_{t+1}\delta_{t+1} f q d(\delta_{t+1}) d(s_{t+1})\right] \\ &= \frac{1}{1+r}\left\{g\left[\int_{\neg\psi} k_v s_{t+1}\delta_{t+1} f q d(\delta_{t+1}) d(s_{t+1}) + k_m \int_{\psi} f q d(\delta_{t+1}) d(s_{t+1})\right]\right. \\ &\quad \left. + (1-g)\left[\int_{\neg\varphi} k_v s_{t+1}\delta_{t+1} f q d(\delta_{t+1}) d(s_{t+1}) + k_m \int_{\varphi} f q d(\delta_{t+1}) d(s_{t+1})\right]\right\} \end{aligned} \tag{13}$$

Since $r > 0$, $g \leq 1$, and the expected values of s_{t+1} and δ_{t+1} both equal 1, the term in square brackets on the left-hand side of the equation must be strictly positive. Since the terms on the right-hand side sum to a strictly positive number, one has that $k_\ell > 0$.

To find the upper bound on k_ℓ consider the conditions that would maximize the amount a bank would lend. The largest possible period $t+1$ payoff to the bank would occur if, at m equal to infinity, the home always remained saleable. In this case the bank repossesses the home in period $t+1$ for sure and thereby acquires its full value. Clearly, the actual loan must be based on the bank's expectation that it will get less than this. Setting m to infinity and g to one in Equation (9) yields $k_\ell < (k_\ell + 1)/(1+r)$, solving for k_ℓ yields $k_\ell \leq 1/r$. To prove that the inequality is strict in the presence of moral hazard, note that setting k_m such that $g = 1$ cannot be optimal. If $g = 1$, then raising k_m will allow the bank to lend strictly more and thus increase the maximum bid by a homeowner. Q.E.D.

Proposition 6 *In equilibrium homeowners set c such that* $g < 1$. *Thus, lots become vacant with some positive probability.*

Proof The proof is by contradiction. Suppose that $g = 1$. By assumption, when $g = 1$, its derivative $g' = 0$. Thus, if $g = 1$, the left-hand side of the equilibrium Equation (8) reduces to the probability that $P_{t+1} > m$. For the first-order conditions to hold, this probability must equal zero. However, if P_{t+1} never exceeds m, then the solution to the homeowner's first-order condition must set care to zero. But if care equals zero, then $g < 1$, a contradiction. Q.E.D.

Proposition 7 *Developers accumulate dilapidated homes when expected income growth,* s_t, *exceeds some critical bound* s^*, *and they construct new housing when it is below* s^*.

Proof Developers accumulate inventory by delaying construction. Thus, the goal is to show that development takes place when $s_t < s^*$. Given the current value of the state variables s_t and w_t, it pays to rebuild a dilapidated house in the current period if

$$
\begin{aligned}
&[k_\ell s_t + 1]w_t - hs_t w_t \\
&\quad \geq \frac{1}{1+r}\left\{\int_\xi [k_\ell s_{t+1} + 1]\delta_{t+1} - hs_{t+1}\delta_{t+1}]s_t w_\ell f q \, d\delta_{t+1}\, ds_{t+1}\right. \\
&\qquad \left. + \int_{\neg\xi} k_v s_{t+1}\delta_{t+1} s_t w_\ell f q \, d\delta_{t+1} ds_{t+1}\right\} \qquad (14)
\end{aligned}
$$

where the left-hand side of the equation equals the value obtained from immediate construction and the right-hand side the value from delaying construction one period and then building optimally. To prove the proposition, one must first sign the following expression

$$
\begin{aligned}
\Xi = (h - k_\ell)(1+r) &+ \int_\xi [(k_\ell s_{t+1} + 1)\delta_{t+1} - hs_{t+1}\delta_{t+1}] f q \, d\delta_{t+1}\, ds_{t+1} \\
&+ \int_{\neg\xi} k_v s_{t+1}\delta_{t+1} f q \, d\delta_{t+1} ds_{t+1}. \qquad (15)
\end{aligned}
$$

The last two terms in (15) are proportional to the value from delaying construction one period and then building optimally. Now consider the suboptimal strategy of waiting one period and then building in the next period in all states of nature. Since this strategy, by definition, must have at least a weakly lower payoff, one has that

$$\Xi \geq (h - k_\ell) + \int [(k_\ell s_{t+1} + 1)\delta_{t+1} - h\delta_{t+1}s_{t+1}] f q \, d\delta_{t+1} ds_{t+1}$$
$$= (h - k_\ell)(1 + r) + k_\ell + 1 - h = 1 + r(h - k_\ell). \quad (16)$$

From Proposition 3, the value of k_ℓ lies below $1/r$ and therefore $\Xi \geq rh \geq 0$. Having signed Ξ, one can now rearrange (14) to show that the developer will rebuild a dilapidated home if $s_t \leq (1 + r)/\Xi$. Q.E.D.

Proposition 8 *The price ratio for homes in good condition to those in dilapidated condition declines with s_t. Mathematically,*

$$\frac{d\left[\frac{P_t}{V_t}\right]}{ds_t} = -\frac{k_v}{s_t^2 k_\ell^2} < 0. \quad (17)$$

Proof The price of a home in good condition equals $s_t k_\ell + 1$, while one in dilapidated condition sells for $s_t k_v$. Take the ratio and differentiate to prove the proposition. Q.E.D.

Proposition 9 *The expected volume of current housing construction increases with the previous period's return to housing. Formally, let n_t equal the number of homes built in period t then $dE[n_t | r_{ht-1}]/d(r_{ht-1}) > 0$.*

Proof Since a rigorous version of the proof is algebraically tedious and not particularly informative, only the highlights are given here. The goal is to show that for any number of dilapidated homes in period $t - 1$, say o_{t-1}, the expected value of n_t is increasing in r_{ht-1}. If this is true for all $o_{t-1,}$ then it is true unconditionally as well. Thus, all probability statements given from here on out should be read as conditional on o_{t-1}.

Developers will rehabilitate the homes they have in inventory in period t if $s_t < s^*$. Thus, in period $t - 1$, expected housing construction in period t equals $E_{t-1}[n_t] = \text{pr}(s_t < s^*)E_{t-1}[o_t]$. To prove the proposition, one now needs to calculate $E_{t-1}[o_t]$. If $s_{t-1} > s^*$, then any inventory held by the developers in period $t - 1$ will be rehabilitated (see Proposition 7) and o_t will simply equal the number of occupied homes that become dilapidated. Thus, $E_{t-1}[o_t|s_{t-1} < s^*] = (1 - g)N$. Here g is written without any arguments since, in equilibrium, c_t is set by homeowners to keep g constant over time. If $s_{t-1} > s^*$, then any inventory held by the developers in period $t - 1$ will not be rehabilitated (see Proposition 7) and o_t equals the number of occupied homes that become dilapidated plus the inventory of dilapidated homes held by the developers. Thus, $E_{t-1}[o_t|s_{t-1} > s^*] = o_{t-1} + (1-g)(N - o_{t-1}) = o_{t-1} + (1-g)N$. Note that these two results imply that one expects more development in period t when $s_{t-1} > s^*$ than when $s_{t-1} < s^*$. In other words, the amount of housing development in period t is correlated with s_{t-1}. This basically drives the result in the proposition since housing returns in period t are also positively correlated with s_t.

Next, use the relationship between housing prices and the state of nature to calculate s_{t-1} in terms of r_{ht-1} and the state variables to produce

$$s_{t-1} = r_{ht-1}[\delta_t(s_t k_\ell + 1) - r_{ht-1}k_\ell]^{-1}. \tag{18}$$

By taking the expectation of both sides and employing some algebra, one can now show that $dE_{t-1}[s_{t-1}]/dr_{ht-1} > 0$. This should not come as much of a surprise since Equation (11) shows that expected housing returns are positively related to the current value of s. At this point, we have shown that expected housing construction in period t increases in s_{t-1} and that the expected value of s_{t-1} increases in r_{ht-1}. Some additional algebra then shows that expected level of housing construction therefore also increases in r_{ht-1}. Q.E.D.

Proposition 10 *The optimal mortgage contract sets* $m(P_{t+1}, V_{t+1}, 0) = V_{t+1}$ *if the house becomes dilapidated. If the house remains in good condition then setting* $m(P_{t+1}, V_{t+1}, 1) = w_t s_t k_m$ *is weakly optimal. (Note: here the equilibrium value of* k_m *will lie below the value derived in the state independent setting of Section 1.)*

Proof From the limited liability assumption, m can never exceed the value of the house. Thus, the family's problem is exactly the same as in (3) except the integrals are now over all possible states and m may depend upon the realized state of nature. Thus, the first-order condition describing the solution to the family's problem is also identical to the one on Equation (4) except that again the integral should be taken over all states of nature. From (4) it is clear that the family's effort can be maximized by setting m equal to V whenever the home becomes dilapidated. This condition must therefore constitute part of the optimal contract. To see why, note that the equilibrium contract must maximize the amount the bank will contribute to the purchase of the house. By increasing m in the event that the home becomes dilapidated, the bank earns more on average for a given level of care and the family will in fact increase its care level, further boosting the amount the bank will willingly contribute. Thus, $m(P_{t+1}, V_{t+1}, 0) = V_{t+1}$.

With the above result in hand, the equilibrium conditions describing the bank's contribution to the purchase of the house simplify to

$$\ell(1+r) = g(c/s_t w_t) \int m(P_{t+1}, V_{t+1}, 1) f(\delta_{t+1}) q(s_{t+1})\, d\delta_{t+1}\, ds_{t+1}$$
$$+ [1 - g(c/s_t w_t)] \int V(s_{t+1}, w_{t+1}) f(\delta_{t+1}) q(s_{t+1})\, d\delta_{t+1}\, ds_{t+1}, \qquad (19)$$

and

$$\max_m g(c/s_t w_t) \int m(\cdot) f(\delta_{t+1}) q(s_{t+1})\, d\delta_{t+1}\, ds_{t+1}$$
$$+ [1 - g(c/s_t w_t)] \int V(s_{t+1}, w_{t+1}) f(\delta_{t+1}) q(s_{t+1})\, d\delta_{t+1}\, ds_{t+1}. \qquad (20)$$

With $m(P_{t+1}, V_{t+1}, 0) = V_{t+1}$, the amount of care taken by the family and the value of the loan agreement to the bank depend only upon the expected value of m in the event the home remains in good condition. This implies any two functions of m that have the same expected value when $\theta_h = 1$ produce the same equilibrium loan. Now consider any loan contract in which m is independent of P_{t+1} and V_{t+1} when $\theta_h = 1$ but has the same expected value when $\theta_h = 1$. Since this loan has the same value to the bank, it must be no worse than the alternative state-dependent loan. Therefore, there exists a contract in which m is independent of P_{t+1} and V_{t+1} when $\theta_h = 1$ that is weakly optimal. The proof that $m(P_{t+1}, V_{t+1}, 1) = w_t s_t k_m$ is that contract follows along the same lines as the proof in the Equilibrium section, and is thus omitted here. Q.E.D.