# 15 An Introduction to Formal Semantics

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## 1 Introduction

When people talk, they generally talk about things, events, and situations in the world. They are able to do this because they represent connections between the expressions of their language and extra-linguistic phenomena in a fully systematic way. The meaning of a sentence in a language is, to a large extent, dependent upon the ways in which the words and phrases from which it is constructed can be related to situations in the world. Speakers of a language are able to communicate effectively with each other because they have internalized the same rules for pairing the lexical items of the language with non-linguistic elements, and they use the same procedures for computing the meaning of a syntactically complex phrase from the meanings of its parts. Therefore, speakers will, in general, converge on the same sets of possible language–world connections which they assign to the sentences in their discourse. Formal semanticists seek to understand this aspect of linguistic meaning by constructing precise mathematical models of the principles that speakers use to define those relations between expressions in a natural language and the world which support meaningful discourse.[1]

Consider an example. Assume that two students in a class are discussing the class's progress on a term paper. One student asserts (1a) and the second responds with (1b).

(1) a. John has finished his paper.
    b. No one in the class has finished his / her paper.

For the second speaker to understand (1a), he / she must be able to pick out the person corresponding to *John*. He / She must also know what property *finished his paper* expresses and recognize that the first speaker is claiming that the person corresponding to *John* has this property. If (1b) is true, then it implies that (1a) is false by virtue of the fact that (1b) states that no person in the class has the property of having finished his / her paper. Therefore, assuming that the second speaker understands both (1a) and (1b), then he / she recognizes that asserting (1b) involves making a statement which is incompatible with the one made by the first speaker.

To competent speakers of English all of this is thoroughly obvious. This is because we have already internalized the semantics of English, which we rely on in understanding the partial (and informal) description of the semantic competence required to interpret the simple dialogue in (1). But consider what is involved in developing a complete theory of this semantic competence which renders formal and explicit our tacit knowledge of linguistic meaning rather than presupposing it. Such a theory will specify what sort of properties verb phrases (VPs) like *finished his paper* refer to, and it will model properties in general as formal objects of a kind which can apply to the entities of the sort identified by *John*. It will also capture the important semantic distinctions and similarities between proper names like *John* and quantified noun phrases (NPs) such as *no one*. Specifically, while *John* selects an individual, *no one* does not. On the other hand, both kinds of expression can combine with the predicate *finished his paper* to yield a meaningful statement. It is also necessary to explain the difference in the anaphoric relation which holds between the pronoun *his*(/ *her*) and the subject NP in (1a) and (1b).

A complete semantic theory will apply not only to the sentences in (1), but to all syntactically well-formed sentences of the language. Specifically, it must explain our capacity to assign interpretations to an unbounded number of grammatical sentences. Given that we can only represent a finite number of primitive semantic elements, this capacity requires the recursive application of rules to the meanings of expressions in order to derive interpretations for larger phrases.[2] There is, then, a direct formal analogy between the syntactic component of the grammar, which employs recursive procedures to generate a (potentially) infinite set of sentences from smaller lexical and phrasal units, and the semantics, which combines the meanings of these units into the interpretations of the sentences in which they are contained.

In the following sections I will look at some of the central questions which arise in constructing a formal semantic theory for natural language, and I will briefly indicate several of the major lines of research which formal semanticists have pursued in their attempts to answer these questions.

## 2 Meanings and Denotations

Semanticists have traditionally focussed on theories of meaning which apply to sentences that make statements, and are taken to be either true or false. The assumption underlying this approach is that this type of sentence provides a paradigm of the sort of relationship between linguistic expressions and the world which is at the core of linguistic meaning. An additional assumption is that if it is possible to construct a successful account of the meaning of declarative sentences used to make statements, then this account can be generalized to non-declarative sentences, like interrogatives that are employed for asking questions, and imperatives which communicate commands.[3]

It is possible to locate the beginnings of modern formal semantics in the work of the German logician, Frege, who created the foundations of first-order logic.[4] We have identified one of the key tasks of a semantic theory as the specification of a systematic correspondence between categories of expressions in a language and types of entities in the world. The main syntactic categories which Frege identifies in natural language correspond to the types of first-order logic. These types are (i) individual terms (names of individuals, and variables that occur in the same positions in sentences that names do), (ii) predicates (terms for properties and relations), (iii) connectives (*and*, *or*, *if . . . then*, and *not*) for building composite sentences and negations out of component sentences, and (iv) quantifiers that are linked to variables (bind the variables). Proper names, like *John*, and definite descriptions like *the Prime Minister* are treated as individual terms that occupy the positions of arguments in predicate terms. VPs like *sings* and *introduced the bill* are one-place predicates in that they apply to single arguments to yield statements.

Frege claims that for each logical type an expression of that type can take a certain sort of entity as its *denotation* (the thing that it stands for). Individual terms denote individuals in the world (more precisely, in the domain of discourse shared by the speaker and his / her hearers). If one knows how a declarative sentence like (2a, b) stands in relation to the world, then one knows whether it is true or false.

(2) a. John sings.
    b. The Prime Minister introduced the bill.

In this sense, the primary semantic value of a declarative sentence is its truth or falsity, and Frege takes declarative sentences to denote truth-values. One-place predicates denote functions from individuals to the truth-values true or false. Every function is a mapping from a set of arguments (its domain) to a set of values (its range). Therefore, the function $f$ which a one-place predicate denotes can be represented as the set of objects in $f$'s domain for which $f$ yields the value true. The VP *sings*, for example, denotes the set of things in the domain of discourse of which *sings* is true. (2a) is true if and only if (iff) the individual which *John* denotes is an element of the set that *sings* denotes, and similarly for (2b).

This schema for category–type correspondence extends naturally to sentences formed with logical connectives like *and*, *or*, *if . . . then*, and negation, as in (3).

(3) a. John sings and Mary dances.
    b. John sings or Mary dances.
    c. If John sings then Mary dances.
    d. John doesn't sing.

Two-place connectives denote functions from pairs of truth-values to a truth-value. So *and* maps two true sentences into the value true, and every other combination of truth-values into false. (3a) is true iff both *John sings* and *Mary dances* are true. *Or* maps any two false sentences into the value false, and any other pair of values into true. (3b) is true iff at least one of the disjuncts connected by *or* is true. *If . . . then* is false if the antecedent (the sentence immediately following *if*) is true and the consequent is false, and true otherwise. It follows that (3c) is true iff either *John sings* is false or *Mary dances* is true. Finally, a negated sentence is true iff the sentence to which the negation applies is false. (3d) is true iff *John sings* is false.

What about quantified NPs like *nobody* in (1b), and the subjects of (4)?

(4) a. Someone sings.
    b. Everyone dances.

Unlike individual terms, they do not denote individuals in the domain, but they do seem to occupy the same grammatical category as these terms. How, then, do we interpret them? Frege revolutionized logic by treating quantifiers as second-order functions, or, equivalently, second-order property (set) terms (see note 4 for the distinction between first- and second-order terms). On this view, (1b) and (4a, b) are not statements in which a predicate is applied to an argument, but quantified sentences in which a term that corresponds to a property of a set applies to a predicate (a term that denotes a set). (4a) is true iff the set of things that sing has at least one element, and (4b) is true iff everything in the domain of people dances. (1b) is equivalent to (5).

(5) It is not the case that someone in the class has finished his / her paper.

This sentence is true iff the set of people in the class who have finished their respective papers is empty.

First-order logic has two basic quantifiers, *every* and *some*. Each of these quantifiers can be expressed as an operator that is prefixed to a sentence and associated with variables which appear in the argument positions of predicates in the sentence. The symbol commonly used for *some* is $\exists x$ (*for some x*), and for *every* it is $\forall x$ (*for every x*). The symbols frequently used for negation, conjunction, and implication are ~ (*it is not the case that*), & (*and*), and $\rightarrow$ (*if . . . then*), respectively. Let's substitute *some student* for *someone* in (5) in order to give explicit expression to the restriction of the quantifier *some* to the set of students in the domain. Then we can represent (5) in first-order logic as (6a), which is equivalent to (6b).

(6) a. $\sim\exists x(\text{student}(x) \ \& \ \text{finished x's paper}(x))$
    b. $\forall x(\text{student}(x) \rightarrow \sim \text{finished x's paper}(x))$

(6a) states that it is not the case that there is an object x in the domain which is both a student and finished its own (x's) paper. (6b) states that for every x in the domain, if x is a student, then it is not the case that x finished x's paper. Notice that each occurrence of the variable *x* is interpreted relative to the quantifier prefixed to the sentence where the variable appears. The quantifiers ∃x and ∀x *bind* the variable *x* in (6a) and (6b), respectively.

On Frege's view individual terms and variables, unlike quantifiers, are arguments of predicates. Therefore, (1a) is expressed in first-order logic by (7).

(7) finished john's paper (john)

Notice that the anaphoric relation between the pronoun *his* and *John* in (1a) is captured by substituting the denotation of *John* for the pronoun in (7). By contrast, the anaphoric dependence of *his* (*her*) upon its quantified NP antecedent *no one* in (1b) is represented by using a bound variable for the pronoun in (6a, b).

Definite descriptions pose an interesting problem for a theory which attempts to explain the meaning of an expression in terms of its denotation. The definite descriptions *the former Governor of Arkansas* and *the President of the United States* denote the same object, Bill Clinton. Therefore, if we substitute one for the other as the argument of the predicate *plays the saxophone*, the truth-value of the resulting statement should not be affected. In fact, (8a) and (8b) do have the same truth values.

(8) a. The former Governor of Arkansas plays the saxophone.
    b. The President of the United States plays the saxophone.

However, the two descriptions do not have the same meaning, and (8a) and (8b) assert different statements. *The former Governor of Arkansas* identifies the person who was the previous governor of Arkansas, but who no longer holds this position, and *the President of the United States* denotes the individual who is the current president at a particular point in time. The difference in meaning can be brought out clearly by evaluating (8a, b) relative to a particular point in time. During the 1992 American presidential election campaign, (8a) was true, as Clinton was the former Governor of Arkansas, but not yet the President. (8b), however, was false, because George Bush was the President.

The observation that the denotation of an expression does not exhaust its meaning led Frege to factor meaning into the two components of *denotation* and *sense*. He characterizes the sense of an expression as the principle for determining its denotation. Therefore, two terms with the same sense will always have identical denotations, but as (8) indicates, the converse does not hold. Frege does not give a precise description of the formal entities which correspond to senses. Carnap (1947) substitutes *extensions* for denotations and *intensions* for senses. Extensions correspond closely to Frege's denotations. We can take the extension of an expression E to be the entity which it denotes, where this entity is of the kind appropriate for E's logical type. The extension of a declarative sentence is its truth-value, of a name an individual object, and of a predicate a set of objects (or, in the case of a relation, a sequence of objects).

The intension of an expression E is essentially a rule for identifying E's extension in different situations. Carnap characterizes intensions as functions from possible worlds to denotations, where a possible world can be thought of as the result of specifying the properties and relations which hold for the objects of a domain in a way that defines a complete state of affairs for the entities of the domain. The actual world is one of many (in fact, an infinite number of) possible worlds. The intension of an expression takes a possible world as an argument and yields the extension of the expression in that world as its value. Therefore, the intensions of *the former Governor of Arkansas* and *the President of the United States* identify (i) the person who satisfies the property of being the previous governor of Arkansas and (ii) the person who is currently the President of the United States, respectively, in each world. These two denotations converge on the same individual in the actual world, but are distinct in other possible worlds (and times). Similarly, the intension of the VP *plays the saxophone* picks out the set of objects which play the saxophone for each world. The intension of a sentence assigns it a truth-value in each possible world. We obtained (8b) from (8a) by substituting one description for another with the same extension but a distinct intension. The substitution produced a sentence with the same extension (truth-value) in the actual world (at the present time), but a different intension (proposition).

We observed that one of the main tasks of semantic theory is to explain how speakers compute the meanings of complex phrases from the meanings of their parts. Frege adopts the principle of *compositionality* as a condition of adequacy on any account of meaning. Compositionality requires that the meaning of any well-formed phrase in a language be a function of the meanings of its syntactic components. This condition implies that, for any phrase P, given the meanings of the constituents of P, there is a function which maps these meanings into the meaning of P. This principle has enjoyed wide acceptance throughout the history of semantic theory. Clearly, if an account of meaning satisfies compositionality, it specifies the way in which the interpretations of complex structures are generated from their constituents. However, as we will see in section 5, it is possible to construct non-compositional semantic theories which also fulfill this task.

On the Frege–Carnap approach, the principle of compositionality yields two distinct sub-principles: (i) the extension of a phrase is a function of the extensions of its parts; (ii) the intension of a phrase is a function of the intensions of its parts; truth functional connectives produce complex sentences that satisfy (ii). So, for example, the truth-value of (3a) is a function of the truth-value of the two conjuncts of *and*.

(3) a. John sings and Mary dances.

However, verbs like *believe*, which map propositions into properties (sets) of individuals are problematic. Unlike truth functional connectives, *believe* is sensitive to the intension as well as the extension of the sentence which it takes as its grammatical complement. Substituting one complement sentence for another with the same truth-value but a different proposition can alter the extension, as well as the intension of the entire VP.

In addition to the Frege–Carnap view there is another approach, which dispenses with intensions and seeks to construct a theory of meaning solely in terms of the contributions which expressions make to the truth (i.e. extension) conditions of sentences. This approach is developed by Davidson, and it takes as its starting point Tarski's (1933) definition of truth for first-order languages.[5] Tarski constructs a recursive definition of the predicate *true-in-L* for a class of first-order languages similar to the first-order language characterized by Frege. The definition proceeds stepwise first to elementary sentences constructed from individual terms (constants or names, and variables) and predicates, next to compound sentences formed by applying truth functions to other sentences, and finally to quantified sentences. For each sentence S of type T in language L, it specifies the truth conditions for S in terms of the relations which must hold among the denotations of the constituents of S. As a result, Tarski's truth definition generates appropriate truth conditions for the full set of well-formed sentences of L.

Davidson regards Tarski's truth definition as the paradigm of a semantic theory.[6] If to know the meaning of a declarative sentence is to know its truth conditions, then Tarski's definition gives an explanation of sentence meaning in terms of a precise and fully systematic account of the connections between sentences and the world. It does this in a way which exhibits how the interpretations of sentences are built up from the interpretations of their constituents.[7] Davidson's general strategy is to associate the sentences of a natural language with first-order logical forms to which a Tarskian truth definition can apply.

Frege and Carnap on one hand, and Davidson on the other, share the assumption that the sentences of natural language are analyzed in terms of the types of first-order languages, specifically, individual terms, k-place predicates (predicates that take k number of arguments), truth-functional connectives, and first-order quantifiers like *every* and *some*. Montague (1974) discards this assumption, and establishes a far richer and more expressive type system for intensional semantics.[8]

The basic framework which Montague adopts for developing a formal syntax and semantics for natural language is categorial grammar.[9] In this system a small number of syntactic categories are taken as basic. All other categories are functions from input expressions of a certain category to output expressions of a given category. Assume, for example, that we take sentences and expressions which denote individuals (i.e. names) as basic, and that we indicate the former category by *t* (for truth-value) and the latter category by *e* (for entity). Categorial grammarians represent functional categories as slashed expressions in which the argument term appears to the right of the slash and the output term is to the left. A VP and a common noun are both a t/e (a function from names to sentences), a transitive verb is a (t/e) / e (a function from names to VPs), a verb like believe, which takes a sentential complement, is (t/e) / t (a function from sentences to VP's), an NP is a t / (t/e) (a function from VP's to sentences), and a determiner is a (t/(t/e)) / (t/e) (a function from common nouns to NPs. In each case, a slashed category expression combines with a term of the kind indicated to the right of the slash in order to produce a term of the sort which appears to the left of the slash.

Consider the sentences in (9a, b).

(9) a. Mary sings.
    b. John likes Mary.

If we take *Mary* as name of category e, then *sings*, which is an intransitive verb of type e/t combines with the e term *Mary* on its left to produce a t term (sentence). Similarly, the transitive verb *likes* in (9b) is of category (t/e) / e. It combines with the e term object *John* on its left to yield an intransitive verb (VP) *likes John* of type e/t. This e/t term takes the e term *John* on its left to give a t term as its value.

Montague establishes a strict correspondence between the syntactic categories and semantic types (denotation types) of the grammar. The correspondence is expressed as a homomorphism, which is a mapping that assigns a single semantic type to each syntactic category. Sentences denote truth-values, and predicates (VPs and common nouns) denote functions from individuals to truth-values (equivalently, sets of individuals). For all other categories where $f$ is a syntactic function of the form $a/b$, the semantic value (denotation) of $f$ will be a function from the intension of $b$ ($f$'s argument) to the extension of $a$ ($f$'s value). So, for example, *believe* is an element of the category of functions from sentences to VPs, and it denotes a function from sentence intensions (propositions) to sets of individuals. This set contains the people who stand in the belief relation to the proposition expressed by the complement of *believe*. Montague grammar defines the category-type correspondence recursively for every expression of the language in a way which satisfies the principle of compositionality. Therefore, the meaning of every phrase in the language is a function of the meanings of its parts. Moreover, given the functional nature of syntactic categories and semantic types, it is possible to generate as many of each as one requires to accommodate complex syntactic structures in natural language. Each functional category will always map into a corresponding semantic type that specifies the set of possible denotations for the expression. Although there are, in principle, an unbounded number of functional categories and types, only a finite (and fairly small) number are used in the grammar of a language.

Two important differences between the Montague and Davidsonian approaches concern (a) the analysis of modification and (b) the treatment of NPs. Consider modifiers of common nouns, like the adjective *green* in *green house*, and modifiers of VPs, like the temporal adverb *on Thursday* in *arrived on Thursday*. On the Davidsonian view, modifiers are predicates which apply to individuals. A modified common noun is taken to be the conjunction of several predicates. (10a), for example, is analyzed as (10b), which states that there is an object x such that x is a house, x is green, and Mary has x.

(10) a. Mary has a green house.
b. ∃x(house(x) & green(x), & has(mary, x)).

Adverbs are also taken as predicates, and they are applied to events, which are included in the domain of entities.[10] (11a) is interpreted as (11b), which asserts that there is an event e that has the property of John arriving in e, and e occurred on Thursday.

(11) a. John arrived on Thursday.
b. ∃e(arrived(j,e) & on_Thursday(e)).

For Montague both common nouns and VPs are predicates. Syntactically, modifiers are functions from predicates to predicates, and semantically they are functions from predicate intensions to predicate extensions (sets). In (10a) *green* denotes a function which takes the intension of *house* as its argument and yields the set of green houses as its value. Similarly, in (11a) the function which *on Thursday* denotes applies to the intension of *arrived* to give the set of things that arrive on Thursday.

Davidson's account is attractively simple and straightforward. It reduces all modification to first-order predication. However, it encounters two problems. First, it assigns a semantic type to modifiers which is quite remote from their syntactic role. Syntactically modifiers are functions that apply to expressions to produce expressions of the same category. Adjectives and relative clauses apply to nouns to create modified nouns, and adverbs apply to VPs to create modified VPs. However, Davidson's analysis treats modifiers as semantic predicates that have the same kind of denotation as the predicates they modify. So, for example, in (10b) both the noun *house* and its modifier *green* are taken as one-place predicates. Similarly, in (11b) the verb *arrived* corresponds to a two-place predicate, and its adverb *on Thursday* is analyzed as a one-place predicate.

Second, the analysis does not extend to modifiers that produce expressions whose meanings cannot be taken as the conjunction of two predicate extensions. The adjective *toy* and the adverb *allegedly* in (12a, b), respectively, are examples of such non-extensional modifiers.

(12) a. John has a toy car.
b. Mary allegedly submitted her paper.

(12a) cannot be paraphrased as there is an x such that x is a toy, x is a car, and John has x. The sentence means that John has an object which resembles a car in certain respects, but which is not a car. Similarly, (12b) cannot be taken to assert that there is an event e in which Mary submitted her paper, and e allegedly occurred. If (12b) is true, then there may have been no event of Mary submitting her paper. Non-extensional modifiers require a different kind of semantic representation. They cannot be analyzed as predicates that apply to objects and events. Therefore, Davidson's approach does not provide a unified treatment of modification.

Montague's account avoids both difficulties. The semantic type of a modifier is a function which works in strict parallelism with its syntactic function. Syntactically it is a function from predicates to predicates, and semantically it denotes a function from the intension of its syntactic argument to the extension of its syntactic value. An adjective denotes a function from the intension (property) of the noun to which it applies to the set of objects that the modified noun denotes. An adverb has as its denotation a function from the intension (property) of the VP which it modifies to the set of objects that provide the extension of the modified VP. This account covers non-extensional modifiers by virtue of the fact that the function that a modifier denotes applies to predicate intensions rather than extensions. In (12a) the denotation of *toy* applies to the intension of *car* rather than the set of cars (the extension of *car*) to give the set of toy cars (not the set of things which are both toys and cars). In (12b), the denotation of *allegedly* takes the intension of *submitted her paper*, not the set of submitted papers as its argument. It yields the set of (female) things which allegedly submitted their respective papers as the extension of the modified VP.

The disadvantage of Montague's treatment of modification is that it does not express the fact that when an extensional modifier applies to a predicate, it does produce a predicate whose interpretation is equivalent to the conjunction of two predicates. In order to capture this property of modification, it is necessary to add a set of rules to the semantic part of the grammar which insure that (10a) implies that Mary has a house, and it is green, and (11a) implies that John arrived and his arrival was on Thursday. Therefore, while Montague's approach offers a unified account of modification, it does so at the cost of a more complicated treatment of extensional modifiers.

Turning to the interpretation of NPs, we have already observed that Davidson follows Frege in taking proper names to be terms that denote individuals and appear as arguments of predicates, while analyzing quantified NPs as operators which bind variables in argument positions. Therefore, (13a) and (14a) are assigned the logical forms in (13b) and (14b), respectively.

(13) a. John sings
b. sings(john)

(14) a. Every student sings.
b. $\forall x(\text{student}(x) \rightarrow \text{sings}(x))$

The advantage of this view is that it associates sentences like (13a) and (14a) with first-order formulas for which a Tarskian truth definition is available. The semantic intepretation of the sentence follows directly from its logical form. Notice, however, that while names and quantified NPs appear in the same syntactic roles (subject, object, indirect object, object of a preposition, etc.), they are mapped into distinct semantic types.

Because names and quantified NPs occupy the same syntactic roles (subject, object, object of a preposition, etc.), Montague treats them as members of a single syntactic category. He characterizes them as functions which take VPs as arguments to produce sentences (i.e. they are functions of the sort t / (e/t)). Recall that all elements of a given category receive the same semantic type in accordance with the general principle that specifies the category–type correspondence. It follows from this principle that all NPs denote functions from VP (predicate) intensions to truth-values. Predicate intensions are properties of individuals, and, as we have observed, a function from entities to truth-values is equivalent to the set of those entities to which it assigns the value true. Therefore, the function which an NP denotes can be represented by a set of properties (the set of properties for which it gives the value true). Recall that Frege treats quantifiers as second-order properties, i.e. as sets of sets. If we simplify Montague's account slightly by taking NPs as functions from predicate extensions (sets), rather than predicate intensions, to truth-values, then NPs denote sets of sets. For Montague, all NPs are, in effect, quantifiers. This semantic type is referred to as the class of *generalized quantifiers* (GQs), where a GQ is a set of sets of individuals.[11]

It is clear how an NP like *every student* can be interpreted as a generalized quantifier. It denotes the set of sets (or properties) each of which contains (at least) every student. (14a) is true iff the set of things that sings is an element of this set of sets. The set of singers is an element of the set of sets denoted by *every student* iff the set of singers contains the set of students as a subset, which is equivalent to the assertion that every thing which is a student sings. The truth conditions that Montague's GQ analysis assigns to (14a) are equivalent to those of the first-order sentence in (14b).

But it is not so obvious how proper names can be accommodated in this system. Montague's solution to this problem is to treat a name as denoting not an individual, but the set of sets containing an individual (the property set of an object). (13a) is true, then, iff the set of singers is an element of the set of sets containing John, which holds iff John is an element of the set of singers. As there is a one-to-one correspondence between the property set of an individual and the individual itself, these truth conditions reduce directly to those for (13b).

The GQ analysis sustains a uniform semantic representation of NPs. However, it does so at the price of certain complications. These become particularly clear in the case of NPs in non-subject position, like *Mary* in (15a) and *every paper* in (15b).

(15) a. John likes Mary.
b. Max read every paper.

The truth conditions of these sentences can be expressed by the first-order sentences in (16), where *likes* and *read* are naturally represented as denoting relations between individuals.

(16) a. likes(john, mary)
b. $\forall$x(paper(x) $\rightarrow$ read(max, x))

However, if *Mary* denotes a GQ, then *likes* denotes a function from GQs to sets.[12] This function must be characterized as applying to Mary's property set to yield the set of objects that like Mary as the denotation of *likes Mary*. Similarly, *read* in (15b) maps the GQ denoted by *every paper* into the set of objects which read every paper. Therefore, we are forced to adopt the counter-intuitive idea that transitive verbs stand for relations between individuals and sets of sets (GQs) rather than the more natural view that they denote relations between individuals.

An important advantage of the GQ approach is that it covers NPs like *most students*, which cannot be reduced to restricted first-order quantifiers like *every / some student*. To see this, consider what sort of logical form would correspond to (17).

(17) Most students sing.

Assume that *most*(x) is a variable binding operator like $\exists$x and $\forall$x, and that C is a truth-functional connective. Then the logical form for (17) will be an instance of the schema (18), with an appropriate connective substituted for C.

(18) most(x)(student(x) C sings(x))

But there is no truth-functional connective which can be substituted for C to yield a first-order sentence with the correct truth conditions for (17). The reason for this is that *most*(x) quantifies over the entire domain of objects, while in (17) the natural language determiner *most* expresses a relation between the set of students and the set of singers which cannot be captured by a truth-functional connective. If we use & for C, then (18) states that most objects in the domain are both students and singers. Alternatively, if we take C to be $\rightarrow$, then (18) asserts that for most objects x, if x is a student, then x sings. (17) does not make either of these claims. It states that the majority of objects in the set of students are singers. In fact, there is no first-order sentence whose truth conditions give the intended interpretation of (17).[13]

Taken as a GQ *most students* denotes the set of sets which contain more than half the set of students. (17) is true iff the set of singers is in this set. This condition holds iff the the number of students who sing is greater than half the number of students. Clearly, these are the correct truth conditions for (17). The existence of quantified NPs like *most students* shows that the meanings of some expressions in our language cannot be fully expressed in terms of the truth conditions of first-order sentences, and it is necessary to use more powerful systems, like GQ theory, to model the semantics of natural language.

## 3 Dynamic Semantics: beyond Static Sentence Meanings

Until now we have been concerned with the interpretation of sentences as static and independent units of meaning. This perspective allows us to focus on the way in which the meanings of a sentence's constituents contribute to its truth conditions. But, in fact, we generally encounter a sentence as a part of a discourse, where we understand it on the basis of preceding contributions to the conversation. When we situate sentence meanings in a discourse, they are no longer static objects, but active devices that have the capacity to inherit semantic content from previous sentences, modify it, and pass on the new information to the next sentence in the sequence.

The simple two-sentence discourse in (19) illustrates this dynamic aspect of meaning.

(19) John came in. He sat down.

We understand *he* in the second sentence as referentially dependent upon *John* in the first. We also impose an ordering relation on the events described by these sentences, so that we take John to have sat down after he entered. The interpretation of *He sat down* depends upon the information introduced by *John came in*.

Now consider the discourse in (20).

(20) A man came in. He sat down.

Although it resembles (19), there is an important difference. The proper name *John* denotes an individual, but the indefinite NP *a man* does not. Notice, also that because the pronoun occurs in a different main clause than the indefinite, we cannot treat it as a variable bound by an existential quantifier. In general, pronouns can only be interpreted as bound by a quantifier in the same clause. In (21a), *his* can be understood as a variable bound by the quantifier corresponding to *every boy*, as in (21b).

(21) a. Every boy handed in his paper.
b. $\forall x(\text{boy}(x) \supset \text{handed in x's paper}(x))$
(For every x, if x is a boy, then x handed in x's paper.)
c. Every boy arrived. He had a good time.
d. $\forall x(\text{boy}(x) \supset \text{arrived}(x)).\ \text{had\_a\_good\_time}(x)$
(For every x, if x is a boy, then x arrived. x had a good time.)

However, such an interpretation is not available for *he* in (21c). The quantifier in (21d) cannot bind the variable *x* in the following sentence, which is out of its scope. Therefore, *x* is free (unbound by the quantifier) in the second sentence of (21d). This sentence says that x had a good time without placing any restrictions on the values of *x*. We could have used *y* instead of *x* in the second sentence of (21d), which would give *had_a_good_time*(*y*), without changing the meaning of (21d).

The interpretation of *A man came in* in (20) makes available a possible referent which *he* can be used to identify in the next sentence. However, it is not clear precisely which part of the meanings of these two sentences creates this entity.

The cases in (22) provide examples of a similar but more complex anaphoric relation between a pronoun and an indefinite NP.[14]

(22) a. Every man who owns a donkey beats it.
b. If a man owns a donkey, he beats it.

As with the pronoun in (20), *it* is not within the scope of its antecedent, the indefinite NP *a donkey*, in either (22a) or (22b). This NP is contained either in a relative or subordinate clause rather than in the main clause where *it* appears. *He* is not within the scope of *a man* in (22b) for the same reason. However, both pronouns appear to function like variables bound by the universal quantifier *every*. On their most natural readings, (22a, b) assert that for every man x and every donkey y, if x owns y, then x beats y. The quantified NP subject in (22a) and the antecedent *if* clause in (22b) give rise to the representation of a set of ordered pairs <a, b> such that a is a man, b is a donkey, and a owns b. For each such pair, *he* in (22a) identifies a, and *it* in both sentences selects b. The problem is that because the pronoun *it* is anaphorically dependent upon the indefinite *a donkey* in (22a, b) it does not correspond to a variable bound by a universal quantifier. It is not obvious, then, how it is possible to interpret (22a, b) as equivalent in truth conditions to a sentence in which *it* is bound by a universal quantifier corresponding to *every donkey*.

There are three main approaches to dynamic anaphora, and I will briefly sketch each one in turn. The first is discourse representation theory (DRT).[15] In this framework an indefinite NP is treated not as a quantified NP, but as an expression which introduces a discourse referent that satisfies the content of the indefinite description. In (20) *a man* introduces an object u, which satisfies the predicate *man*, into the store of information available within the discourse. The sentence also applies the predicate *came in* to u. Therefore, the first sentence of (20) adds the conditions *man*(u) and *came in*(u) to the discourse information store. As u is now accessible at future points in the discourse, it is possible to use a suitable pronoun to refer to it. The second sentence of (20) contributes the condition *sat down*(u), which is obtained by taking u as the value of *he*. The conjunction of these conditions on u yields a discourse representation structure that holds iff there is a man who came in and that man sat down, which is the desired reading of the sequence.

Applying this approach to (22b), the two indefinite NPs in the antecedent clause introduce two distinct discourse referents u and v, and the conditions *man*(u), *donkey*(v), and *owns*(u,v). These referents and conditions are accessible to the consequent clause, where u and v are substituted for *he* and *it*, respectively, to produce the condition *beats*(u,v). However, the relation between the two clauses is not that of a simple sequential conjunction, as in (20), but a conditional connective. Therefore, it is necessary to interpret the combined discourse structure as asserting an *if . . . then* relation between the conditions of the antecedent and that of the consequent. On the preferred reading of (22b), the conditional sentence is within the scope of the implied adverb of universal quantification *in every case* (or *always*). Applying this quantifier to the conditional discourse structure gives a set of conditions that hold iff for every case, if there is a pair containing a man and a donkey which he owns, then the first element of the pair beats the second. This is the required interpretation for (22b). Assume that the universal quantifier *every* of *every man who owns a donkey* in (22a) sets up a universal conditional relation between the conditions imposed by the modified noun *man who owns a donkey* and those of the VP *beats it*, and that it also introduces a variable x into both sets of conditions. The antecedent of this conditional contains *man*(x), *donkey*(u), and *owns*(x,u), and the consequent adds *beats*(x,u). This discourse representation structure specifies the same interpretation as the one for (22b).

The second approach to dynamic anaphora is the dynamic binding account.[16] It retains the traditional view of indefinites as existentially quantified NPs. In addition to the classical logical connectives and quantifiers it introduces dynamic counterpart operators whose scopes can extend beyond single clauses. The dynamic existential quantifier $\exists^d x$ has the effect of introducing a discourse referent associated with the variable x which can be inherited by the informational state (discourse model) that serves as the input to a subsequent sentence. The dynamic conjunction $\&^d$ passes the referents in the information state produced by its first conjunct to the interpretation of the second. These dynamic operators are used to represent (20) as (23a), where the dynamic existential quantifier occurs in the first dynamic conjunct of the sentence. The interpretation assigned to this formula has the same truth conditions as (23b), in which a static (classical) existential quantifier has scope over all the conjuncts.

(23) a. $\exists^d x(\text{man}(x) \;\&\; \text{came\_in}\ (x)) \;\&^d\; \text{sat\_down}(x)$
(for some$^{\text{dynamic}}$x[x is a man and x came in] and$^{\text{dynamic}}$ [x sat down])
b. $\exists x(\text{man}(x) \;\&\; \text{came\_in}\ (x) \;\&\; \text{sat\_down}(x))$
(for some x[x is a man and x came in and x sat down])

The dynamic implication $\rightarrow^d$ holds between two sentences A and B for a given set R of discourse referents iff every information state which A produces for R gives rise to one which successfully interprets B. The connective $\rightarrow^d$ can be combined with the dynamic existential quantifiers $\exists^d x$, $\exists^d y$ to represent (22b) as (24a), where the dynamic quantifiers occur in the antecedent of the conditional sentence and dynamically bind the variables in the consequent. (24a) has the same truth conditions as (24b), in which the entire conditional is within the scope of two static universal quantifiers.

(24) a. $\exists^d x \exists^d y(\text{man}(x) \; \& \; \text{donkey}(y) \; \& \; \text{owns}(x,y)) \rightarrow^d \text{beats}(x,y)$
(for some$^{dynamic}$x and for some$^{dynamic}$y[if$^{dynamic}$ x is a man and y is a donkey and x owns y], then$^{dynamic}$ [x beats y])

b. $\forall x \forall y((\text{man}(x) \; \& \; \text{donkey}(y) \; \& \; \text{owns}(x,y)) \rightarrow \text{beats}(x,y))$
(for every x and for every y[[if x is a man and y is a donkey and x owns y], then [x beats y]])

(24a, b) are true iff for every pair <a,b> such that a is a man, b is a donkey, and a owns b, a beats b. However, 24a corresponds directly to (22b) in that it represents both indefinite NPs in (22b), *a man* and *a donkey*, as (dynamic) existentially quantified NPs rather than as universally quantified NPs as in (24b). Therefore, this analysis provides an explanation for the fact that, in sentences like (22b), pronouns which are anaphorically dependent upon indefinites behave like variables bound by universal quantifiers.

By defining a dynamic universal quantifier $\forall^d x$ and combining it with $\exists^d y$ and $\rightarrow^d$, it is possible to obtain (25) for (22a). (25) has the same truth conditions as (24a, b). In this formula, *every man* corresponds to a restricted dynamic universal quantifier and *a donkey* to a restricted dynamic existential quantifier.

(25) $\forall^d x((\text{man}(x) \; \& \; \exists^d y(\text{donkey}(y) \; \& \; \text{owns}(x,y))) \rightarrow^d \text{beats}(x,y))$
(for every$^{dynamic}$x [[if$^{dynamic}$ x is a man and for some$^{dynamic}$y[y is a donkey and x owns y]], then$^{dynamic}$ [x beats y]])

As in the case of (24a) and (22b), (25) corresponds directly to (22a) in that the indefinite *a donkey* is represented by a (dynamic) existential quantifier rather than a universal quantifier (as in (24b)). Therefore, the dynamic binding account of donkey anaphora also permits us to account for the fact the pronoun *it* in (22a) is understood as bound by a (classical) universal rather than a (classical) existential quantifier.

While DRT uses indefinites to introduce referents into a discourse and dynamic binding relies on dynamic operators to pass information concerning discourse referents from one sentence to another, the third approach locates the mechanism for dynamic anaphora in the interpretation of the pronoun which takes a quantified NP as its antecedent. This sort of pronoun, referred to as an E-type pronoun, effectively functions like a pointer to a description that refers back to an entity (or collection of entities) in the set that is determined by its quantified NP antecedent.[17] Taking *he* in (20) and (22b), and *it* in (22a) and (22b) as E-type pronouns gives interpretations of these sentences corresponding to (26a) and (26b).

(26) a. A man came in. The man who came in sat down.
b. Every man who owns a donkey beats the donkeys he owns.
c. If a man owns a donkey, the man who owns a donkey beats the donkey he owns.

So in (20), for example, the E-type pronoun *he* is interpreted by the description *the man who came in*, as in (26a).

Another way of understanding an E-type pronoun is to treat it as corresponding to a function which applies to objects in an appropriately specified domain to give values in a set defined in terms of the denotation of its antecedent NP. The antecedent of *he* in (20) and (22b) is *a man*, which is not within the scope of a quantified NP. Therefore, the E-type function associated with *he* maps any object in the domain of discourse onto an element in the set of men who own donkeys. In (22a, b) the antecedent of *it* is *a donkey*, which is in the scope of *every man* and *a man*, respectively. It denotes an E-type function from men who own donkeys to (collections of) the donkeys which they own.

The three approaches discussed here use different formal techniques for modeling dynamic anaphora. However, common to all of them is the view that a major part of understanding the meaning of a sentence is knowing its possible influence on the informational structure of a discourse in which it appears.

## 4 Meanings and Situations: beyond Possible Worlds

In section 2 I described the intension of an expression as a function from a possible world to the extension of the expression. A world is the result of assigning the objects of a domain to properties and relations in such a way as to produce a complete state of affairs containing these objects. There are at least some cases where it seems to be necessary to use situations rather than worlds to specify the interpretation of a sentence.[18] A situation is a smaller and more fine-grained object than an entire world. It can be contained in larger situations, and it is, in effect, the specification of part of a world (equivalently, a partial specification of a world).

To see the role of situations in representing meaning let's return to the analysis of generalized quantifiers. In section 2 we characterized the denotation of an NP as a GQ (a set of sets). For quantified NPs, we can, equivalently, take the determiner of the NP as denoting a relation between the set denoted by the noun to which the determiner applies and the predicate set of the VP. For example, the GQ corresponding to *every student* is the set of sets each of which contains the set of students. Alternatively, *every* denotes the relation that holds between any two sets A and B when A is contained in B. On both conditions, (14a) is true iff the set of students is a subset of the set of singers.

(14) a. Every student sings.

Similarly, the determiner *the* denotes the relation that holds for two sets A and B when the unique element of A is a member of B. Therefore, (27) is true iff there is a single woman and she dances.

(27) The woman dances.

If the intension of *the woman* takes the actual world (or any world which resembles it) as its argument, then it will yield the set containing the empty set as the extension of the NP. This is because it is not the case that the set of woman has only one element in the actual world. As the relation denoted by *the* does not hold between the set of women and the set of dancers, (27) is false in the actual world. It will only be true in a world containing a unique woman. But this is the wrong result. There are surely cases where an assertion of (27) is literally true in the actual world by virtue of the fact that the speaker is referring to a particular woman, despite the existence of other women in the world.

Instead of treating a property as applying to an object in a world, we can localize the relation to a situation within a world. This will give us statements of the form *Mary is a woman in s*. We can express this relation between a statement and a situation s by saying that s supports the information that Mary is a woman. If we identify a situation s containing a unique woman u and interpret *woman*, relative to s, as denoting the singleton set containing u, then (27) is true if u dances, even though u is not the only woman in the world. This s is the resource situation which we use to determine the GQ that *the woman* denotes.

Imagine a conversation in which I am telling you about two successive visits to the theater. On the first trip I saw a production of a musical with one female actor, and on the second I saw a comedy which also featured one female actor. Using each play as a resource situation I assert (28a) in describing the first production, and (28b) in my account of the second.

(28) a. The woman sang.
 b. The woman did not sing.

Assume, also, that the same actress appears in both plays. It is still the case that both (28a) and (28b) are true. Although the two resource situations identify the same person, each situation supports one of the assertions.

Cooper (1996) uses resource situations to characterize the class of GQs denoted by NPs in natural language. He also points out that it is necessary to distinguish between the resource situation in which the restriction (common noun) set of a GQ is fixed and the situation in which the entire sentence containing the GQ expression is evaluated. (29) brings out the distinction clearly.

(29) Everyone spoke to John.

The quantificational situation q which supports (29) includes John. Therefore, if we identify it with the resource situation r for setting the restriction set of the relation denoted by *every*, (29) implies that John spoke to himself. This consequence is avoided if r and q are distinct. We could, for example, take r to be properly contained in q, so that the restriction set is a subset of the set of people in q.

Cooper also argues that the quantificational situation must be distinguished from the individual situations i in which the property expressed by the VP applies to each of the elements of the restriction set. He invokes cases in which perception verbs, like *see*, take quantified complements to motivate this claim.

(30) a. John saw everyone leave the concert.
b. John saw each person leave the concert.

(30a) can be true in a situation in which John saw all of the people at a concert leave the hall, but he did not observe each person leave individually. By contrast, (30b) is true only if he saw each person leave. This difference in interpretation consists in the fact that the truth conditions for (30b) require the identification of q and i while those for (30a) do not.

Conditional donkey sentences in the scope of quantificational adverbs like *usually* provide another case in which situations play a central role in determining the meaning of quantifier terms. (31) allows at least two different interpretations.

(31) Usually if a man owns a donkey, he beats it.

On one reading, (31) says that for most pairs <a,b> where a is a man, b is a donkey, and a owns b, a beats b. Given this interpretation, (31) is true in the following state of affairs. There are 10 donkey owners, 9 of whom each owns a single donkey, and one who owns 20. The 9 men who each own a donkey do not beat it, but the one donkey owner who has 20 beats all of them. There are 29 distinct pairs of men and donkeys they own. The man who owns 20 is the first element of 20 pairs, with each of his donkeys as the second element of one of these pairs. The 9 other owners and their donkeys contribute the remaining 9 pairs. The sentence is true because the first element beats the second in 20 out of 29 of these pairs. On the second reading, 31 claims that most men who own donkeys beat the donkeys they own. With this interpretation, the sentence is false in the situation described here, as it requires there to be more than 5 men who beat the donkeys they own.[19]

It is possible to account for these interpretations by treating adverbs like *usually* as quantifiers that denote relations between sets of situations.[20] *Usually* denotes a relation that holds between two sets of situations A and B iff most of the elements of A are also in B. The different readings are generated by varying the size of the situations in the restriction set that corresponds to the antecedent of the conditional sentence. If this set contains only minimal situations involving a donkey owner and a single donkey, then (31) asserts that most situations consisting of a man and a single donkey which he owns are situations in which he beats that donkey. This yields the first reading. When the restriction set contains maximal situations involving a man and all of the donkeys he owns, then (31) states that most situations in which a man owns donkeys are situations in which he beats the donkeys he owns. This provides the second reading. The first interpretation is symmetrical in that *usually* quantifies over situations defined by pairs of donkey owners and individual donkeys. The second is asymmetrical as *usually* effectively quantifies only over donkey owners.

By using situations to specify the extensions of predicates and quantificational expressions it is possible to represent aspects of interpretation which cannot be captured in classical intensional semantics.

## 5 Underspecified Representations: beyond Compositionality

As we observed in section 2, the condition of compositionality requires that the meaning of any expression P be computable by a function which, given the meanings of P's syntactic constituents as its arguments, yields P's meaning as its value. We have also seen that Montague grammar satisfies this condition by characterizing the relation between the set of syntactic categories and the set of semantic types as a homomorphism which maps each syntactic structure into a single denotational type.[21] In this framework the meaning of an expression is fully determined by (a) its syntactic structure and (b) the meanings of its constituents.

In order to sustain a homomorphism of this kind, the function which specifies the mapping from syntax to semantics must apply to expressions with fully specified syntactic representations and yield unique semantic values. Therefore, syntactic and semantic ambiguity are eliminated by the mapping which the function specifies. Ambiguous lexical items are divided into words which stand in a one-to-one correspondence with the distinct senses of the original term. The verb *run*, for example, becomes a set of verbs each of which is assigned a denotation corresponding to one of *run*'s meanings (move quickly, operate or administer something, flow, function, etc.).

(32) is ambiguous between two scope interpretations of the quantified NP *a painting* relative to the intensional verb *seek*.

(32) John is seeking a painting.

If *a painting* receives narrow scope relative to *seeks*, then John wants there to be some painting or other which he finds. If it has wide scope, then there is a particular painting which he is looking for. Montague generates these readings from distinct syntactic structures. The narrow scope reading is obtained when *a painting* originates *in situ* as an argument of *seeking*. For the wide scope reading, *a painting* is generated outside of the sentence *John seeks it* and is substituted for the pronoun. The VP of the first structure denotes the set of things which stand in the seek relation to the intension of the GQ denoted by *a painting*. This set is the value that the function denoted by *seeks* assigns to the intension of *a painting*. On the second syntactic derivation, *a painting* is interpreted as a GQ which applies to the predicate set containing the objects that John is seeking. This derivation yields the interpretation that there is a painting x and John is seeking x.

In fact, it is possible to construct a semantic system that is non-compositional, but relates the meaning of an expression systematically and incrementally to the meanings of its parts.[22] This is achieved by allowing the mapping from syntax to semantics to be a relation which assigns more than one meaning to an expression under a single syntactic representation. In such a system the verb *run* could be paired with a disjunction of meanings corresponding to each of its senses. (33) would be represented as having one syntactic structure, with a VP headed by a single verb *run*, which is associated with at least two distinct semantic representations.

(33) John ran the marathon.

On one, John was a runner in a race, and on the other he administered it. On this view, (33) would be represented by a single syntactic structure which is mapped to a set containing two interpretations, each providing a distinct set of truth conditions. To obtain a disambiguated reading of the sentence it is necessary to select one element of the set.

A more interesting case of non-compositional interpretation involves mapping a syntactic structure into a set of alternative scope readings. There are at least two ways of doing this. On one approach, quantified NPs can either be taken as GQs *in situ* (in the argument positions where they appear in the syntactic structure of the sentence) or interpreted through the device of quantifier storage.[23] When storage applies to an NP, a variable meaning is substituted for the argument position which it occupies, and the GQ is placed in a stored part of the meaning of the expression where the NP appears. The non-stored meaning of the expression, which includes the variable in the original argument position of the NP, is combined with the meanings of larger expressions until a point is reached where a predicate set is specified. The GQ can be released from storage at this point and applied to the predicate. As we have seen, if *a painting* in (32) is interpreted *in situ*, it is within the scope of the verb *seeks* and the narrow scope reading results. If it is placed in storage, the set of objects x such that John seeks x is computed as the interpretation of the open sentence (predicate) *John is seeking x*. The GQ denoted by *a painting* is released from storage and applied to this set to yield the wide scope reading of the sentence. Unlike Montague's analysis, this account assigns a single syntactic structure to

(32) where *a painting* is always in object position. The structure is associated with two distinct scope interpretations obtained by different procedures.

On the second approach, sentences containing scope-taking expressions are assigned schematic semantic representations in which the scopes of these terms are left unspecified.[24] In the representation for (32), for example, the scope relation between *a painting* and *is seeking* is undefined. Similarly, *a student* and *every program* are unordered for relative scope in the representation assigned to (34).

(34) A student checked every program.

The second treatment of scope ambiguity is similar to the first in that it also associates a syntactic structure with a set of alternative scope interpretations. However, it implies a more far reaching revision of the compositional view of semantic interpretation. This approach takes the meaning of an expression to be a partial representation R defined in terms of a minimal set of conditions C on the interpretation of R. To obtain a more specified meaning one adds new constraints to C to restrict the set of interpretations with which R is compatible. A compositional semantics provides a homomorphism for mapping unambiguous syntactic structures into fully specified semantic values. An underspecified semantics, by contrast, establishes a relation between syntactic structures and partial semantic representations whose parameters characterize sets of possible values. These sets can be further restricted by adding constraints to the representation.

## 6 Conclusion

Initial attempts to construct a formal semantic theory for natural language used the syntax and truth definitions of first-order languages as a model. Therefore, they associate the categories of natural language with the semantic types of first-order logic. Montague introduced a richer type system which permits a direct mapping of complex functional categories into corresponding types. It also expresses the interpretation of higher-order expressions, such as non-first order generalized quantifiers. Dynamic semantics then moved beyond the static meaning of an individual sentence taken in isolation to representing semantic content in terms of the way in which a sentence transforms the information state inherited from previous sentences in a discourse. Situation semantics replaced the interpretation of expressions relative to a possible world with evaluation in a situation, where the latter is a more finely structured and partially specified entity than the former. Finally, underspecified semantics discards the condition of compositionality to construct a more flexible mapping between syntactic structure and semantic interpretation. This approach sustains a systematic connection between the meaning of a phrase and the meanings of its parts while using partially defined representations to capture ambiguity and under-determined interpretation.

It is important to recognize that as new paradigms of semantic representation have emerged, the leading ideas of the earlier programs have not disappeared. They have continued to survive in various formulations and to exert influence on successive generations of theorists, many of whom attempt to solve semantic problems by integrating the insights of earlier models into new frameworks.

In considering the recent history of semantic theory, it becomes clear that the past twenty-five years have seen considerable progress in the application of increasingly sophisticated formal techniques to the explanation of a wide range of semantic phenomena. This work has opened up new areas of investigation and yielded promising results which have turned formal semantics into a well-grounded and exciting domain of linguistic research.

## NOTES

I am grateful to Ruth Kempson, Gabriel Segal, and the editors of this volume for helpful comments on earlier drafts of this paper. I am solely responsible for any shortcomings which remain.

1 There are numerous introductory texts on formal semantics, each highlighting different issues and tending to represent a particular theoretical paradigm. Chierchia and McConnell-Ginnet (1990), and Heim and Kratzer (1998) are two recent texts which offer interesting background and perspectives on the field. The papers in Lappin (1996a) provide introductions to current research in the major areas for formal semantics.

2 Rules are recursive if they can apply to their own output an unlimited number of times. By virtue of this property recursive rules can generate an infinite number of structures.

3 For a discussion of the relation between the semantics of declarative and non-declarative sentences see Lappin (1982). For analyses of the semantics of interrogatives see Karttunen (1977), Hamblin (1973), Ginzburg (1996), Higginbotham (1996), and Groenendijk and Stokhof (1997).

4 See Frege (1879), (1891), and (1892). A logic is first-order when all of its predicates (property terms) apply only to individuals in the domain of discourse (the domain of the logic). A higher-order logic contains predicates which apply to properties or sets of individuals (and possibly other higher order entities). So, for example, "green" is a first-order predicate that applies to physical objects, while "partially ordered" is a higher-order predicate of sets.

5 See note 4 for the notion of first-order terms and first-order logic. A first-order language is a formal language all of whose predicates are first-order. We can say that a logic is a formal language which has additional principles that identify a set of sentences in that logic as true.

6 See Davidson (1967a) and the papers in Davidson (1984). For applications of Davidson's program within linguistic semantics see Higginbotham

(1985), May (1991), and Larson and Segal (1995). Sher (1991) and (1996) extends Tarskian semantics beyond first-order systems.

7 Interestingly, Tarski expressed skepticism about the prospects for developing formal truth definitions for natural languages. He claimed that their terms are often vague or ambiguous. Morever, they permit self-reference in a way which generates paradox, as in the famous liar paradox *This statement is false*, understood as referring to itself. Davidson, like most semanticists, attempts to get around these reservations by adopting an incremental program on which a formal truth definition is first constructed for a representative fragment of a natural language and then extended to progressively larger sets of sentence types.

8 Dowty et al. (1981) provides a very clear and detailed introduction to Montague grammar.

9 For recent introductions to Categorial Grammar see Moortgat (1988), Morrill (1994), and Jacobson (1996).

10 See Davidson (1967b) for this analysis of adverbs. Higginbotham (1985) proposes a Davidsonian treatment of modifiers within the framework of Chomsky's (1981) government and binding model of syntax.

11 For discussions of generalized quantifiers in natural language see Barwise and Cooper (1981), Keenan and Moss (1985), Keenan and Stavi (1986), van Benthem (1986), Westerståhl (1989), Keenan (1996), and Keenan and Westerstahl (1997). For a comparison of the Davidsonian and the GQ approaches to the semantics of NPs see Lappin (1996b) and (1998).

12 I am again simplifying the account by taking transitive verbs to denote functions on the extensions rather than the intensions of NPs. See Cooper (1983) for a treatment of transitive verbs as functions of this kind.

13 See Barwise and Cooper (1981) and Keenan (1996) for this result.

14 Geach (1962) introduced these sorts of cases into the modern semantics literature. The pronouns which are dependent upon indefinite NPs in (22) are generally referred to as *donkey pronouns*, and the anaphoric relation in these structures is described as *donkey anaphora*.

15 DRT was first proposed by Kamp (1981). An alternative version of this theory is presented in Heim (1982). For a recent model of DRT see Kamp and Reyle (1993).

16 The version of dynamic binding which I am summarizing here is essentially the one presented in Groenendijk and Stokhof (1990) and (1991). For an alternative account see Chierchia (1995). Groenendijk et al. (1996) propose a theory of update semantics based on dymamic binding. Kempson et al. (forthcoming) develop a deductive approach to dynamic semantics which has much in common with all three approaches discussed here.

17 Evans (1980) initially proposed the idea of an E-type pronoun. Cooper (1979), Lappin (1989), Heim (1990), Neale (1990), Chierchia (1992), and Lappin and Francez (1994) suggest different E-type accounts of donkey anaphora.

18 Barwise and Perry (1983) introduced a situation-based theory of meaning into formal semantics. For more recent work in situation semantics see Barwise (1989), Barwise et al. (1991), Cooper et al. (1990), Gawron and Peters (1990), Aczel et al. (1993), Cooper et al. (1994), and Cooper (1996). The treatment of generalized quantifiers in terms of situation theory discussed here is based on Cooper (1996).

19 Explaining these distinct readings for (31) is known as the *proportion problem* for conditional donkey sentences

with quantificational adverbs of non-universal force. See Kadmon (1990), Heim (1990), Chierchia (1992), and Lappin and Francez (1994) for discussions of this problem.

20 Heim (1990), and Lappin and Francez (1994) pursue this approach. Lappin and Francez analyze quantificational adverbs as generalized quantifiers on sets of situations.

21 A homomorphism is a functional mapping from a domain A to a range B in which several elements of A can be associated with one object in B. Montague's category–type correspondence is a homomorphism because in some cases the same semantic type is assigned to more than one syntactic category. For example, both common nouns and predicates denote sets of individuals.

22 See Nerbonne (1996) for a non-compositional approach to semantics in a constraint-based framework. My discussion of compositionality in this section owes much to his treatment of the issue. Zadrozny (1994) shows that any mapping from syntax to semantic interpretation for a language can be formulated as a function, and so can be expressed compositionally. However, such functions may be non-systematic in the way in which they specify the dependence of a phrase's interpretation on the meanings of its constituents. Specifically, they may involve a case by case listing for subsets of the relevant ordered pairs of meanings for which the functional relation holds.

23 See Cooper (1983), Pereira (1990), Pereira and Pollack (1991), and Dalrymple et al. (1991) for accounts of quantifier storage. Lappin (1991) and (1996b) gives arguments for using storage rather than a syntactic operation of quantifier raising to capture wide scope readings of quantified NPs.

24 See Reyle (1993) and Copestake et al. (1997) for different versions of this view.