18

From Logic to Ontology: Some Problems of Predication, Negation, and Possibility

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1 Negation and Nonexistence

Russell contrasted considering things "from a logical point of view" with linguistic and philosophical points of view, where 'philosophical' meant 'ontological.' This fits with his suggesting 'Philosophical Logic' for the title of Wittgenstein's *Tractatus*. While Wittgenstein supposedly responded that the phrase was nonsense, ontological issues raised by logic and 'logical form' are now a basic part of philosophical logic. One of the oldest problems concerns negation. In a passage in *The Sophist* Plato writes: "When we assert *not-being* it should seem, what we assert is not the *contrary* of *being*, but only something *other*." (Taylor 1971: 164). Taken with other passages, this suggests Plato considers 'x is not-F' as (1) '(f)(f ≠ F **if** fx)' or (2) '(y)(y ≠ x **if** Fy).' [I use '**if**' to avoid explicit use of a truth-functional conditional and obvious problems posed by using '⊃' to eliminate '¬'.] While Taylor takes Plato to construe 'not-F' in terms of "being something other than what we call 'F'," he construes 'other' in a restricted sense that involves F belonging to a group of incompatible properties. Owen rejects such an interpretation of Plato and takes 'x is not-F' as 'all attributes of x are different from F', rather than as 'some attribute of x *excludes* F' (Owen 1986: 131, 114–15). Many Plato scholars consider such readings problematic, but Owen, following (1), analyzes not-F in terms of the Platonic forms of *difference* (≠) and *sameness* (=). Forgetting 'types' and questions about how Plato construes relations, obvious problems arise if '=' and '≠' are used to define '¬', as the equivalences – (a) '¬(f = g) **iff** f ≠ g' and (b) '¬(f ≠ g) **iff** f = g' – are not derivable. Consider (b). Using (1), '¬(f ≠ g)' becomes '(R)(≠(R, ≠) **if** R(f, g))'. But we can neither derive 'f = g' from that nor that from 'f = g', though '[(R)(≠(R, ≠) **if** R(f, g))] **iff** f = g' may seem obviously true. The same holds for (a) and '[(R)(≠(R, =) **if** R(f, g))] **iff** f ≠ g', using (2) in place of (1). '¬¬Fa **iff** Fa' poses a related problem.

Bradley and Bosanquet suggested '¬Fx' be construed as '(∃f)(fx & f is incompatible with F),' while Demos took '¬p' as '(∃q)(q is true & q is incompatible with (in opposition to) p).' Russell argued that Demos did not avoid *negative facts*, as incompatibility is a form of negation, being the Scheffer stroke function, and that Demos' view generated a problematic regress, since 'p and q are in opposition' means 'p and q are not both true' (Russell 1918, 1919). But in the 1925 edition of *Principia* he wrote: "Given all true atomic propositions, together with the fact that they are all, every other true proposition can theoretically be deduced by logical methods" (Whitehead and Russell 1950: xv). Since atomic facts ontologically grounded true atomic propositions, this took a set of atomic facts and the general fact that the set contained *all* the *atomic facts* to avoid negative facts, though he spoke of *atomic propositions*, not atomic facts. A negation of an atomic proposition was true if it followed from the statement of the general fact and the 'list' of true atomic propositions. This followed his rejecting conjunctive facts due to 'p, q ⊨ p & q.'

Russell's theme has been revived in recent years by views appealing to a 'meta-fact' about atomic facts or a *class* or *totality* of atomic facts or both. The simplest version of such a view recognizes a domain of *all* atomic facts, a class, as the ontological ground for true negations of atomic propositions. The class of all atomic facts is taken to suffice as the truth maker for such negative truths, since it is purportedly *not* a fact *that* an excluded atomic fact *is not* in the class. That is a consequence of an ontological analysis of classes taking a class, say {a, b}, to suffice as the truth ground for statements like 'a ∈ {a, b}' and '¬c ∈ {a, b},' as opposed to holding that a relation, ∈, obtains or does not between the term and the class. Moreover, one can argue that classes are presupposed by standard systems of logic, since the logical variables and quantifiers presuppose domains, which are classes, or 'ranges' of application. But recognizing such a 'range' implicitly recognizes a domain (class) comprised of all and only things satisfying some condition. As classes of particulars and properties correspond to individual and predicate variables and quantifiers, respectively, the sentential variables can be taken to correspond to a domain of facts, rather than to the 'truth values' used in the 'evaluations' in logic texts. This requires rejecting the 'substitutional' account of quantification as untenable. On such an account, the quantifier sign '(∃x)', for example, is read in terms of 'There is a name (constant)' rather than 'There is an object (individual).' Supposedly one can then 'semantically ascend' to talk of signs, instead of things, and avoid *ontological commitments* to non-linguistic objects in the domain of a quantifier. Semantic 'ascension' has led to tortured attempts to prove that formal systems can have non-denumerably many proper names. But that is of no import, for one can simply assume that there are sufficiently many individual constants. The real problem is the assumption that for *every* object there is a corresponding sign – a claim that involves quantifying over *objects* as well as signs. The standard response, going back to a 1968 argument of Belnap and Dunn, is the pointless and problematic claim that such a use of an objectual quantifier can be construed substitutionally, involving a further quantification over objects that is then treated substitutionally, and so on *ad infinitum*.

Conjunctive and disjunctive facts may be avoided, as true conjunctions and disjunctions are so in virtue of the truth or falsity of component atomic sentences. But negation raises a unique problem. The difference is reflected, first, by there being no standard logical rule for negation corresponding to 'p, q ⊨ p & q,' and, second, by an evaluation assigning *one of* T or F, but *not both*, to the atomic sentences in a standard bivalent logic. This latter point can be taken to reflect the traditional logical laws of excluded middle and non-contradiction and their special status, though all tautologies, being logically equivalent, are 'equal.' Such laws provide a basis for the use of truth tables, as the 'law of identity' is presumed by any coherent system of signs. But the truth table for negation does not explicate the meaning of '¬'. Nor does it resolve questions about negative facts and the ontological correlate of the negation sign. What a standard truth table shows is: (1) that '$\neg$' is taken as the sign for negation; (2) that every sentence of the schema is taken to be true *or* false; and (3) that 'or' in (2) is used in the exclusive sense since no sentence is both true and false. Some logical signs, and concepts, are basic, and so-called *elimination* and *introduction* rules neither provide analyses of them nor resolve ontological issues raised by them. Recent purported explications of the *meaning* of the quantifier signs, stemming from Wittgenstein's Tractarian views, by means of such rules also do not do what they purport to (Celluci 1995; Martin-Löf 1996). Such rules merely codify the *interpretation* of the quantifier *signs*, as truth tables do for truth functional *signs*, linking them to *generality* and *existence.*

Russell appealed to a general fact about all true atomic facts *and*, implicitly, a class ('list') of atomic facts to ground the truth of true negations. Later, others took such a class of atomic facts to suffice while some held that a general fact alone sufficed. All such attempts fail to resolve the issue of negative facts. We can see why by returning to the attempt to take the truth ground of '$\neg$ Fa' to be a class, D, of atomic facts *and* a general fact, (p)(p $\neq$ Fa), with D giving the range of 'p.' '(p)(p $\neq$ Fa)' states that no fact is a's being F. But such a general fact involves an apparent negation. Limiting the discussion to a miniature world (model) with D = {Ga, Fb}, we can take Russell's *list* in terms of '(p)(p = Ga $\vee$ p = Fb),' stating that Ga and Fb are all the atomic facts, without a negation. But that is still problematic. As '($\exists$x)($\exists$y)(x $\neq$ y & (z)(z = x $\vee$ z = y))' states that there are only two particulars, we can state that a and b are the only particulars by '(x)(x = a $\vee$ x = b).' That entails, with an additional name 'c,' that '($\exists$x)(x = c)' entails '(c = a $\vee$ c = b).' But as '(x)(x = a $\vee$ x = b)' does not entail '$\neg$($\exists$x)(x = c),' '(p)(p = Ga $\vee$ p = Fb)' does not entail either '$\neg$($\exists$p)(p = Fa)' or' '$\neg$ Fa' is true.' All that follows that is relevant is '($\exists$p)(p = Fa) $\models$ (Fa = Ga $\vee$ Fa = Fb),' which entails '(Fa $\neq$ Ga & Fa $\neq$ Fb) $\models$ $\neg$($\exists$p)(p = Fa),' *assuming* that we can instantiate to 'Fa,' as we assumed about 'c' above. Stating that the nonexistence of Fa grounds the truth of '$\neg$Fa' thus involves 'Fa $\neq$ Ga & Fa $\neq$ Fb' or '(p)(p $\neq$ Fa),' as well as the apparent implicit use of 'Fa' to represent a nonexistent fact.

The issue raised by '(p)(p $\neq$ Fa)', or instantiating to 'Fa' from '(p)(p = Ga $\vee$ p = Fb),' recalls Meinong's nonexistent objects and nonsubsistent *objectives*, since Fa does not exist. The correspondence theory of truth, taking facts as truth grounds for sentences (propositions), that Moore set forth in lectures of 1910–11 raised the issue that was put cryptically by Wittgenstein (1961: 4.022). An atomic statement, or a 'thought' that a is F, represents a situation – *shows* its sense – whether or not it is true, and *states* that it obtains. As showing or *representing* is a relation, between a statement or thought and a *situation*, that obtains whether or not the represented situation does, since the thought must have the same sense whether it is true or not, a problem arises. Moore avoided the issue by saying that his talk of the 'non-being' of a fact was merely an unavoidable way of speaking, while taking the *being* of the fact that-p to *directly prove* the truth of 'the belief that-p.' But holding, like Russell, that "Fa" is true **iff** the fact that-a is F exists', his use of the clause 'that-a is F' pointed to the implicit recognition of facts as *possibilities* (situations) which may *obtain* (exist) or *not*. Thus *correspondence* was an ambiguous concept. In one sense 'Fa', whether true or not, corresponded to a *possibility*; in another sense, if true, it corresponded to an *existent* fact.

2 Designation and Existence

Carnap (1942: 24, 50–2) considered the issues of truth and reference in terms of the semantics of 'designation'. Consider (1) 'a' designates Theaetetus; (2) 'F' designates the property of flying; (3) 'Fa' designates the state of affairs that Theaetetus is flying. Carnap took (1)–(3) as semantical 'rules' for a schema. With *designates* as a semantical relation, (3) is true even if 'Fa' is false. (1)–(3), as semantical rules, do not express matters of fact. That such rules are rules of a particular schema is a matter of fact. The same sort of distinction applies to ordinary language variants of (1)–(3) – 'Theaetetus' designates Theaetetus, etc. Considered as statements about the usage of terms, they express matters of fact, but, properly understood, they are semantic rules. Taking the signs as interpreted signs – symbols, in the sense of Wittgenstein's *Tractarian* distinction between a sign and a symbol, there is, in a clear sense, an *internal* or *logical* relation involved in such rules. (1)–(3) express *formal* or *logical truths*, since the *symbols*, not signs, would not be the symbols they are without representing what they represent. This incorporates a 'direct reference' account of proper names and the direct representation of properties and relations by primitive predicates. This was involved in Russell's notion of a "logically proper name" or label that functioned like a demonstrative, as opposed to a definite description that 'denoted' indirectly, via the predicates in the descriptive phrase. In the last decades of the century, with the decline of interest in and knowledge of the work of major early twentieth-century figures, petty debates have erupted about priority. One of the most absurd concerns whether Barcan or Kripke originated Russell's account, which was set out in the first decade of the century and adopted by many since. The absurdity has been compounded by the misleading linking of Russell with Frege in what some speak of as the 'Frege–Russell' account of proper names, which ignores Russell's attack on Frege's account in the classic "On Denoting" (Russell 1956a; Hochberg 1984). The direct reference account was ontologically significant for Russell and others who took the primitive nonlogical constants (logically proper names and predicates), representing particulars and properties (relations) respectively, to provide the ontological commitments of the schema (Bergmann 1947; Hochberg 1957). This contrasted with Quine's taking quantification as the key to ontological commitment – "to be is to be the value of a variable" – which allows a schema limited to first order logic to contain primitive predicates while avoiding properties, by fiat. That fits Quine's replacing proper names by definite descriptions, involving either primitive or defined predicates. For one only then makes ontological *claims* by means of variables and quantifiers, and predicates retain ontological innocence (Quine, 1939, 1953). If primitive predicates involve ontological commitments, as in Carnap's (2), attempting to eliminate all directly referring signs via descriptions faces an obvious vicious regress, aside from employing an *ad hoc* and arbitrary criterion.

Wittgenstein simply ignored the problem about (3) by giving (1) and (2) the role of (3), as Russell was to do in the 1920s under his influence. This was covered over by his speaking of the 'possibilities' of combination being 'internal' or 'essential' properties of the 'objects' that were combined. Carnap's (3), which articulates Moore's view, makes explicit reference to a possible fact or situation. Russell had suggested using his theory of descriptions to avoid reference to possible facts, as well as to nonexistent objects (Russell 1905). He developed that idea in 1913 (Russell 1984; Hochberg 2000), but abandoned the book, partly due to Wittgenstein's influence. Russell replaced (3) by "'Fa' is true **iff** the fact *consisting of* Theaetetus and the property of flying exists," thereby avoiding a designation relation connecting a sentence to a purported state of affairs. What he suggested is more explicitly rendered by:

(3R) 'Fa' is true ≡ Fa ≡ E! (p)(T(a, p) & A(F, p) & *f*(φx, p)),

with 'T,' 'A,' and '*f*' for 'is a term in,' 'is attributed in,' and 'is the form of' and Ωx as the form of monadic first-order exemplification. (3R) is a tripartite biconditional that is an interpretation, but *not* designation, rule *and* a 'rule of truth,' specifying a truth maker, that avoids *possibilities* and Meinongian nonsubsistent objectives. The relations T, A, and *f* do not raise the same problem, since atomic sentences, unlike names and predicates, are not *designators*, as they are in Carnap's (3). Since they do not designate atomic sentences are not taken as names of *situations* in (3R), as Wittgenstein does take them in the *Tractatus*, despite his claim to the contrary. We can now express the non-existence of the *purported* fact that-a is F by:

(3N) '¬Fa' is true ≡ ¬Fa ≡ ¬E! (ι p)(T(a, p) & A(F, p) & *f*(φx, p)).

The question that arises is whether recognizing the class of atomic facts allows for specifying a truth maker in terms of (3N) without recognizing negative facts. One might argue we can do so since an ontological ground for taking such statements to be true is acknowledged: the class or domain of atomic facts taken as the correlate of the sentential variables. It is tempting to argue that it is no more a further fact that no such member of the class exists than it is a further fact that such a fact does exist, if the sentence 'Fa' is true. As there is no need to hold that when an atomic fact exists there is an additional fact, the fact that the atomic fact exists, there is no need to recognize the fact that an atomic fact does not exist, a negative fact, when the atomic fact does not exist. This is supposedly reinforced by recognizing that what makes a statement of class membership true or false is not a relational fact involving the relation of class membership, but simply the class itself. One can apply the same idea in the case of true negations, by taking 'Fa' to be false *given* the class of atomic facts. If the appeal to a set of facts, taken as the domain of atomic facts, is viable we can avoid negative facts. But there is a simple argument against such a view. We cannot say, where '¬Fa' is true, that *the fact* Fa does not belong to the totality or *is not* or that the fact Fa is not identical with any of the atomic facts by using 'Fa' or the expressions 'that-Fa' or 'the fact Fa' to *designate* a nonexistent fact. Rather, we can only *describe* such a fact and purport to *denote* it by a definite description to make such a claim.

The claim that the fact Fa does not belong to the class of atomic facts thus involves a description of that fact and a statement of the form '¬((ι p)(T(a, p) & A(F, p) & *f*(φx, p)) ∈ D),' and not one like '¬c ∈ {a, b}.' We cannot simply appeal to a class or domain as the truth ground for either 'The fact Fa does not exist' or '¬Fa.' For such attempts to dispense with negative facts involve implicit claims that amount to:

(N′) $(q)(q \neq (\iota\, p)(T(a, p) \,\&\, A(F, p) \,\&\, f(\Omega x, p)))$,

where the variables ‘q’ and ‘p,’ as earlier, range over existent atomic facts. (N′) serves the purpose of a list or corresponding universal disjunction or reference to the domain D, while avoiding problems raised by infinite lists or disjunctions. If ‘≠’ is a primitive sign, as *diversity* is taken by some to be *phenomenologically* basic, as opposed to *identity*, then (N′) becomes:

(N″) $(\exists_u p)((T(a, p) \,\&\, A(F, p) \,\&\, f(\Omega x, p)) \,\&\, (q)(q \neq p))$,

using the subscripted ‘u’ for ‘uniqueness.’ This will obviously not do as an expression of the truth ground for ‘¬Fa,’ since it states that *there exists* a fact, a’s being F, that is diverse from every fact. Making sense of (N″) requires accepting both existent (actual) and merely possible facts, different senses of ‘exists’ and different variables to range over such respective domains, as in some ‘free’ intensional logics. This is not acceptable to one seeking to avoid negative facts by appealing to classes or totalities. Yet, to reject diversity as basic, and treat ‘≠’ in terms of ‘¬’ and ‘=’, treats (N′) as ‘$(q)\neg(q = (\iota\, p)(T(a, p) \,\&\, A(F, p) \,\&\, f(\Omega x, p)))$’, and hence as:

$(q)\neg(\exists_u p)((T(a, p) \,\&\, A(F, p) \,\&\, f(\varphi x, p)) \,\&\, (q = p))$.

This returns us to the problematic use of an embedded negated existential claim that either repeats what we must account for or leaves us with the issue of negative facts.

The rejection of negative facts by simple appeals to classes or totalities is not viable. Accepting them, however, poses a problem as to their analysis (Hochberg 1999: 193f). But there is an alternative. Consider the following derivation, with ‘p’ and ‘q’ ranging over monadic atomic facts (thus simplifying matters by omitting reference to the form φx):

(DN)
1 $a \neq b$
2 $F \neq G$
3 $(q)(q = (\iota\, p)(T(b, p) \,\&\, A(F, p)) \vee q = (\iota\, p)(T(a, p) \,\&\, A(G, p)))$
4 $(p)(q)[\{(x)(y)(f)(g)((x = y) \,\&\, (f = g) \,\&\, T(x, p) \,\&\, A(f, p) \,\&\, T(y, q) \,\&\, A(g, q)) \equiv p = q\}]$
5 $(\exists x, y, f, g)(x = a \,\&\, y = b \,\&\, F = f \,\&\, G = g)$

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6 $\neg E!\,(\iota\, p)(T(a, p) \,\&\, A(F, p))$.

Since (4) states that monadic (first order) atomic facts are the same **iff** their constituents are the same, (DN) is a valid argument. Hence, as ‘$\neg E!\,(\iota\, p)(T(a, p) \,\&\, A(F, p))$’ is taken to be equivalent to, or a transcription of ‘¬Fa,’ we have derived the latter. For, assuming ‘$E!\,(\iota\, p)(T(a, p) \,\&\, A(F, p))$,’ we can instantiate (3) to (6) ‘Fa = Fb ∨ Fa = Ga,’ using the atomic sentences to abbreviate the corresponding descriptions of the relevant purported facts. But, by (4), (6) is false, so we arrive at ‘¬Fa’, that is ‘$\neg E!\,(\iota\, p)(T(a, p) \,\&\, A(F, p))$.’ We thus ‘ground’ the truth of ‘¬Fa’ *without* appealing to a negative fact by the use of ‘$\neg E!(\iota\, p)(T(a, p) \,\&\, A(F, p))$’ as an implicit premise. (DN) differs in this crucial way from using a generalization like '$(q)\neg(q = (\iota p)(T(a, p) \& A(F, p))$,' as a premise, to arrive at '$\neg E!\ (\iota p)(T(a, p) \& A(F, p))$.' In the latter case, since the premise and conclusion are trivially equivalent, we merely assume the negation to be derived and thereby acknowledge, rather than avoid, negative facts. But (DN) requires (1) and (2), which can be taken as recognizing basic and specific facts of diversity. This raises two issues: Is diversity or identity the fundamental concept? Are facts of diversity, or denials of identity, negative facts? In any case, (DN) can be seen as illustrating a sense in which Plato was right.

3 Logical Truth, Modality, and Ontology

We avoid conjunctive facts since '$p, q \models p \& q$' justifies taking the facts that ground the truth of the conjuncts as the truth makers for a true conjunction. But what ontologically grounds logical entailments and logical truths? To hold there is no ground can lead one to follow logical positivists and rule out the question as a pseudo-question, along with other 'metaphysical' questions, and to viewing logic as a matter of 'convention' or as involving only 'internal' questions relative to a system. The conventionalist move has variants other than the Viennese one. There is the French fashion that includes 'being responsible for your own birth,' as *we* impose our concept of 'birth' on Being (Sartre), and "your child not being your child without language" (Lacan), and the anglo-American variants emphasizing 'world making,' 'ways of life,' 'webs of belief,' 'rules,' 'normative aspects' and 'social contexts.' All of them, linked one way or another to holism and German Idealism, have a hollow ring, as does Hume's speaking of the 'necessity' of logical entailment in terms of a psychological *determination* to proceed from one idea to another. Employing model theory (set theory) to provide 'semantics' for logical systems does not change the basic issue, despite familiar problems that lead some to believe that logic rests on axiomatic systems that require 'arbitrary' (hence conventional) restrictions to avoid paradoxes. Neither positivism nor conventionalism fits the obvious fact that coherent discussion of the issues assumes fundamental and familiar logical truths and rules. In a different context, Moore expressed the basic theme behind the logical realist's rejection of the three-headed Hydra of conventionalism–idealism–psychologism: the task is not to prove the obvious but to clarify the grounds for it being so.

Ontologically grounding logical truth is traditionally linked with the explication of 'necessity' and 'possibility' and the question of whether there are necessities other than logical ones. Concern with modalities dates from Aristotle through the medieval period to the present. The logical positivists, following a theme in Russell and the early Wittgenstein, sought to explicate 'necessary truth' in terms of *logical truth*. The latter notion was sometimes considered in purely formal or 'syntactical' terms. Logical and mathematical truths were taken to be so since they were theorems of certain calculi. This led to Carnap's distinguishing 'external' from 'internal' questions and declaring the former 'pseudo-questions.' One could only consider questions about logical and mathematical truth as questions about formulae being theorems of some system. Aside from the inadequacy of such a view, given Gödel's incompleteness result (Gödel 1986; Lindström 2000), it is philosophically inadequate in a basic sense. Consider standard propositional logic, which is complete. It is a system of logical truths in virtue of the concepts of truth, falsity, and negation and the logical 'laws' that truth tables are based on. Speaking of logical or mathematical truth solely in terms of theorems of some formal system takes one nowhere.

Carnap subsequently sought to explicate the notions of logical truth, necessity, and possibility, by extending his 1942 system of semantics to modal logic. The development of modern modal logic is taken to begin with Lewis' and Langford's work on propositional modal logic – their addition of modal signs ('◊' for 'possible') to propositional calculi and employing a modal conditional of *strict implication* 'p ɜ q.' But Carnap took the modal concepts to be 'unclear' and 'vague' requiring an explication of the notion of 'logical necessity.' He sought to provide one in terms of *logical truth*, taken as a meta-linguistic 'semantical concept' (Carnap 1947: 174). It is fashionable, but more myth than fact, to date quantified modal logic from Barcan's March, 1946 paper (Hughes and Cresswell 1996: 255; Boolos 1993: 225) that was received on September 28, 1945, having been extracted from a doctoral dissertation in progress. The same journal published Carnap's paper (1946) on quantification and modalities in June, having received it November 26, 1945. Carnap's paper was based on *Meaning and Necessity*, a book he had worked on in 1942, completed a first version of it in 1943 and, after an extensive correspondence with Quine and Church, published in 1947 (Carnap 1947: vi). It was the third of a series of books on logic and semantics done in the 1940s. In both earlier works of the trilogy he mentioned his work on a system of quantified modal logic in the 1943 manuscript (Carnap 1942: 85, 92; Carnap 1943: xiv).

Carnap's 1946 paper contains one of the earliest semantics for a system of modal logic that he altered and developed in the 1947 book. Barcan's paper consists of simple derivations from assumptions. One *assumption*, the *Barcan formula*, was among the *theorems* for which Carnap offered semantical proofs in 1946 and 1947. Semantics for modal systems, of the general kind now associated with Kripke's name, occur earlier in the work of Kanger (1957) and Bayart, while Carnap is often said to have 'anticipated' them. Based on a theme in the *Tractatus*, reflecting the idea that a necessary truth is true in all possible worlds, Carnap introduced 'state descriptions,' as sets or lists of *sentences*, which we can consider, in terms of the miniature model we used for discussing negation, as sets of *possibilities*. In our simple case there are the sets: {Fa, Gb, ¬Fb, ¬Ga}, {Fa, ¬Gb, Fb, ¬Ga}, etc. While 'Fa' would be true for only some such sets or 'worlds,' 'Fa ∨ ¬Fa' would be true in all, and hence *necessarily* true. Thus '**N**(p ∨ ¬p)' is a theorem by a *rule* for '**N**' ('necessary'). Carnap's system is, in effect, the one known as S_5, obtained by the addition of '◊p ⊃ **N**◊p' to S_4, 'characterized' by '**N**p ⊃ **N**(**N**p)'. One adds these to a system, often called 'T', usually obtained by taking '**N**(p ⊃ q) ⊃ (**N**p ⊃ **N**q)', '**N**p ⊃ p' and all valid formulae of standard propositional logic as axioms, along with rules like substitution, *modus ponens*, and 'necessitation' (if a formula is a theorem then the formula preceded by '**N**' also is).

In S5 the modal concepts are not relativized to a possible world. The essential conceptual change made after Carnap was that the modal concepts were relativized, so certain 'things' (including 'worlds') were possible *relative to* some possible worlds but not others. For example, a 'world,' w, with domain {a, b} can be said not to be 'accessible from' one, w*, with domain {b}, and Fa is not then 'possible' relative to w* (whether 'Fa' is then rejected as a formula, taken to be without a truth value, etc. is irrelevant). This illustrates the simple idea behind the later modifications of Carnap's semantics that led to formally characterizing different modal systems in terms of logical characteristics (transitivity, symmetry, etc.) of a relation, on the set of worlds, and constructing models for alternatives to S5, such as S4, T, etc. But, from an ontological point of view, we merely have various axiom systems about unexplicated and ungrounded modal concepts or overly rich ontologies (if one speaks literally of 'worlds'), though different systems appear to fit, more or less, different uses of 'necessity' and 'possibility' – logical necessity, causal necessity, etc. One problematic mutation has been the construal of causal notions in terms of a primitive counter-factual relation, p□→q (had p occurred q would have), and a triadic *similarity* relation, S, between possible worlds – w is more similar to (closer to) w′ than w″ is – where conditions for S provide a 'semantics' for '□→'. Such attempts are notoriously vague, either turning in transparent circles by illicitly employing causal notions or introducing arbitrary stipulations (conditions) relativizing S (closer in what way?). The appeal to possible worlds as entities is often denied by claiming that talk of such worlds is merely a way of speaking, as Moore once said about his referring to nonexistent facts. But philosophical honesty requires literal talk or the admission that one merely speaks fancifully about linguistic structures, models, and connections among them. Recent revivals of Carnap's construal of state descriptions as sets of sentences take the form of construing possible worlds as sets of sentences. This leads some to think that, as sets are 'abstract entitites,' they deal with 'metaphysics' and ontology. Such pointless patterns invariably involve problematic uses of the term 'sentence' and ignore the fact that atomic sentences require interpretation rules like Carnap's (3). Such rules introduce the basic problems posed by possible facts and possible worlds that are obviously not resolved by talking about sets of sentences. *Modes* of facts (*possibility*, *actuality*) and possible facts can ontologically ground talk of possible worlds, taken as sets of facts where at least one element is a *possible* but not *actual* fact. But such modes, as modalities, are neither clarified nor codified by the various modal logics. The same is true of (1) the use of 'possible' involved in considering the possibility of further objects, properties, and worlds; (2) the sense of 'possible' in the phrase 'possible world'; and (3) the 'modality' involved in categorial *necessities* – F necessarily being a property, etc. – based on *exemplification* and presupposed by standard systems of logic as well as modal systems. The philosophical problems posed by modal logics might have led to their demise but for connections to intuitionistic logic (Gödel 1933) and 'reinterpretations' of '**N**' and '◊' in terms of *provability* and *consistency*, in the development of a logic of *provability* related to Gödel's incompleteness results (Boolos 1993; Lindström 1996).

Predication and the categorially *necessary* distinction between terms and attributes are at the core of two logical paradoxes – the Bradley–Frege paradox and the Russell paradox. Both stem from mistaken ontological analyses of *exemplification* and confusing properties with propositional functions. The first results from taking exemplification as a relation connecting a term (terms) and a property (relation) to form a fact (or proposition). It then seems that a further relation must connect the exemplification relation itself to the term(s) and property (relation), and so on *ad infinitum*. This led

Frege, and later Russell in the case of relations (sometimes properties as well), to insist that no such relation was needed as 'concepts' were 'incomplete' or 'unsaturated' functions that required being completed rather than being connected to terms (arguments). It led Bradley to hold that exemplification was paradoxical and Sellars to argue that realism about properties and relations was thereby refuted. The problem, which bears on an earlier discussion, disappears on the analysis employing (3R). Consider the fact $(\iota\, p)(T(a, p)\ \&\ A(F, p)\ \&\ f(\varphi x, p))$. No *connection* of exemplification is involved. Monadic exemplification is a *logical form*, Øx. We also have recognized *logical relations*, T, A and f, between a fact and its term, attribute, and logical form, *but they are not exemplified* and do not give rise to *further facts* or *possibilities*. A relation of exemplification is not illicitly used in clauses like 'T(a, p)', as there is no further fact that a and the described fact *exemplify* T. For, by Russell's theory of descriptions, '$T(a, (\iota\, p)(T(a, p)\ \&\ A(F, p)\ \&\ f(\varphi x, p)))$' – a stands in T to the fact that a is F – reduces to '$E!\ (\iota\, p)(T(a, p)\ \&\ A(F, p)\ \&\ f(\varphi x, p))$' – the fact that a is F exists. This is why T, A, and f can be said to be *logical relations* without simply dismissing the problem. They are not relations that combine terms into further facts. Hence no regresses arise. The point can be reinforced. Assume the fact exists and we 'name' it '[Fa]'. 'T(a, [Fa])' is true since [Fa] exists, and not in virtue of a further relational fact, as '$a \in \{a, b\}$' is true given $\{a, b\}$. The point also applies to *part-whole* relations and mereological calculi.

The Russell paradox for properties arises from taking $\neg\varphi\varphi$ (non-self-exemplification) as a property, the so-called Russell property, and deriving a paradox from asserting that such a property exists. Thus one avoids it by avoiding the existential claim. But there is a further point. One need not even consider such an existential claim, since '$\neg\varphi\varphi$' is a dyadic *abstract*. Thus the attempt at self-predication involves the purported sentence '$\neg\varphi\varphi(\neg\varphi\varphi)$'. Since '$\neg\varphi\varphi$' is dyadic, as are '$\varphi x$' and '$x = x$', even when the terms are the *same*, '$\neg\varphi\varphi(\neg\varphi\varphi)$' is not even well-formed, being like 'R(R)' where 'R' is a dyadic predicate. To consider '$\neg\varphi\varphi(\neg\varphi\varphi, \neg\varphi\varphi)$' is pointless. For, to arrive at the familiar paradox, one must employ a conversion rule that allows replacing both occurrences of the variable 'φ' in the *predicate* '$\neg\varphi\varphi$' by the *subject* sign '$\neg\varphi\varphi$' to arrive at '$\neg\neg\varphi\varphi(\neg\varphi\varphi)$'. But that involves replacing a monadic predicate variable by a dyadic predicate, which mixes logical 'types,' in one of Russell's unproblematic uses of 'type.' This is so irrespective of illicitly assuming that '$\varphi\varphi$' and '$\neg\varphi\varphi$' represent relations or properties or forms – as some pointlessly and problematically take '$x = x$' and '$\neg x = x$' to represent dyadic relations or monadic properties – of *self-identity* and *non-self-identity*. In the latter case we have, at most, the *dyadic relations* of identity and diversity. Such philosophically problematic moves are aided by the formal device of forming 'abstracts' – as in the case of lambda-abstracts. Thus, using '$(\lambda x,y)(x = y)$' for the identity relation or 'function,' one forms '$(\lambda x)(x = x)$' to represent the monadic function of self-identity. One then easily moves to '$(\lambda\varphi)\varphi\varphi$' and '$(\lambda\varphi)\neg\varphi\varphi$' to arrive at the purported Russell 'property' or function. The device is misleading, first, as such functions cannot be confused with properties, and, second, as forming such signs has no ontological significance whatsoever, unless one postulates that corresponding, and problematic, entities exist. Such issues aside, the basic distinction between monadic and dyadic predicates prevents the Russell paradox for properties without resorting to a hierarchy of types or similar restriction, which removes a ground for claiming arbitrary restrictions are required to avoid logical paradoxes.

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