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Symbolic Logic and Natural Language

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Initially the connection between the formal notation of symbolic logic and ordinary sentences of natural language might seem opaque. Why on earth would anyone want to draw a parallel between the technical and abstract endeavors of formal logicians and what seems more properly an object of study for linguists? However, it has been a common assumption of twentieth-century Anglo-American philosophy that symbolic logic can reveal something important about language. The reasons for this assumption are, in actual fact, not very hard to see.

Arguments (1) and (2) are deductively valid inasmuch as it is impossible for their premises (their first two sentences) to be true and their conclusions (their last sentence) false:

(1) If the Yankees won, then there will be a parade.
The Yankees won.
So, there will be a parade.

(2) If Socrates is a man, then he is mortal.
Socrates is a man.
So, he is mortal.

Moreover, the reason that (1) is valid does not seem to be independent of the reason that (2) is: both seem valid because they share a common form. Each begins with a conditional statement, followed by another premise that asserts the condition part (the *antecedent*) of the first premise, and concludes with its *consequent* part. By virtue of sharing this form, both arguments (and countless others) are not only valid but are valid *in virtue of this shared form*.

Though (1) is about the Yankees and parades, and (2) is about men and their mortality, when our concern is with inference (i.e. issues about which sentences can be validly deduced, or 'follow,' from which others), it seems best to abstract away from any particular *content* and concentrate instead on *structure*. The structure underlying an argument (and hence, the structure underlying the sentences making up that argument) in virtue of which it has its inferential properties is known as its *logical form*. We arrive at statements of logical form by replacing certain expressions (so-called nonlogical ones) with schematic letters and combining these with symbolic representations of the logical components of the argument;[1] for instance, (1) and (2) share the logical form:

$A \supset B$
A
$\therefore B$[2]

(with '$\supset$' representing the logical component 'if . . . then,' 'A' and 'B' standing for propositional claims, and '$\therefore$' indicating the conclusion). The logical representation of a sentence then might be thought of as *a structure that determines from which sentences it can be validly deduced, and which sentences can be validly deduced from it and other premises*.

The notion of logical form has become commonplace in philosophical discussions of language (at least in the analytic tradition), but theorists are not always explicit about the kind of relationship they envisage between natural language sentences and statements in logical form, or about the role they expect such symbolizations to be playing. Our aim in this chapter, then, is to explore these questions; in Section 1 we will concentrate on the question of constraints on logical representations, while in Section 2 we concentrate on the nature of the relationship between natural language and logical form.

1 What are the Constraints on Formal Representations?

Given what we have said so far, the only constraint that must be respected in mapping natural language sentences onto a symbolic notation is that whatever form we assign to a sentence, relative to an argument, must underwrite the logical properties of that argument. However, this condition can lead to some *prima facie* surprising results; to see this, let's look at Frege's system of predicate logic. Frege's logical system was designed to be able to cope with the sort of generality evidenced in sentences like 'All men are mortal' or 'Some girl is happy' (i.e. claims which tell us about the range of objects in the world which possess certain properties, rather than telling us any particulars about specific objects which possess those properties). He attempted to achieve this end with two explicit quantifier symbols, '$\forall$,' '$\exists$' (introduced to stand for the English counterparts 'all' and 'some' respectively), which could combine with predicates (e.g. 'is a man') and variables (given by lower case letters from the end of the alphabet like 'x' or 'y') in order to represent general claims.

A standard practice for representing a universal sentence like 'All men are mortal' in the language of predicate logic is as '$(\forall x)(\mathrm{Man}(x) \supset \mathrm{Mortal}(x))$'; which says in 'logicians' English': *for all things, x, if x is a man then x is mortal*. Although such a claim, if true, entails something about individual males, it does not assert anything about one particular man. If corresponding representations for 'Socrates is a man' and 'Socrates is mortal' render the inference from the first two sentences to the third in argument (2) valid in virtue of form, our techniques have achieved their end.

Suppose, though, that someone complains about the occurrence of the symbol '$\supset$', the notational counterpart, recall, for an English conditional (typically, an 'if . . . then' statement). Unlike the first premises in (1) and (2), the universal English sentence 'All men are mortal' makes no (overt) mention of conditionality, so why should its symbolic representation do so? This is one respect in which we may find the Fregean representation of natural language sentences surprising.

A second area of divergence between the surface appearance of natural language sentences and their formal representations in Fregean logic comes with respect to numerical claims like 'One girl is happy' or 'Fifteen men are mortal.' Frege's suggestion is that, for all such counting quantifier expressions (like 'one' or 'fifteen'), we use combinations of '$\forall$' and '$\exists$' claims to deliver the logical forms of sentences containing them. So, for example, we can use an instance of the existential quantifier '$\exists$' to symbolically represent that there is (at least) one thing satisfying the given predicate (e.g. '$(\exists x)(Man(x))$'). If we introduce another instance of the existential quantifier (e.g. '$(\exists y)(Man(y))$') and then state that the two existential claims are about distinct objects (e.g. by using a non-identity claim '$y \neq x$'), the final product, symbolized as '$(\exists x)(\exists y)((Man(x)\ \&\ Man(y))\ \&\ y \neq x)$' can be used to symbolize the English sentence 'There are (at least) two men.'

Obviously, we can go on 'counting' indefinitely, simply by introducing more existential quantifiers and more non-identities to those already introduced. This might seem a rather laborious way of symbolically representing numerical claims, especially those involving large numbers (imagine the length of the logical representation of 'One hundred and one Dalmatians came home' on this model!); but the technique does allow the Fregean system to formulate many more quantificational claims than we might have envisaged at first, particularly given the limited base of '$\forall$' and '$\exists$'. Despite containing only two basic quantifier expressions, the Fregean system can express, and therefore, formalize, any natural language claim involving a counting quantifier. In short, though Frege's system may introduce more parsimony than the project of codifying logical inferences asks for or demands, if it achieves this end (i.e. if it captures all the inferences that need to be captured), it's hard to see what project is jeopardized by doing it with a minimum of logical symbols.

The conditional form of universal statements in predicate logic, and the complexity of statements involving count quantifiers, might be surprising to us but nothing so far said would require withdrawing our proposed symbolizations based on this sort of consideration. We have been assuming that symbolic representations function merely to codify logical properties and relations involving natural language sentences. If these symbolic representations contain elements not obvious in their natural language counterparts, why should it matter as long as the right inferences are captured in virtue of these assigned forms?

One reason for concern will be addressed in Section 2; for the moment we'll assume that the only self-evident constraint on an adequate symbolization is that it captures correct logical inferences. It should be obvious that this condition can serve to rule out certain suggestions about the logical form of natural language sentences – those which fail to preserve logical inferences will be ruled out. However, it may also turn out that this constraint is insufficient to chose between alternative logical renditions of a natural language sentence; and when this happens, we might, perhaps, expect there to be further constraints which come into play to help us choose. To see this, let's consider a particular example: for in the realm of definite descriptions we can see both the constraint to capture logical inferences and the potential need for an additional constraint in play. Questions about the appropriateness of any such additional constraint will then lead us, in Section 2, to consider how we should construe the relationship between natural language sentences and logical form.

Case study: Representing definite descriptions

The idea we will explore in this section is as follows: perhaps finding an adequate symbolic notation for natural language is difficult not because of an absence of *any* symbolic system which looks like it might be up to the job, but because of a *surplus*, each of which is *prima facie* promising. One thought might be that what we need, when faced with alternatives, is a way to choose amongst them. We can tie down the main point here with reference to a well-explored example, viz., definite descriptions. These are expressions of the form 'the F,' where 'F' is a complex or simple common noun, as in 'The woman' or 'The woman who lived in New Jersey.' These expressions have been a focus for philosophical logicians, in part because of divergent intuitions about their linguistic status, and accordingly, about which logical inferences they participate in. One can find in the literature a myriad of different accounts of the logical form of definite descriptions, but we'd like to explore just three which will help to demonstrate the constraints involved in a choice of symbolization.

The first proposal is Frege's, who treated definite descriptions as members of his class of referring terms. The details of his larger philosophy of language are inessential here; what matters is that, according to him, definite descriptions are akin both to names (like 'Bill Clinton' or 'Gottlob Frege') and indexical expressions (like 'I,' 'you' and 'today,' which depend on a context of utterance for a referent).[3] Each of these expressions is treated identically within his system: each is assigned a designator which appears in predicate assignments. 'I am happy,' 'Bill Clinton is happy' and 'The president of the US is happy' can all be symbolized in Frege's notation as 'Ha' (with 'H' symbolizing the predicate 'is happy,' and 'a' designating the object picked out by each referring term).

Famously, Russell, disputed Frege's analysis, arguing that definite descriptions are not proper names, but instead belong to an alternative logical category in Frege's system: viz., the class of quantifier expressions.[4] At first his suggestion might seem odd, for the Fregean quantifiers were explicitly introduced to play roles equivalent to 'all' and 'some,' and *prima facie*, whatever the role of 'the' in our language, it isn't playing either of these. However, Russell's contention is that we can symbolically represent definite descriptions as complex entities constructed out of these two primitive Fregean quantifiers. That is to say, he suggests that we can treat the definite article 'the' in a way analogous to the account Frege gave for counting quantifiers like 'two.'

Informally, a sentence of the form 'The F is G' is represented, according to Russell, as making a uniqueness claim, viz., there is one and only one F that is G.[5] So a sentence of the form 'The tallest man is happy' will be analysed as stating:

(3) There is a tallest man; and
(4) there is only one tallest man; and
(5) whoever he is he is happy.

Collectively these three claims are symbolized within Frege's logical system as (DD):

(DD) $(\exists x)(\text{Tallest man } (x) \,\&\, (\forall y)((\text{Tallest man } (y) \supset y = x) \,\&\, \text{Happy}(x)))$

Whereas a sentence containing a genuine referring term is represented by Frege with a simple formula (as in 'Fa'), for Russell, a sentence with a definite description requires a complex logical symbolization like (DD).

Finally, let's introduce a third relatively recent development in quantification theory, which gives us our last account of the logical form of descriptions. Many contemporary theories of quantification, such as the 'Generalized Quantifier' (hereinafter, GQ) theory of Higginbotham and May (1981), and Barwise and Cooper (1981), recommend altering the way the relationship between a quantifier expression and the rest of the sentence it appears in is handled, as well as acknowledging many more primitive quantifier expressions than the two inherited from Frege.[6] The first point relates to a feature of Fregean quantification we noted at the outset: viz., that it needs to introduce the logical connective '⊃' into the formal representations of sentences containing 'all'. The reason for this is that the Fregean quantifiers '∀' and '∃' are *unary* or 'free-standing' expressions: they act autonomously to bind a variable, which can then go on to appear in property assignments. It is this independent nature of the quantifier which leads to the need for a logical connective: we treat 'All men are mortal' as containing a free-standing quantifier – '∀(x)' – and then say of the variable which appears next to (and hence which is bound by) the quantifier that: 'if it is a man, then it is mortal.'

However, advocates of a theory of quantification like GQ reject this autonomy for quantifier expressions; they maintain that quantifier expressions are ineliminably bound to the common noun they modify. That is to say, rather than treating 'all' and 'men' as separable units within the logical form of 'all men are mortal,' they suggest we should treat 'all men' as a single, indissoluble unit, which acts together to bind a variable which then appears in the predicate assignment 'is mortal.' In the GQ system of quantification, then, this kind of claim can be represented along the following lines: '[All (x): Man (x)] Mortal (x).' On this kind of model, quantifier expressions are said to be *binary* or *restricted*, requiring a common noun to act in tandem with a quantifier to bind a variable.

Unlike predicate logic, GQ is a second-order logical system: roughly, this means that the objects quantifiers are taken to range over are sets (of objects), rather than their constituents (i.e. the objects themselves). Logical rules for GQ quantifiers are given in terms of numerical relations between sets; for example a GQ quantifier might tell us about the number of objects in common between two sets (i.e. the set of objects in the intersection of two sets). The intuitive idea here is easy enough to see: for instance, the sentence 'All men are mortal' can be understood as telling us that there is no object which belongs to the first set, that is the set satisfying the general term 'men,' which does not also belong to the second set, that is the set of things satisfying the general term 'mortal.' In other words, the number of men that are non-mortal is zero. The GQ rule for 'all' captures this numerical claim: take X to be the set of 'F'-things and Y to be the set of 'G'-things, then a sentence of the form 'All F's are G' is true just in case there are zero objects left over when you take the set Y away from the set X (i.e. that everything in X is also in Y). Similarly, for a quantifier like 'some'; the GQ rule for 'some' is that a sentence like 'Some man is mortal' is true just in case the number of objects in the intersection of the set of men and the set of mortal things is (greater than or equal to) one.

This leads us on to a second area of difference between GQ and predicate logic relevant for our concerns, for GQ theorists reject Frege's technique for handling counting quantifiers. Rather than analysing expressions like 'three' and 'nine' (which seem to play the grammatical and inferential roles of quantifiers), with complex combinations of '∀' and '∃' statements, GQ theory introduces symbols for them in the formal language. For instance, it represents the quantifier 'three' by requiring that (at least) three objects fall within the intersection of two sets X and Y in order for 'Three F's are G' to be true. The result is that GQ contains a logical element for each numerical expression in the natural language that can modify a count noun. Again, however, if GQ is capable of capturing all relevant inferential properties, no *a priori* reason exists for resisting introducing additional logical items (with their additional rules of inference – a technical topic we do not need to discuss here).

Advocates of GQ can, then, agree with Russell (in opposition to Frege) that definite descriptions are best represented as quantifier phrases, yet disagree that their best symbolic representation is given by anything like (DD). The definite article 'the' in GQ is symbolically represented by its own quantifier, which for ease of translation we might represent by the symbol '[The x]'. '[The x: Fx] Gx' is true just in case exactly one object lies in the intersection of the 'F' and 'G' sets. This end result is similar to Russell, for both systems treat phrases of the form 'The F is G' as being true just in case there is exactly one thing which is F and it is also G;[7] but the GQ theorist can obtain this same semantic result without treating the natural language phrase 'the' as possessing a complex, multiply quantified logical form.

To recap: we now have three distinct proposals for symbolizing sentences with definite descriptions: the Fregean treatment, in which they are handled as akin to sentences with proper names; the Russellian analysis where they are treated as combinations of universally and existentially quantified claims; and GQ, where the definite article is treated as a quantifier phrase, which requires a common noun to be complete, and which maps on to its own unique element in the formal language. The question now is: 'how do we decide between all these alternative accounts?'

Recall, first, our initial adequacy constraint on symbolic representations: viz., that they capture logically valid inferences involving the expression in question. One way of understanding the objections Russell leveled at Frege's account of definite descriptions, then, is that the latter's proposal fails this constraint (i.e. there are logically valid inferences Frege's notation fails to capture by virtue of symbolizing definite descriptions as singular terms). For instance, in 'Everyone wants John,' the quantifier expression 'everyone' is its subject, 'John' its object, and 'wants' its transitive verb. This sentence is unambiguous, having only one possible translation into the formal system of predicate logic. The sentence 'Everyone wants the winner,' on the Fregean assumption that 'the winner' is a referring term, ought then to be unambiguous as well. Both should be symbolically representable in predicate logic as 'Rab.' But the definite description sentence is ambiguous; it has two readings, one in which there is a particular person everyone wants, and another where everyone wants whoever has the property of being the winner, regardless of whom he or she turns out to be.[8]

The difference between these readings is sometimes indicated by saying that in one the description takes *wide scope* over the rest of the sentence, and in the other it takes *small scope*. This feature of definite descriptions – that they enter into what we might call 'scope ambiguities' – likens them more to quantifier expressions (since it is a hallmark of expressions containing 'all' or 'some' that they display scopal ambiguity) and less to singular referring terms. Indeed, if we symbolically represent them as singular referring terms (as Frege did), we have no way to explain this logical phenomenon.[9] In short, Frege's treatment of definite descriptions is flawed; to capture all the inferential properties of sentences with the expression 'the F' we need to assign it more structure than the Fregean analysis does. Thus a quantificational theory of descriptions is preferable over the Fregean approach; but what are we to say about the debate between the Russellian and GQ theorist? Since the two approaches *agree* about inferential properties expressions of the form 'The F is G' possess, their disagreement cannot emerge from the failure of either approach to accommodate such inferential properties. Instead, it seems the GQ theorist assumes that it is permissible to invoke wider features of our symbolization to decide between competing approaches. That is to say, GQ theorists object to the Russellian approach to definite descriptions on the grounds that Fregean logic is inadequate for formalizing natural language as a whole. To see why the advocate of GQ might think this, we need now to take a slight diversion through the analysis of quantifier phrases, before returning again to the issue of definite descriptions.

An initial point GQ theorists have pressed in their favor is that other expressions in natural language look intuitively to be playing the same logical role as 'all' or 'some' (or 'the'), but provably resist analysis in terms of the primitive Fregean quantifiers '∀' and '∃'. The problem is that, although any quantifier making a specific numerical claim can be logically captured by a complex construction of Fregean quantifiers, some quantificational elements in natural language make no such claims. Consider 'many,' 'most,' and 'few'. These quantifiers are like traditional Fregean quantifiers inasmuch as sentences like 'All men are mortal' and 'Most men are mortal' seem to share grammatical makeup, and convey general claims about the extension of certain properties, rather than making referential claims about a particular individual. Furthermore, both apparently display the same kinds of ambiguity in linguistic contexts when nested inside other quantifiers. 'Every boy loves many girls' is ambiguous between there being one single privileged set containing many girls which are loved by all boys, and it being the case that, for each boy, there are many girls he loves, though each boy may love a different set of girls. Since these expressions intuitively seem so much like those expressions that Frege originally chose to symbolically represent as quantifiers, why not treat them as such? But how can we accomplish this end armed only with '∀' and '∃'?

To see the problem that the Fregean system faces, let's run through its options for an expression like 'most'. First, we might try representing 'Most girls are happy' with either (6) or (7), thereby equating 'most' with one of the two existing quantifier phrases:

(6) $(\exists x)(\mathrm{Girl}(x)\ \&\ \mathrm{Happy}(x))$
(7) $(\forall x)(\mathrm{Girl}(x) \supset \mathrm{Happy}(x))$

(6) states only that some girl is happy, and (7) that all girls are happy, and neither of these is what we need. (6) doesn't even logically imply the 'most' statement, and the 'most' statement does not logically imply (7).

Alternatively, we might try representing 'most' as expressing a specific numerical claim, since we know that expressions making these sorts of claims can be captured by complex combinations of '$\forall$' and '$\exists$'. Perhaps 'most' tells us that some specific number of happy girls is greater than the number of unhappy girls; for example (8).

(8) $(\exists x)(\exists y)((Girl(x) \,\&\, Happy(x)) \,\&\, (Girl(y) \,\&\, Happy(y)) \,\&\, x \neq y) \,\&\, (\exists z)((Girl(z) \,\&\, {-}Happy(z)) \,\&\, (\forall w)((Girl(w) \,\&\, {-}Happy(w)) \supset w = z)))$

(8) states that there are at least two happy girls and only one unhappy girl; but intuitively, our original sentence does not logically imply (8). (8) provides a circumstance in which our original sentence would be true, but it does not adequately logically capture what the original sentence means (after all, 'Most girls are happy' would also be true if five girls were happy and one unhappy, and in countless other situations as well).

So, neither '$\forall$', nor '$\exists$', nor some combination of them, seems adequate for capturing 'most'; but now we are in a position to see that the problem lies not merely in our limited range of quantifiers, but in the very form that Fregean quantifiers take. The problem is that in order to logically represent 'most' correctly we need to see it as having an intimate connection to the common noun it appears concatenated with (i.e. 'girls' in 'most girls'). Unlike with 'all' and 'some,' we cannot simply 'hive off' the quantifier expression for analysis (as the Fregean system does) and see it as binding a variable which then appears in predicate assignments, tied together by one of our sentential connectives. We can see that this is so by allowing the advocate of predicate logic to introduce a brand new quantifier expression, to add to '$\forall$' and '$\exists$'.

Let's use the symbol 'Σ' and simply stipulate that it stands for 'most'. However, although we are extending the Fregean system by one new quantifier, we will retain the general picture of how quantifiers and predicates relate; that is to say, 'Σ', like '$\forall$' and '$\exists$', will be a unary (free standing) quantifier. So with 'Σ' we can construct the following kinds of formulae:

(9) $(\Sigma x)(Girl(x) \,\&\, Happy(x))$

(10) $(\Sigma x)(Girl(x) \supset Happy(x))$

The problem with this suggestion is, first, that (9) states that 'Most things (in the world?) are happy girls', a sentence which is false just in case girls are not the largest set of objects in the world; so (9) seems an incorrect analysis of our original sentence. Sentence (10), on the other hand, states 'Most things are, if girls, then happy', and the logical rule for conditional statements tells us that if its first claim (its antecedent) is false, then the whole 'if . . . then . . .' claim will be true (regardless of the truth or falsity of the second claim, the consequent). Yet the antecedent in (10) will be false on almost all occasions, for what it claims is that given most objects, they are girls. So, if the majority of objects are not girls, this is sufficient to falsify the antecedent claim, and this in turn is sufficient to make the whole conditional claim true. So (10) turns out to be true just in case girls do not form the majority of objects in the domain; on this construal, 'Most girls are unhappy' turns out to be true as well!

The problem with both (9) and (10) is that they issue in claims of truth or falsehood based on considerations about the wrong sets of objects: (9) is false and (10) true just in case there are less girls than boys and boats and trains, etc., all combined. Yet we wanted a much more specific condition for the truth or falsehood of our original claim, viz., that more *girls* be happy than unhappy.

What the failure of (9) and (10) demonstrates is that we *cannot* symbolically represent 'Most girls are happy' as containing two acts of predication, bound together by a sentential truth-functional connective, and concerning a variable previously bound by a distinct quantifier. Instead, what we need is to represent one predicate as an ineliminable part of the quantifier expression itself. Suppose we treat 'most girls' as an indissoluble unit that binds a variable *then* available for the predicate assignments 'are happy.' Then we can formulate a sentence like 'Most girls are happy' as: [Most (x): Girls (x)] Happy(x)', which yields precisely the interpretation we were after – it tells us that, given the set of girls, the majority of this set are happy. However, to adopt this kind of proposal is precisely to reject the Fregean form of quantification for sentences involving 'most,' in favor of something like the GQ proposal which treats quantifiers as binary expressions (i.e. as requiring both a quantifier phrase, like 'most,' *and* a common noun to yield a complete expression).

Returning, finally, to the central debate about definite descriptions, we are ready to draw a moral for logically representing these expressions. Advocates of GQ argue that since English has expressions which logically play the same role as straightforward quantified noun phrases, and yet which *cannot* be successfully formalized using either '∀' or '∃', combined with various sentential connectives, we must reject the Fregean system of quantification as inadequate for capturing logical inferences in natural language. Since *some* intuitively quantified expressions in natural language require a non-Fregean system of quantification, the conclusion drawn is that *all* quantified expressions in natural language require a non-Fregean system.

In effect, the GQ theorist is assuming that our original constraint on an adequate formalization (viz., that it capture inferential properties of a sentence) is insufficient. In addition, the formalization must belong to a formal system adequate for symbolizing other natural language expressions of the same type. There remains the question of how to spell out the notion of 'same type,' but as a first approximation, we might appeal to similarity in grammatical distribution and inferential properties (such as whether or not the expression can be concatenated with a common noun to form a larger phrase, and whether or not the expression gives rise to scope ambiguities in suitably complex contexts, like those containing other quantifiers or intentional verbs). Because the Russellian symbolization of 'The F is G' uses a logical system inadequate for expressions of the same type, like 'Most Fs are G,' it is held to be inadequate *simplicities*, despite capturing all the logical inferences definite descriptions support in natural language. The GQ analysis of definite descriptions is therefore alleged to be preferable over its Russellian competitor, because GQ is judged preferable over the Fregean quantificational system which at most adequately treats '∀' and '∃'.

Note that, if we accept this line of argument, a traditional and persistent objection to Russell's theory of descriptions actually carries over to the formalization of all quantified claims in predicate logic. This objection is that the Russellian theory 'butchers' surface grammatical form, seeing an apparently simple sentence like 'The F is G' as possessing a vastly complex underlying content involving two distinct acts of quantification, a conditional, a conjunction and an identity claim. Yet this apparently runs counter to our intuitions about the grammar and structure of the original sentence. The GQ analysis avoids this worry, positing a simple underlying logical form; but interestingly it also suggests that this traditional objection can be leveled against other logical form claims. The GQ theorist challenges us to explain why, since we cannot represent 'Most girls are happy' as containing a conditional or conjunctive element, we should represent 'All girls are happy' and 'Some girls are happy' as containing a conditional or conjunctive element.

In response to this kind of attack, advocates of the Fregean system might question the crucial GQ assumption: why should we accept that a symbolization in a logical system, L_1, for a sentence, s_1, of a natural language, N, is adequate only if L_1 is capable of symbolizing all sentences of the same type as s_1 in such a way that the inferential properties of those sentences are preserved? The Fregean who rejects this assumption might recommend that we hold on to the predicate logic analysis for 'all,' 'some,' 'the' and other counting quantifiers, and employ a GQ analysis only for 'non-standard' quantifiers like 'most' and 'few' – ones that are probably not definable in terms of the notation of the others.

One counter-response to this, of course, is that if we had *started* our formalization of quantifier phrases by concentrating on expressions like 'most' we would have needed a GQ-type analysis from the outset, and this would have rendered the Fregean treatment otiose, since we could have handled all quantifiers within a single system of notation. Thus, the proposal that we adopt two different systems of quantification to handle the class of quantifier phrases in natural language might seem to go against some quite general philosophical principle, such as posit only the minimum set of explanatory items needed to explain the data. However, the issue here is perhaps not as settled as the GQ theorist presumes: there are technical costs involved in moving from the Fregean system to the richer GQ system (which we cannot explore here), and there may still be reasons that the Fregean can bring to light to license special treatment for 'all' and 'some.'[10] At the very least, we should note that the general philosophical principle appealed to above cuts both ways – also telling in favor of the more austere two-quantifier Fregean system, as against the 'quantifier profligacy' of GQ.

So, which approach should we adopt here? Who has got the constraints on symbolic representations right? We have seen that a minimum condition on an adequate formalization for a natural language expression is that it capture all the inferential properties of that expression; a stronger condition is that the logical language be adequate for capturing all the inferential properties of expressions of that type; and a (perhaps) maximal condition is that the logical language be adequate for capturing all the inferential properties of all the expressions of that language. Which of these conditions of adequacy we choose to accept, and how we see them as playing out in practice, will help us decide which kind of logical representations we accept.[11] However, we might begin to think now that perhaps we can simply sidestep this entire debate: for why can't we simply allow that a natural language sentence like 'the F is G' has *multiple* adequate logical representations? Why should we presume that there must be, in the end, just one single 'ideal' notational language which, in some sense, 'really' gives the logical form of natural language sentences? Considering these questions takes us on to the issue of how we should construe the relationship between logical form and natural language.

2 What is the Relationship between a Natural Language Sentence and its Formal Representation?

The debate in the previous section between Russell's theory of descriptions and a GQ analysis seemed so important because of an implicit assumption that one or other of these accounts (or, perhaps, neither) gave the unique correct logical form for the natural language expression. Indeed, this was precisely the assumption made by Russell, who held that the formal language of *Principia Mathematica* was the unique, ideal formal language within which to reveal the true underlying logic of our language.[12] It is because of this kind of assumption that the need to choose between formally equivalent representations (like the quantificational account of definite descriptions, given by Russell, and GQ) seemed so pressing. But perhaps the assumption is mistaken; indeed, it is explicitly rejected by the twentieth-century American philosopher and logician W. V. O. Quine.

Quine claims that the sole purpose in symbolically representing a natural language sentence in a regimented language is "to put the sentence into a form that admits most efficiently of logical calculation, or shows its implications and conceptual affinities most perspicuously, obviating fallacy and paradox" (Quine 1971: 452). There will be different ways of doing this even with the same system of representation – since any logically equivalent formulation will do, and there will be infinitely many such sentences. Consequently, talk of *the* logical form of a natural language sentence even within a single system of symbolic notation might be misguided. The American philosopher Donald Davidson, following Quine, sees logical form as relative to the logic of one's theory for a language (see Davidson 1984: 140).

This kind of approach avoids the central worry we have been pressing, for we have no compunction to treat 'All men are mortal' as in any sense *really* containing a conditional. It just so happens that one symbolic representation adequate for capturing inferential properties of this sentence treats it in this way. Yet a liberal approach to logical form faces its own problems; for there is one crucial aspect of our language that creates a serious worry for anyone who believes that the notation we adopt in logically regimenting our language is more a matter of taste than fact.

The productivity of natural language

One key aspect of natural language so far ignored but relevant to any question of admitting multiple logical forms (for a single sentence) is that natural languages have no upper bound on their number of non-synonymous expressions. This is because they abound with constructions that generate meaningful complex expressions out of simpler ones. Grammatical sentences can be formed in English by concatenating two sentences with either ‘and’ or ‘or’; for example (13) and (14) are concatenations of (11) and (12):

(11) John left.
(12) Mary stayed.
(13) John left *and* Mary stayed.
(14) John left *or* Mary stayed.

Our language also exploits relative clause construction to create complex expressions from simpler ones. For example, new definite descriptions can be devised from old ones by adding restrictive relative clauses on head nouns, as in (15)–(17).

(15) The man left.
(16) The man *whom* I met yesterday left.
(17) The man *whom* I met yesterday *who* was eating breakfast left.

Though we can list only *finitely* many members of any of these various classes of grammatical English constructions, a casual look should convince you that each is unbounded. New sentences are formed by conjoining old sentences; new descriptions by adding relative clauses on the head nouns of old ones. These are but a few of the devices that render our language limitless.

Obviously, after performing these operations several times, say, conjoining a few sentences or relativizing a few clauses, speakers inevitably fail to comprehend the products. This is not a feature of English, however, but merely of how our minds and memories are organized. Suppose that English speakers cannot comprehend sentences with more than seven relative clauses. Would it follow that sentences with eight relative clauses are ungrammatical? Not at all. If increased memories or processing powers allowed us to understand eight clause sentences, this would merely enhance an already intact linguistic competence.

The relevance of unbounded classes to our current debate is this: since members of each of these infinite sets of sentences stand in indefinitely many distinct logical relations to one another, no mere (finite) list can correctly or adequately complete the task of codifying the set of logical inferences of a natural language. There are too many inferences. Therefore, a theory must be devised about how to symbolically represent them, in order that various inferential relations are ‘captured’ correctly. Once we see how such theories are devised, we see why, though there may be indefinitely many logically equivalent formulations for any single natural language sentence, some are preferable over others – and not *just* on pragmatic grounds of overall simplicity.

First, note that it’s no accident that both (18) and (19) are contained in (20).

(18) John left.
(19) Mary stayed.
(20) John left *and* Mary stayed.

Indeed, it’s the non-accidental occurrences of (18) and (19) in (20) that accounts for their logical inter-relatedness. We see this in asking what is it about (20) in virtue of which it logically implies (18) and (19)? The answer is that (20) is true as a matter of meaning alone in English just in case its components, (18) and (19), are true. Indeed, generally, sentences devised from other sentences with the word 'and' have this logical property as a matter of meaning alone. An excellent candidate that explains this logical relation between conjunctions of sentences and their simpler components is meaning rule (A).

(A) A conjunction is true just in case its conjuncts are true.

With enough such rules, we can show how every complex expression bears logical relations to simpler ones (and vice versa), relative of course to logical relations among their simpler expressions, say, down to primitives – where primitive expressions stand in no logical relation to one another.

As noted above, complex nouns can also be constructed out of simpler nouns and relative clauses. From a primitive expression like 'man,' a complex expression like 'man who loves a woman' can be formed, which in turn can be used to form a more complex expression like 'man who loves a woman who hates a dog' and so on. Rules are required to show how logical properties of such complex expressions are determined by those of simpler nouns and relativizations on these nouns.

To figure out what such a rule might be, consider the primitive 'man' and the more complex 'loves a woman.' The relative pronoun 'who' can grammatically conjoin these expressions. How is the logical role of the complex 'man who loves a woman' predicted from whatever logical roles its component parts might have? The complex expression 'man who loves a woman' is true of an individual just in case that individual has its simpler components 'man' *and* 'loves a woman' true of him as well. A perfectly fine rule, then, that enables us to project from the logical roles of simpler components to those of the complex expression built up by relativization is meaning rule (R).

(R) A construction of the form – X who Y – (where X is a noun and Y is the rest of the relative clause prefaced by 'who') is *true of* an individual just in case both X and Y are true of this same individual.

It is both interesting and surprising how much (R) resembles (A). Like (A), (R) also sees the components of complexes as making a conjunctive contribution. Conjunction by itself explains that complex relativizations are true of something just in case their components are as well, and this biconditional rule suffices to explain the logical relations between the complex relativization and its simpler components.

When the project of symbolically representing natural language sentences into a formal notation is seen from the perspective of the unboundedness of natural language, and therefore, the unboundedness of inferential relations among sentences of natural language, the idea of allowing all logically equivalent notations to be employed in representing the same sentence becomes harder to swallow. Codifying known inferences might be a project independent of any particular choice of formal notation, but our logical representations must *also* make the right projections and predictions for inferential relations. For this to be possible, we need a tighter connection between natural language sentences and their formal representations than mere codification. To have any hope that a system of symbolic representation will work, we need to assume that natural language sentences *actually possess* some kind of formal structure on the basis of which we can project out to the explanations of the inferential properties of novel linguistic items.

So, it seems that the laissez faire approach to logical form, which sees it merely as a tool of codification, encounters difficulties when confronted with productivity. If we are to be able to account for the limitless nature of our language, it seems that we must posit structure inherent within natural language sentences, and the intriguing proposal is that these same structures can account for the logical properties of the sentences of the natural language. This realization reinstates the seriousness of the debate between opposing accounts of the logical form of definite descriptions which closed the first section of this chapter: it is not, it seems, sufficient for us simply to admit multiple adequate logical representations for a sentence of the form 'The F is G,' for we need to know which form captures its inherent logical form.[13] This in turn brings us back to the question of which constraints are correctly placed on our choice of logical form for a sentence and whether such constraints will guarantee a unique logical form for each sentence of natural language. Although we haven't answered these questions in this essay, we do hope to have shown why it is important to ask them. As noted earlier, the notion of logical form has become commonplace in the philosophical arena at the turn of the twenty-first century; however, if symbolic logic is really to advance our understanding of language, we need to be very clear from the outset about the relationship envisaged between the two realms.

Notes

1 Exactly how this replacement takes place will vary, however, depending on the logical system in play; for instance, propositional logic will replace each whole proposition with a schematic letter, whereas (as we will see below), a system like predicate logic will introduce schematic letters for sub-propositional elements.

2 This argument form is so common as to have a special name: *modus ponens*.

3 Referring terms were to be handled by Frege's notions of *sense* (the mode of presentation of an object) and *reference* (the object), see Frege (1879).

4 Russell (1905).

5 Of course, we need to take into account context as well. If someone says, 'The man left', what he said might be taken to be true in a context, say, where there are two women and only one man, even perhaps some small children.

6 Higginbotham and May (1981), and Barwise and Cooper (1981).

7 Not all logical representations of definite descriptions agree on the claim of uniqueness; cf. Szabo-Gendler (forthcoming).

8 We might think of the difference as turning on whether or not the property of 'being the winner' figures essentially in the agent's wanting.

9 We might also wonder how, on a Fregean analysis, we capture the valid inference from the truth of 'The man who broke the bank at Monte Carlo died' to the truth of 'Some man died.' (Replacing 'broke the bank at Monte Carlo' with any other meaningful complex noun will also underwrite the inference, and so it's an inference we want to accommodate in virtue of logical form.)

10 The technical issues concern the fact that certain properties of Frege's logical system (i.e. 'closure' and 'completeness') are lost in the move to GQ; these are quite complex technical properties that need not concern us here.

11 The latter point matters, for the advocate of GQ *could* argue that our first and second conditions collapse with respect to quantifier phrases, since sentences like 'any girl is happy' and 'some girl is happy' seem to be logically connected (the former apparently entailing the latter), and this fact could prove difficult, if not impossible, to accommodate given different methods of representation for the two sentences. Again, however, the details of this debate go beyond our present concerns.

12 A similar assumption was also made by the early Wittgenstein, though he remained agnostic on the choice of ideal language, merely holding that natural language items actually possessed some particular logical form, which would be revealed by a process of logical analysis.

13 Of course, the discussion about definite descriptions is meant as a paradigm example of the issues discussed here, not as the only case of them. For another particularly clear example, consider prepositional phrase modification and the apparently valid move from 'John buttered some toast in the kitchen' to 'John buttered some toast.' The question is: what kind of logical form might capture this inference and will the form suggested be acceptable as the genuine underlying logical form of the sentence? One suggestion for the logical form of such sentences is Davidson's 'event' approach (see Davidson 1967), which posits an 'extra place' in the logical form for an event variable; but a common objection to such an approach is that it diverges too far from the surface form of the original sentences. The debate is thus parallel to that had in the text concerning definite descriptions and the conditional representation of 'all' statements: we have approaches which predict the right inferential relations, but which may be deemed unsuitable as the 'real' logical form of the sentence in question due to divergences from surface form.

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