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Bertrand Russell (1872–1970)

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Russell was the most important British philosopher of the twentieth century. At the start of the century he helped to develop the new theories that transformed the study of logic at this time, but his greatest contribution was not to logic itself. Instead it lay in developing and demonstrating the philosophical importance of this new logic and thereby creating his "logical-analytic method," which is the basis of the analytical style of philosophy as we know it today. The result is that we can still read Russell's writings as contributions to contemporary debates. He is not yet someone whose works belong only to the history of philosophy. He lived to be nearly 100 and there is every reason to expect that some of his writings will have an active life and age at least as great as his. A classic instance is provided by his introduction to philosophy, *The Problems of Philosophy* (his "shilling shocker" as he liked to call it), which, though published in 1912, remains one of the best popular introductions to the subject.

Early life

Despite the fact that as a philosopher Russell remains almost a contemporary, in other respects his life now seems very distant from us. His family, the Russells, was one of the great Liberal families of British politics: his paternal grandfather, Lord John Russell, had been Prime Minister twice during the first half of the nineteenth century, and Russell describes meeting Mr. Gladstone several times. His parents, Viscount Amberley and his wife Kate, were friends with John Stuart Mill, who agreed to act as an honorary godfather to their young son Bertrand. Mill in fact died during the following year, too soon for Bertrand to make his acquaintance. Much more traumatic for the young child, however, was the death of both his parents soon afterwards, so that in 1876 he was left at the age of 4 in the care of his grandparents, indeed of just his grandmother after his grandfather's death in 1878. Russell described his lonely childhood in his *Autobiography*. His rebellious elder brother Frank was sent away to school, but Bertrand ("a solemn little boy in a blue velvet suit," 1967: 30) was educated at home, brought up in a constricting atmosphere whose narrow limits were fixed by his grandmother's strict Presbyterian beliefs. The only refuge that the young Bertrand found was in the privacy of his own thoughts; he kept a secret diary in code in which he set down his growing doubts about religious orthodoxy. There can be little doubt that Russell's troubled later emotional life (he had four marriages) was affected by this childhood. In his writings there are many passages in which he refers indirectly to it; for example, when writing in 1916 about the difficulties of marriage, he remarks that "The fundamental loneliness into which we are born remains untouched, and the hunger for inner companionship remains unappeased" (1916: 191).

From an early age his precocious talent in mathematics had been recognized, and in 1890 he went to Trinity College, Cambridge to study mathematics. Despite his delight in escaping from his grandmother, however, he soon found himself dissatisfied with the antiquated teaching of mathematics at Cambridge. So in 1893 he switched to the study of philosophy, a subject into which he had been initiated through membership of the Cambridge "Apostles" (a private society largely dedicated to the discussion of philosophy) and friendship with the philosopher J. M. E. McTaggart, then a young Fellow of Trinity. In 1894 he obtained a first class result in his final examinations and almost immediately started work on a dissertation in the hope of winning a prize fellowship at Trinity College. At the same time he married, and then travelled with his first wife, Alys, to Germany. In his *Autobiography* he recounts a moment of clear-minded future resolution during this honeymoon:

> During this time my intellectual ambitions were taking shape. I resolved not to adopt a profession, but to devote myself to writing. I remember a cold, bright day in early spring when I walked by myself in the Tiergarten, and made projects of future work. I thought that I would write one series of books on the philosophy of the sciences from pure mathematics to physiology, and another series of books on social questions. I hoped that the two series might ultimately meet in a synthesis at once scientific and practical. My scheme was largely inspired by Hegelian ideas. Nevertheless, I have to some extent followed it in later years, as much at any rate as could have been expected. The moment was an important and formative one as regards my purposes. (1967: 125)

This passage (though written with the benefit of hindsight) is remarkably prophetic; Russell had no settled profession (he held teaching positions for only about ten years) and for most of his life he made his living by writing, in which he had remarkable proficiency. He wrote about seventy books, which do indeed form two series: there are about twenty books of philosophy, mostly on "the philosophy of the sciences"; and many of the rest concern "social questions," though it cannot be said that the two series meet in a synthesis. His most popular book was *A History of Western Philosophy*, despite the fact that this is an unreliable and distinctly idiosyncratic book in which Russell devotes most space to ancient and medieval philosophy.

Breaking with idealism

During the 1890s the dominant school of philosophy in Cambridge, as in Britain generally, was idealist. Under McTaggart's influence Russell chose to work within a broadly idealist framework (as the Tiergarten testament quoted above shows), and the project he selected for his fellowship dissertation was that of providing a revised a priori foundation for geometry, one that would take account of the possibility of non-Euclidean geometries in a way that Kant's famous account does not. Russell argued that Kant's conception of the conditions of the possibility of experience had been too limited. What is important, and thus a priori, is that space be of a constant curvature, but it is not an a priori matter just what its curvature is – for example whether it is zero, as Euclid maintained, or positive, as Riemann proposed. Russell was duly elected to a six-year prize fellowship at Trinity College in 1895 and in 1897 he published a revised version of the dissertation, *An Essay on the Foundations of Geometry*.

As well as offering a Kantian foundation for geometry Russell argued in a Hegelian fashion that within the abstract conception of points in space characteristic of geometry there are "contradictions," which can only be resolved by incorporating geometry into an account of the physical structure of space. This led him into a study of the foundations of physics and in particular to a study of the problems associated with the continuity of space and time. He initially approached these matters with the presumption that these problems arise from a conflict between, on the one hand, the fact that individual points or instants differ only in respect of their relations and, on the other, the requirement that "all relations are internal," which he took to imply that differences in the relationships between things are dependent upon other differences between these things. Russell summed up the conflict here as "the contradiction of relativity": "the contradiction of a difference between two terms, without a difference in the conceptions applicable to them" ("Analysis of Mathematical Reasoning," *Papers*, 2: 166). The presumption that there is a contradiction here was a commonplace among the idealist logicians of the period, such as F. H. Bradley, and was central to their denial that there are any relational truths and thus to their metaphysical monism. It was therefore by calling this presumption into question that Russell made his break with idealism. The key to this was his affirmation of the independent reality of relations, which he proposed in his 1899 paper "The Classification of Relations" (see *Papers*, 2). Once this move was made, the alleged "contradiction of relativity" is dissipated and Russell was free to approach the issues raised by the continuity of space and time afresh.

Although Russell's papers from this period show him finding his own way to this anti-idealist thesis, he always acknowledged the decisive importance of G. E. Moore's writings at this time ("It was towards the end of 1898 that Moore and I rebelled against both Kant and Hegel. Moore led the way, but I followed closely in his footsteps," 1995a: 42). G. E. Moore was two years younger than Russell. Having been drawn from the study of classics to that of philosophy at Trinity College partly through Russell's influence he graduated in 1896 and completed his own, successful, dissertation for a prize fellowship in 1898. It is in this dissertation that Moore works out his own break with idealism. His basic claim is that of the unqualified reality of the objects of thought, propositions, as things is in no way dependent upon being thought about. Moore further maintained that there is no reason to duplicate ontological structures by hypothesizing the existence of facts for true propositions to correspond to. Instead there are just propositions and their constituents: the world just comprises the totality of true propositions, and an account of the structure of propositions is an account of the structure of reality itself. One implication of this is that the structure of space is independent of our experience of it and, therefore, of the conditions under which experience of it is possible. So Moore was very critical of Russell's neo-Kantian account of geometry, and Russell quickly came to agree with Moore on this matter (see MOORE).

Russell read Moore's dissertation at an early stage, and immediately accepted many of Moore's central points, including in particular his conception of a proposition. As we shall see, many of his later difficulties can be traced back to this. But at the time Russell was exhilarated by the possibilities that this new realist philosophy opened out before him:

> But it was not only these rather dry, logical doctrines [concerning the reality of relations] that made me rejoice in the new philosophy. I felt it, in fact, as a great liberation, as if I had escaped from a hot-house on to a wind-swept headland. (1995a: 48)

The principles of mathematics

This tremendous sense of intellectual liberation quickly became focused on a new project, which was to dominate Russell's thought and life for the next ten years: the "logicist" project of demonstrating that "all mathematics is Symbolic Logic." The occasion which fired Russell's enthusiasm for undertaking this project was his visit in July 1900 to the International Congress of Philosophy in Paris, where he heard Peano discuss his formalization of arithmetic using new logical techniques. Peano did not himself seek to provide a purely logical foundation for mathematics; he did not offer logical definitions of the concepts "0," "successor," and "number" which occur in his postulates. But, on hearing him, Russell jumped to the hypothesis that definitions of this kind should be possible, and thus that mathematics is, in the end, just logic. In making this jump Russell was drawing on his recent close study of the philosophy of Leibniz, which, fortuitously, he had undertaken the year before (simply because he stood in for McTaggart who should have been teaching it). Russell recognized that Leibniz had also conceived this hypothesis but had been prevented from demonstrating it, largely because of the inadequacies of the traditional logic to which he adhered. But with the richer resources of the logic employed by Peano (which Russell immediately used to develop a new logic of relations), Russell supposed that Leibniz's logicist hypothesis could now be vindicated.

An important aspect of Russell's new project was the opportunity it provided him to continue his criticisms of idealist philosophy:

> The questions of chief importance to us, as regards the Kantian theory, are two, namely, (1) are the reasonings in mathematics in any way different from those of Formal Logic? (2) are there any contradictions in the notions of time and space? If these two pillars of the Kantian edifice can be pulled down, we shall have successfully played the part of Samson towards his disciples. (1903: 457)

In providing a negative answer to the second of these questions Russell drew on the work of the great German mathematicians of the nineteenth century, Dedekind, Weierstrass, and Cantor, whose work he had discovered a few years earlier but had not then appreciated fully because of his attachment to idealist doctrines. He now devoted a central section of his new book, *The Principles of Mathematics*, to a careful exposition of their philosophy of the infinite, from which he concluded "that all the usual arguments, both as to infinity and as to continuity, are fallacious, and that no definite contradiction can be proved concerning either" (1903: 368). Since the idealist logicians had advanced the opposite view, and then used the infinity of space and time to argue for their unreality, Russell felt that he was here providing a definitive refutation of their position.

In laying out this new philosophy Russell worked at extraordinary speed. He started writing *The Principles of Mathematics* in October 1900 and by the end of the year he had completed a first draft: the book as we now have it is more than 500 pages long and 300 of these come unchanged from that first draft. This period was, he wrote,

> an intellectual honeymoon such as I have never experienced before or since. Every day I found myself understanding something that I had not understood on the previous day. I thought all difficulties were solved, all problems were at an end. (1995a: 56)

But, he continues,

> The honeymoon could not last, and early in the following year intellectual sorrow descended upon me in full measure. (1995a: 56)

The main reason for the onset of this sorrow was his discovery, early in 1901, of "the contradiction," now usually known as "Russell's paradox." This is a contradiction that can be easily demonstrated just at the point at which one seeks to develop elementary logic into set theory in order to show how arithmetic can be established on the basis of logic alone. Russell discovered the contradiction when reflecting upon Cantor's "paradox" that there is no greatest cardinal number. Cantor's paradox rests on the theorem that the number of subsets of a given set *S* is always greater than the number of members of *S* itself. Cantor proves this theorem by deducing a contradiction from the hypothesis that there is a one-to-one correlation between the subsets of *S* and the members of *S*, which would imply, on the contrary, that their numbers are the same. The contradiction arises as follows: consider that subset of *S* whose members are just those members of *S* which do not belong to the subset of *S* with which they are correlated under the hypothesized correlation. Since this so-called "diagonal" set *D* is a subset of *S* it too must be correlated with a member of *S*, say *d*. Cantor now asks whether *d* belongs to *D*: given the way *d* and *D* have been defined, it turns out that *d* belongs to *D* if and only if *d* does not belong to *D*, from which it is easy to derive the explicit contradiction that *d* both belongs to *D* and does not belong to *D*.

The step from Cantor's theorem to Russell's paradox is very simple. Instead of Cantor's hypothetical one-to-one correlation between the members of a set and its subsets, consider instead the non-hypothetical identity relation between anything and itself. Then the analogue of Cantor's "diagonal" set *D* of things that are not members of the set with which they are hypothetically correlated becomes simply the set *R* of things that are not members of themselves. Under the "identity" correlation *R* is of course "correlated" with itself; hence in asking whether *R* belongs to that with which it is correlated we are simply asking whether *R* belongs to *R*. But since *R* just is the set of things which do not belong to themselves, it follows that *R* belongs to *R* if and only if *R* does not belong to *R*. This too immediately gives rise to an explicit contradiction, but in this case the derivation does not depend on a hypothetical correlation which is thereby proved not to exist, but only on the non-hypothetical identity of a thing with itself which cannot be rejected. So in this case there is no obvious positive conclusion to be drawn comparable to Cantor's theorem. There is instead the utterly dismaying implication that there is something seriously amiss in the foundations of logic.

As soon as he had discovered this contradiction Russell communicated it to Frege, whose works he had just recently read properly for the first time and recognized for what they were, namely much the most sophisticated attempt to develop a logicist program of the kind he was also engaged upon. Frege received Russell's letter just as the second volume of his *Grundgesetze* was in press, and added the famous Appendix II, which begins, "Hardly anything more unfortunate can befall a scientific writer than to have one of the foundations of his edifice shaken after the work is finished." Frege did in fact suggest a way of circumventing Russell's paradox, but he could not show that it worked, and it is now known not to. Russell also attempted to find a way around the paradox and ended *The Principles of Mathematics* with a tentative proposal that the definition of the set *R* is ill-formed because a set, being of a different "type" from that of its members, cannot be a member of itself. As we shall see below, this is a proposal to which he returned later; but in the context of the *The Principles of Mathematics* it was not easy for him to advance it, since it conflicts with the conception of logic advanced there, namely that the truths of logic are truths which are absolutely universal, and are therefore not to be restricted by considerations arising from the type of thing under discussion.

The contradiction was not the only problem to delay publication of *The Principles of Mathematics* until 1903 and to dominate his research for the next few years. He also ran into a tangle of difficulties concerning the structure of judgment, with which he continued to grapple thereafter. These difficulties are, broadly, of two kinds: concerning (1) the unity of judgment and (2) the structure of general judgments.

In *The Principles of Mathematics* Russell's treatment of these matters is expressed through a discussion of the structure of propositions, which, following Moore, he takes to comprise both the objects of judgment and the objective structure of the world. So conceived, propositions are not representations which, when true, correspond to a fact. Instead true propositions just are facts – there is no difference between the death of Caesar and the (true) proposition that Caesar is dead. Since his death is something that befell Caesar himself, Caesar is himself a "constituent" of the proposition that Caesar is dead. Indeed the proposition just is a "complex" whose constituents are Caesar and death (which is a "predicate").

The difficulty that now arises concerning the "unity" of judgment is that of explaining how it is that a complete proposition is constituted. Russell discusses this in connection with the proposition that A differs from B. The constituents here are A, B, and difference; but specifying them does not yet specify the proposition in question, for they are also the constituents of the different proposition that B differs from A (because difference is a symmetric relation this is actually a poor case to have taken. The point is much clearer with an asymmetric relation, such as occurs in the proposition that A is larger than B, which is manifestly different from the proposition that B is larger than A although it has the same constituents). Russell sums up his discussion:

> The difference which occurs in the proposition actually relates A and B, whereas the difference after analysis is a notion which has no connection with A and B. (1903: 49)

Furthermore, Russell notes, it does not help if one adds that the difference in the case we want is a difference *of* A *from* B, for all that these additions do is to add further relations to the supposed constituents of the proposition without "actually relating" the relation of difference to the right terms. The problem is that

> A proposition, in fact, is essentially a unity, and when analysis has destroyed the unity, no enumeration of constituents will restore the proposition. (1903: 50)

The point raised here is one that was to come back to plague him. In *The Principles of Mathematics* Russell, having identified it, simply sets it aside for further treatment. This is disappointing, for here there is a straightforward challenge to the Moore–Russell conception of a proposition as a "complex whole" comprised of its elementary constituents. Indeed the point was not new: this difficulty concerning relational judgments had been famously set out by F. H. Bradley in chapter III of *Appearance and Reality* (1893). The person who first saw clearly the way to defuse the issue here was Frege: for through his famous "context" principle (presented in his *Grundlagen*, 1884) that it is only in the context of a sentence that a word has a meaning, he acknowledges the irreducible primacy of judgments instead of regarding them as "complexes" to be constructed out of elementary constituents. We shall see below that Russell's theory of descriptions includes a partial acknowledgment of the context principle; but he himself never generalizes it to solve this problem of the unity of judgment.

The other general type of difficulty that Russell encountered in *The Principles of Mathematics* concerns the structure of general propositions such as the proposition that I met a man. The difficulty here is supposed to come from the fact that, on the one hand, the concept *a man* is a constituent of this proposition; but, on the other hand, such a concept

> does not walk the streets, but lives in the shadowy limbo of the logic-books. What I met was a thing, not a concept, an actual man with a tailor and a bank-account or a public house and a drunken wife. (1903: 53)

Russell's argument here is intuitive and questionable. But it is clear that the tension arises from the dual role of propositions as both objects of thought (and thus constituted of concepts such as *a man*) and situations within the world (and therefore constituted not from general concepts, but from particular men).

Russell's way of resolving this tension is to say that concepts such as *a man* "denote" the things that a proposition in which they occur is "about"; and it is the things that are denoted in this way that "walk the streets" etc. It is difficult at first not to interpret this talk of that which a proposition is "about" as a way of implicitly specifying a "truth-maker" for the proposition distinct from the proposition itself; but this of course would be entirely inimical to Russell's conception of a proposition. Furthermore, Russell develops his account of denoting in such a way as to make this interpretation inappropriate. For he goes on to argue that

> *some man* must not be regarded as actually denoting Smith and actually denoting Brown and so on: the whole procession of human beings throughout the ages is always relevant to every proposition in which *some man* occurs, and what is denoted is essentially not each separate man, but a kind of combination of all men. (1903: 62)

As Russell acknowledges, such a "combination of all men" is a very paradoxical object; in the case of *a man*, what is denoted is supposed to be Smith or Brown or . . . (for the whole human race).

It is in fact clear enough what Russell is seeking to do here: namely, to effect a reduction of general propositions to propositions that involve only disjunctions or conjunctions of singular propositions, for which the tension between propositions as objects of thought and as truthmakers is not so acute. He writes that "the notion of denoting may be obtained by a kind of logical genesis from subject-predicate propositions" (1903: 54), and as Geach has observed, Russell's account is in this respect comparable to medieval theories of *suppositio*. But Russell muddles things by supposing that he needs to hold that there are disjunctive and conjunctive combinations of things denoted by the denoting concepts that occur in the general propositions. These are the "paradoxical objects" which the propositions in question are to be "about." Not only are such objects intrinsically objectionable (as Russell himself acknowledges, 1903: 55n.), this way of coming at the matter makes it impossible to provide a coherent treatment of propositions involving multiple generality, since there is no way of representing scope distinctions, such as the distinction between the two ways of interpreting "Everyone loves someone."

Russell in effect acknowledges this point himself when discussing variables, both free and bound. He would like to handle free variables within his theory of denoting as cases of the denoting concept *any term*; but he can see that this approach does not deal properly with the role of repeated variables. His discussion of bound variables occurs as part of his exposition of Peano's conception of "formal implication" as a universally quantified conditional, as in "for all x, if x is a man then x is mortal." Russell wants to be able to apply his theory of denoting concepts to the quantifier "for all x," as a concept denoting some combination of things which the whole proposition is "about." But he can also see that where there are multiple quantifiers binding different variables (as in both interpretations of "Everyone loves someone") the variables are tied to the quantifiers in a way that blocks off this conception of the denotation of a quantifier. So although he cannot bring himself to say so explicitly, his theory of denoting concepts is inadequate to the new logic of quantifiers and variables upon which the logicist project of *The Principles of Mathematics* is founded.

The theory of descriptions

Two years later, in 1905, Russell published his most famous paper, "On Denoting." He begins by, in effect, developing his earlier discussion of formal implication into a systematic account of the propositions expressed by sentences involving what he now calls "denoting phrases" such as "all men," "a man," and "no man." There is now no talk of denoting concepts; instead he uses the universal quantifier and bound variables to specify the propositions expressed by these sentences by reference to the truth of simpler propositions. Thus he now says that the proposition expressed by "I met a man" is the proposition that propositions of the type "I met x, and x is human" are not always false (*Papers*, 4: 416).

Clearly, much is here assumed, for example the interdefinability of the existential and universal quantifiers. There is also a degree of oversimplification, since, as he recognizes when dealing with multiple quantifiers, he actually needs to specify the variable in the quantifier; in the case above he should have said "are not always false of x." Setting these points aside, what is worth considering is whether Russell offers any general account of quantifiers and variables to replace the theory of denoting concepts that has been tacitly discarded. In "On Denoting" itself Russell is unhelpful: he just says "Here the notion 'C(x) is always true' is taken as ultimate and indefinable" (*Papers*, 4: 416). If, however, one looks ahead to the discussion of the universal quantifier in the introduction that Russell wrote to *Principia Mathematica* (1910), one finds him using the language of "ambiguous denotation" to sketch what is now recognizable as a substitutional account of the quantifier. For he now says that we assert a universal proposition in order to condense the assertion of all the substitution instances "ambiguously denoted" (1910: 40) by the propositional function that occurs in our universal proposition; and the truth of the universal proposition depends on the "elementary truth" of all these substitution instances (p. 42). Since the motivation behind the original theory of denoting concepts was that propositions involving all men, a man, etc. are in some way "about" their instances, it is unsurprising that he ends up with a substitutional treatment of quantification, dealing, of course, with substitutions in propositions, not sentences.

Russell's main topic in "On Denoting" is the structure of propositions whose expression involves definite descriptions, phrases such as "The present President of the USA." In *The Principles of Mathematics* he had applied his theory of denoting concepts to such propositions in order to provide an account of why it is that true identities, propositions expressed by sentences such as "Bill Clinton is the present President of the USA," are of interest to us (1903: 62–4). The problem here is familiar: if we just take it that we have two names for the same thing, and that the proposition's constituents are just the thing thus named twice and the relation of identity, it seems that the very same proposition is also expressed by "Bill Clinton is Bill Clinton," which is of no interest to us. In *The Principles of Mathematics* Russell took it that this problem is solved by the hypothesis that the description "the present President of the USA" introduces a corresponding denoting concept into the proposition expressed through its use, which of course does not occur as a constituent of the proposition expressed by "Bill Clinton is Bill Clinton."

Russell does not explain how this denoting concept occurs as a constituent of the proposition expressed, and he says other things about it on the basis of which it is easy to reopen the old problem. Since the proposition cannot comprise Bill Clinton's identity with the denoting concept in question, it seems that it must comprise Bill Clinton's identity with the thing denoted by the denoting concept, that which the proposition is "about" in Russell's intuitive sense. But this thing is of course just Bill Clinton himself, and we have now come back to the difficulty of showing why this proposition is of any interest to us since it is equally expressed by "Bill Clinton is Bill Clinton." If one looks to Russell's extensive unpublished writings on this matter from the period 1903–5 (see *Papers*, 4), one can, I think, see him identifying this difficulty for his old position. He also begins to think about a different issue, also problematic for his old position, which arises from his critical reaction to some works by the contemporary Austrian philosopher Alexius Meinong, which he studied closely at this time.

In this case the issue concerns the proper treatment of "empty" descriptions, descriptions such as "the present King of France" which, though meaningful, describe nothing that actually exists. Russell interpreted Meinong as advancing a theory of "objects" according to which empty descriptions of all kinds describe an object, even descriptions of impossible objects such as "the round square." In "On Denoting" and thereafter Russell ridiculed this position as conflicting with "the robust sense of reality" which "ought to be preserved even in the most abstract studies" (1919: 169–70); but in truth Meinong's position was a good deal more subtle than Russell appreciated. What is important here, however, is that through thinking about Meinong's work Russell came to appreciate the importance of constructing a theory that would allow for the possibility of meaningful sentences that include empty descriptions – sentences such as "The present King of France is bald." Since Russell took it that the meaning of a name was just the name's bearer, it followed at once that such descriptions were not names. This, he recognized, did not rule out the treatment of such descriptions as introducing denoting concepts into a proposition. But, he argued, there was still a problem: the proposition expressed by "The present King of France is bald" ought to be *about* the present King of France. But there is no such thing; so the theory implies that the proposition is about nothing, which Russell takes to imply that "it ought to be nonsense." But, he continues, "it is not nonsense, since it is plainly false" ("On Denoting," *Papers*, 4: 419).

This argument is condensed, but the way to understand the crucial move from being "*about* nothing" to "being nonsense" is to connect the "*aboutness*" thesis with the thesis that the truth or falsehood of a proposition with a denoting concept depends on the truth or falsehood of the propositions specified by reference to that which the first proposition is *about*. For in the light of this, a proposition *about* nothing will be one for which there are no such propositions to determine its truth or falsehood. So it can be neither true nor false itself; but this is absurd since propositions are inherently either true or false. Hence it follows that the sentence with an empty description fails after all to express a proposition: in which case "it ought to be nonsense."

We now have the two "puzzles" which, Russell says in "On Denoting," an adequate theory of descriptions must solve: (1) Why is it that questions about identity are often of interest to us – how can it be that George IV wished to know whether Scott was the author of *Waverley* but did not wish to know that Scott was Scott? (2) What propositions are expressed by sentences with empty descriptions, such as "The present King of France is bald," and how is their truth or falsehood determined? Before showing how his new theory can solve these puzzles, Russell explains why he rejects the view that one should account for these puzzles by distinguishing between the "meaning" and the "denotation" of a description. He specifically mentions Frege in connection with this view and it is natural to think of him as arguing here against Frege's theory of *Sinn* and *Bedeutung* which was of course precisely introduced to handle the first of Russell's puzzles and seems well suited to handle the second (see FREGE).

Russell's argument in "On Denoting" against the "Fregean" position is notoriously obscure. His conclusion is that there is an "inextricable tangle" in the account this position offers of the relation between the meaning and the denotation of a description

(*Papers*, 4: 422); but in truth it is Russell's own discussion which certainly appears to be an inextricable tangle. My own view is that Russell's argument is vitiated by the fact that he adapts Frege's theory to his own different conception of a proposition, and thereby crucially distorts Frege's actual position. (Russell himself acknowledges the differences here: *Papers*, 4: 419 n. 9.) This can be illustrated by considering a slightly later, and considerably clearer, discussion by Russell of the same issue, in "Knowledge by Acquaintance and Knowledge by Description," in which he is considering the structure of the proposition expressed by "the author of *Waverley* is the author of *Marmion*" (*Papers*, 6: 159). Russell argues here that (1) the proposition involves an identity; but (2) plainly does not involve the identity of the meanings of "the author of *Waverley*" and "the author of *Marmion*"; so (3) it must comprise the identity of the denotations determined by the meanings of "the author of *Waverley*" and "the author of *Marmion*." But these denotations are just Scott himself, so the proposition in question is just the proposition that Scott is Scott. Yet the whole point of the meaning/denotation theory was to preserve the distinction between interesting and trivial identities.

For Frege this argument simply gets off on the wrong foot. The thought expressed by "the author of *Waverley* is the author of *Marmion*" is indeed a thought about identity; but the thought itself is not structured by the relation of identity, but by the sense employed here of this relation, and this sense can then relate the senses expressed by the two descriptions without turning it into a thought that these senses are the same. Russell's argument against the meaning/denotation distinction as applied to descriptions only works insofar as it depends on a non-Fregean hybrid conception of a proposition which violates Frege's distinction between *Sinn* and *Bedeutung*.

The fact that Russell's argument against Frege is a failure does not, of course, vindicate Frege's treatment of descriptions as functional expressions or undermine Russell's own theory of descriptions. The key to this new theory is, as Russell put it later, that descriptions are "incomplete symbols," that is, phrases which "have no meaning in isolation" but are only "defined in certain contexts" (1910: 66). As such, for Russell, descriptions such as "the man" are to be regarded as essentially similar to the other denoting phrases discussed in "On Denoting," e.g. "a man" and "all men." They have "no meaning in isolation" in the sense that there is no thing (not even a concept) that is their "meaning" and that occurs as a constituent of the propositions expressed by sentences in which they occur. Instead they contribute to these propositions in more complex ways by fixing their structure, in the way that Russell conceives of the role of the universal quantifier as described above. It is therefore no surprise that on Russell's new theory of descriptions, the role of descriptions is elucidated by spelling out the quantificational structure of the propositions expressed by sentences in which they occur. This turns out to be a complex matter since Russell sticks to his initial assumption that all such propositions involve only the universal quantifier. But we can grasp the key points of Russell's new theory by providing just the first stage of it, which is that the proposition expressed by "The author of *Waverley* is Scott" is more clearly identified as the proposition expressed by the following sentence:

> For some x, (i) x is an author of *Waverley* and, (ii) for all y, if y is an author of *Waverley* then $y = x$, and, (iii) $x =$ Scott.

Once the proposition is identified in this way solutions to Russell's two puzzles are immediate. There is no reason to identify the complex proposition thus expressed with the proposition that Scott is Scott, for the description "the author of *Waverley*" is not construed as giving rise to a complex name of a constituent within the proposition which turns out to be just Scott. So the distinction between significant and trivial identities is clearly preserved. Secondly, the role of empty descriptions is easily allowed for: the proposition expressed by "The present King of France is bald" is to be identified as that expressed by the sentence:

> For some x, (i) x is a present King of France and, (ii) for all y, if y is a present King of France then $y = x$, and, (iii) x is bald.

It is then unproblematic that there is such a proposition, and that it is "clearly false" as Russell declares that it should be.

The fact that these solutions are so straightforward, and do not depend on the underlying theory of propositions (though they are consistent with it), shows that one can abstract Russell's theory of descriptions from this underlying theory. This is in fact what has largely happened to Russell's theory of descriptions: it is taken to rest on the thesis that descriptions are quantifiers, and as such the Russellian position is best conceived of as one that employs a restricted "definite" quantifier to construe definite descriptions, so that the logical form of "the author of *Waverley* is Scott" can be better captured by construing it as

> For the x who is an author of *Waverley*, $x =$ Scott.

One can then interpret Russell's reduction of the definite quantifier to other quantifiers as a misleadingly expressed way of spelling out the truth-conditions of this sentence. Once the matter is handled in this way, the debate with Frege can be re-opened, as a debate as to whether this way of construing descriptions is preferable to Frege's approach, according to which they are complex singular terms, comparable to functional expressions in mathematics, such that the logical form of "the author of *Waverley* was Scott" is captured by construing it as

> The author of (*Waverley*) = Scott.

This debate was famously revived by Strawson in his attack on Russell. Strawson introduced a range of linguistic data concerning the use of empty descriptions in situations which conflict with our "presupposition" that descriptions are non-empty, and argued on their basis that a Fregean position is in fact preferable to Russell's; in many cases, Strawson argued, we take it that the use of empty descriptions issues in statements that are "neither true nor false" and not "plainly false" in the way that Russell maintained. In fact, however, the linguistic data in this area are indeterminate, and most contemporary discussions focus instead on the relative merits of alternative accounts of the logical behavior of descriptions in complex sentences involving temporal modifiers and counterfactual constructions. Even when these are introduced, however, the issue remains surprisingly open – so much so that it seems to me best to conclude that definite descriptions blur the apparently sharp logical distinction between particular thoughts involving a singular term and general thoughts involving a quantifier.

Returning, however, to Russell himself I want finally to discuss the significance of the theory of descriptions within his philosophy generally. The first point is that Russell felt that his theory showed that there was no need to abandon his one-dimensional conception of meaning in favor of a Fregean theory with its all-encompassing distinction between *Sinn* and *Bedeutung*. Indeed, as we have seen, Russell felt that he could cope better than Frege with the "puzzles" of interesting identities and empty descriptions. This had an important implication for Russell's treatment of names: with no *Sinn*/*Bedeutung* distinction Russell was committed to the view that all identities involving just names are trivial and that there cannot be meaningful empty names. So all putative counterexamples had to be handled by supposing that the names in question were really just descriptions under disguise. The scope of this thesis was massively extended by a point that Russell introduces right at the end of "On Denoting" (p. 427), namely that our understanding of a proposition is based upon our "acquaintance" with its constituents, which are the meanings of the phrases used to express the proposition. For once one adds, as Russell does here, that we are not acquainted with matter and the minds of other people, it follows that our understanding of sentences that include putative names of material objects and other people cannot be achieved by identifying these things as the meanings of the names; instead we are bound to reinterpret the names as descriptions that invoke properties of things with which we are acquainted.

I shall discuss this radical doctrine of "knowledge by acquaintance" below. What I want to stress here is just that it was the theory of descriptions that made this doctrine tenable, since it seemed to offer a way of escaping the doctrine's otherwise unacceptable skeptical implications. The trick here was to suppose that "logical analysis" involving the theory of descriptions could save the appearances of common-sense belief even though its obvious foundations had been removed by the doctrine of limited acquaintance. This move became central to Russell's later "logical-analytic method in philosophy" (1914: v), to which I shall return below.

Another important point is the doctrine of "incomplete symbols," and in particular the central claim of the theory of descriptions that phrases whose meaning at first sight seems to consist in denoting some object turn out, after "logical analysis," not to have such a meaning at all. Instead their meaning is given only "in context": in the broader context of the sentences in which they occur. It is striking that Frege's context principle, which I mentioned earlier when discussing the issue of the unity of judgment, here enters into Russell's theory; but of course it does so only because descriptions are a counterexample to Russell's fundamentally non-contextual conception of meaning. For Russell the appeal to context is appropriate precisely where the demands of logic conflict with superficial grammar and its associated conception of meaning. The idea of such a conflict has been a deeply influential idea in the analytical tradition, giving rise to a very Platonist conception of "logical form" as something characteristically veiled by ordinary language. Wittgenstein rightly identified Russell's seminal role in developing this conception:

> All philosophy is a "critique of language" (though not in Mauthner's sense). It was Russell who performed the service of showing that the apparent logical form of a proposition need not be its real one. (*Tractatus Logico-Philosophicus* 4.0031)

Though by 1914 Russell took the view that "philosophy . . . becomes indistinguishable from logic" (in "On Scientific Method in Philosophy") he would have repudiated this characterization of philosophy as just "critique of language," since for him logic primarily concerns the logical forms of "the various types of facts" (*Papers*, 8: 65); indeed towards the end of his life, in the 1950s, he was very critical of the way in which philosophers seemed primarily concerned with language. As we have already seen, and shall see further below, Russell's logic was always shaped by metaphysical theses (e.g. concerning the nature of propositions) and driven by epistemological concerns (e.g. the doctrine of acquaintance). Nonetheless, his own theory of descriptions, and the uses to which he then put it in his logical-analytic program, did seem to many philosophers to show that it is through the logical analysis of language that philosophers can make progress in resolving old debates. To a considerable extent the whole project of analytical philosophy is founded upon this faith, and to that extent Russell's theory of descriptions remains, as Ramsey called it, "a paradigm of philosophy" (Ramsey 1931: 263n.).

Avoiding the contradiction

Russell's main concern in the years following the publication of *The Principles of Mathematics* was not in fact the theory of descriptions and its implications, which I have been discussing. Instead he was still preoccupied with his logicist project, on which he was now working with the Cambridge mathematician and (later) philosopher A. N. Whitehead, and he was, therefore, confronted by the need to avoid the perplexing contradiction he had discovered in 1901.

His first thought was that the conception of an incomplete symbol he had developed in connection with the theory of descriptions could be put to work to show what was wrong with the paradoxical set *R* of things that are not members of themselves. Russell developed this thought in an ingenious way by interpreting talk of sets in terms of the results of substitutions within propositions and then showed that, under this interpretation, the condition of self-membership cannot be coherently expressed. Regretfully, however, he decided that this approach was not the whole story since it did not resolve paradoxes concerning propositions, such as the liar paradox, which, he felt, were so closely related to his own paradox that there should be a single solution for them all.

He turned next to an idea that arose in the course of a debate with the French philosopher, Henri Poincaré, that these paradoxes arise only because the underlying argument tacitly involves a "vicious circle," in that something which has been defined in terms of a totality is then assumed to belong to this totality. Thus Russell's set *R* is defined in terms of the set of sets which are not members of themselves, and the contradiction is then arrived at by considering whether or not *R* belongs to this very set. Similarly in the case of the liar paradox the crucial move is that whereby the statement made by the liar is taken to be included in the scope of the liar's own statement. So, Russell thought, the way to avoid all these paradoxes is to adhere to the "vicious circle" principle, that "Whatever involves *all* of a collection must not be one of the collection" (1910: 37).

This principle gives rise to a hierarchy of "orders," since anything defined in terms of a collection of things of order n is held to be of order $n + 1$ and therefore not a candidate for membership of the first collection. In developing the idea further, however, Russell went back to his earlier idea that the very definition of the paradoxical set *R* is somehow ill-formed. So he returned to the thought that our normal talk of sets involves incomplete symbols, and thus that sets are only "quasi-things" (1910: 81). But he no longer used his earlier interpretation of this talk in terms of substitutions within propositions: instead he introduced the conception of a "propositional function," a function whose values are propositions, and proposed a way of interpreting talk of sets in terms of propositional functions. Under this interpretation, talk of *a*'s membership of the set of things *x* such that *fx* is interpreted in terms of the truth of the proposition that is the value of the propositional function $f\hat{}x$ (Russell's standard notation for propositional functions) for the argument *a* (i.e. the proposition *fa*). Hence the condition of self-membership entering into the definition of *R* is interpreted in terms of the truth of the proposition that is the value of a propositional function applied to itself as argument. But this, Russell claims, is incoherent: he takes it that the vicious circle principle implies that the propositions that are the values of a propositional function should in no case be specified by reference to the propositional function itself, and thus that

> there must be no such thing as the value for $f\hat{}x$ with the argument $f\hat{}x$ . . . That is to say, the symbol '$f(f\hat{}x)$' must not express a proposition. (1910: 40)

Russell reinforces this point by arguing that although there are functions of functions, in all cases functions must be of a different "type" from that of which they are functions; thus functions of simple individuals cannot themselves be individuals, but must have sufficient complexity to yield complete propositions when applied to individuals. Similarly functions of these functions, such as the quantifiers, have to have the type of complexity required to yield a complete proposition in these cases. Hence, he says, functions cannot be arguments to themselves, for they lack the type of complexity required to yield a complete proposition in this situation.

This line of thought generates a hierarchy of types (individuals, functions of individuals, functions of such functions, etc.) different from the hierarchy of orders generated by the vicious circle principle, which concerns the order of definitions. The resulting theory, the "ramified theory of types," is the result of merging these two hierarchies. There is an element of overkill in this theory, for there are now three reasons why the contradiction does not arise: (1) the vicious circle principle straightforwardly implies that *R* is of a higher order than its members and cannot therefore be a member of it; (2) Russell takes the principle to imply also that the definition underlying *R*, when spelled out in terms of propositional functions that apply to themselves, is ill-formed; (3) Russell also invokes a separate thesis that a function must have a different type of complexity from its arguments if it is to yield a complete proposition as value, which again implies that the definition underlying *R* is ill-formed.

The main aim of the theory, however, was not to avoid the contradiction but to fulfill the logicist project of providing a logical foundation for pure mathematics. This was in a way accomplished by Russell and Whitehead in their massive, though incomplete, trilogy *Principia Mathematica* (1910–13). They had found, however, that their task was obstructed by the complexities of the ramified theory. For example, at a crucial point in the standard theory of real numbers (the least upper bound theorem), the vicious circle principle is violated; hence Russell and Whitehead had, in effect, to set aside this principle by introducing the assumption (the "axiom of reducibility") that wherever a propositional function is defined in terms of a totality to which it is then required to belong there is a way of defining it without reference to that totality, in terms of "predicative" functions that do not involve reference to the totality in question. As their critics observed, if Russell and Whitehead were going to help themselves to this assumption, then they could have simplified things a good deal by formulating the whole theory in terms of predicative functions in the first place. Indeed, after discussion with F. P. Ramsey Russell and Whitehead adopted a proposal of this kind in the second edition to *Principia Mathematica* (1927). The resulting theory, the "simple theory of types," no longer offers a solution to semantic paradoxes such as the liar paradox. But this is a positive gain since the vicious circle principle is anyway not a satisfactory resolution of these paradoxes, which depend on concepts such as truth whose complexities are separate from set theory itself.

But there are other problems, which afflict even this modified theory. Once individuals and functions (or sets) are divided into exclusive types, there has to be a separate, though isomorphic, arithmetic for each type, an idea that is highly counterintuitive. Furthermore the validity of standard arithmetic as applied to individuals requires the assumption that there is an infinity of such individuals. As Russell himself recognized (1919: 141), this assumption, or axiom, is not logical; it is clearly metaphysical (and may well be false).

To say this is to raise the question, central for the logicist project, as to what logic is. In *Principia Mathematica* logic is said to be the theory of formal inference, of inferences that depend merely on the logical form of the propositions involved. The distinction between "form" and "content" is then crucial; Russell takes it that the identification of the logical constants, by reference to which logical form is defined, is only a matter of enumeration. In his abandoned 1913 manuscript "Theory of Knowledge" he appreciates the need for some deeper theory, but remarks, "In the present chaotic state of knowledge concerning the primitive ideas of logic, it is impossible to pursue this topic further" (*Papers*, 7: 99). This is a surprising remark in the light of all Russell's work on logic. But there is no doubt that he had found the experience of coping with his contradiction a chastening experience, which had taught him that even in logic there are no simple answers, and thus that only "patience and modesty, here as in other sciences, will open the road to solid and durable progress" (*Papers*, 8: 73).

To compare logic with other sciences is to invite the question whether logic differs from them except in respect of its subject matter of formal inference, however exactly that be defined. This question is particularly apposite since at least until 1911 Russell affirmed that logic is synthetic, which might suggest that he also thought it is empirical. In fact, however, he held that it is a priori, and rests upon self-evident intuitions concerning the relationships between logical forms, which are universals (1912: ch. X). What is then a little odd is that he generally takes it that logical inference is just a matter of material implication (1910: 8–9), so that although logical inferences must be in fact truth-preserving it is not required that they preserve truth in all possible situations. His views in this area are not easily fitted together. Later, presumably under the influence of his former student Wittgenstein, he describes logical truths as "tautologies," which suggests a move to a conception of them as analytic; but in the "Lectures on the Philosophy of Logical Atomism" (published 1918) he is still very tentative:

> Everything that is a proposition of logic has got to be in some sense or other like a tautology. It has got to be something that has some peculiar quality, which I do not know how to define, that belongs to logical propositions and not to others. (*Papers*, 8: 211)

Modern set theories are based on the work of Zermelo and Von Neumann and avoid the problems that arise for Russell and Whitehead by sweeping away type distinctions. This implies that there is nothing ill-formed about the condition of self-membership, but Russell's paradox is avoided by denying that any well-formed condition (or propositional function, as Russell would call it) determines a set. Indeed it is standard to have an axiom of "foundation," which requires that the membership of a set be founded by being based upon "ur"-elements which do not themselves have members. This axiom (proposed by Mirimanoff in 1917) implies that the condition of non-self-membership does not determine a set and captures the intuition which lies behind Russell's "vicious-circle principle," but without obstructing the development of a systematic set theory that can be interpreted as a foundation for mathematics.

This does not, however, mean that modern set theory provides a vindication of Russell's logicist project. Russell's conception of a propositional function blurs the distinction between standard predicate logic and set theory, but, ironically, Russell's paradox itself shows the importance of maintaining a distinction here, and once this is drawn and set theory is provided with its own distinctive axioms concerning the existence of sets, there is no good reason to count set theory as logic. This does not mean that the logicist project is altogether untenable; for one can abandon set theory in favor of second-order logic and try to use this to provide a foundation for mathematics. Whether the resulting position is satisfactory remains disputed, but even if it is, it is still subject to the implications of Gödel's famous incompleteness theorem, which shows that arithmetic (and thus mathematics) cannot be completely captured within a formalized theory. If logic is just the theory of formal inference, as Russell maintained, then Gödel's theorem provides the ultimate refutation of his project of showing that "all mathematics is Symbolic Logic."

Logical atomism

When Russell was asked in 1924 to provide a personal statement of his philosophical position he chose to entitle it "Logical Atomism" (1956). This is a name that he began to use in 1914 (*Papers*, 8: 65) and then used in his 1918 "Lectures on the Philosophy of Logical Atomism." The rationale for the emphasis here on logic will be obvious; but what is the "atomism"? What are the "logical atoms"?

They are atomic facts. The reference here to "facts" is due to his abandonment by 1914 of his earlier Moorean conception of propositions. He has now adopted a form of the correspondence theory of truth according to which the truth of a proposition, now conceived of as normally a linguistic structure (though Russell also allows for imagistic mental propositions), is grounded in the perfect correspondence of logically simple propositions with atomic facts. I shall explain his reasons for this change of mind concerning propositions below, but since facts are said to be composed of the things that are the meanings of the words occurring in the proposition, it turns out that atomic facts differ little from old-style true elementary propositions, the propositions whose truth formed the basis for the truth of propositions whose expression involves incomplete symbols (see "Lectures on the Philosophy of Logical Atomism," *Papers*, 8: 175).

Atomic facts are facts concerning the intrinsic qualities of, and relations between, particular individuals (*Papers*, 8: 177). But since the individuals and properties that constitute any fact we can talk about must be such that they are the meanings of the words we use, it follows that in practice the identity of atomic facts is constrained by the requirements of Russell's theory of meaning. And the crucial requirement here is that of our *acquaintance* with the things in question:

> A name, in the narrow logical sense of a word whose meaning is a particular, can only be applied to a particular with which the speaker is acquainted. (*Papers*, 8: 178)

Since Russell holds that we are not acquainted with ordinary things, such as Piccadilly and Socrates, but only with sensory particulars which are "apt to last for a very short time indeed" (*Papers*, 8: 181), it turns out that the atomic facts we can talk and think about do not deal with the familiar furniture of life, but for the most part only with the private objects of experience.

It is at this point that Russell's logical atomism makes contact with the doctrine of knowledge by acquaintance, which we encountered earlier, and thereby becomes a form of epistemological atomism. The core of that doctrine is expressed in his fundamental epistemological principle (in "Knowledge by Acquaintance and Knowledge by Description") that

> *every proposition which we can understand must be composed wholly of constituents with which we are acquainted.* (*Papers*, 6: 154)

With what things are we acquainted? Russell's approach is a combination of empiricism and rationalism. Through perception and introspection we gain acquaintance with particulars: with particular colors, noises, feelings, etc.; but through reflection we can also gain acquaintance with the universals of which these particulars are instances. This form of acquaintance involves "conception" (*Papers*, 6: 149–50), and is central to our capacity for intuitive awareness of a priori truths (1912: ch. x).

For our present purposes, the point on which to concentrate is our acquaintance with particulars. Russell uses Moore's general term "sense-data" for these sensory particulars but emphasizes that it is for him an open question whether such particulars may not also exist as unperceived "sensibilia." Sense-data so conceived are not "in the mind," though our awareness of them is immune from error. Initially Russell took the view that they are subjective because their relativity to their subject's position and condition implies that they cannot be combined in a single public world (1912: ch. i); but in 1914 he switched to the view that they are physical elements within private spaces, which are capable of being integrated into a single public world unless they belong merely to dreams and hallucinations. His account of this supposed integration is difficult and the details cannot be discussed here, but it is for him an important application of his "supreme maxim in scientific philosophising" (*Papers*, 8: 11) that

Wherever possible logical constructions are to be substituted for inferred entities.

The basic idea here is said by Russell to be similar to that involved in Ockham's razor, but in fact it is motivated by the need to escape the skeptical implications of his theory of knowledge by acquaintance. For where the apparent objects of putative knowledge are things of which we have no acquaintance, Russell's theory seems to imply that our beliefs can only ever be a matter of uncertain, speculative, inference concerning certain kinds of things "we know not what" whose existence is in some way implied by the existence of the things with which we are acquainted. Russell thinks he can avoid this skeptical result by substituting an alternative way of thinking, which can still be represented as a way of thinking about the apparent objects of knowledge but which, because it is "logically constructed" from thoughts of things (sense-data and universals) with which we do have acquaintance, does not require speculations concerning the existence of any further "inferred entities." This alternative strategy shows how knowledge is possible while respecting the fundamental epistemological principle quoted earlier.

Thus Russell takes it that knowledge of the external world is achieved on the basis of acquaintance with sense-data primarily by the application to these data of the logical principles involved in the theories of descriptions and of classes (though the integration of private spaces into a single public one involves more than logic). The result is that talk and thought putatively about public physical objects is to be provisionally interpreted in terms of logically complex propositions about classes of sense-data; but because classes are themselves "logical fictions" these propositions are themselves to be further interpreted in terms of propositional functions. Only then can we arrive at an indirect specification of the atomic facts, involving only items with which we are acquainted, of which we are aware, and which ground our knowledge of the external world. But, in principle, such a specification can be arrived at; so knowledge is possible, or so Russell supposes. In truth the matter is more tricky than he allows because of the "other minds" problem. As Russell acknowledges, the construction of the external world involves apparent reference to the sense-data of others, for the external world is essentially something that transcends our own sense-data. But since we are not acquainted with the sense-data of others, Russell's fundamental principle implies that we cannot understand any propositions requiring reference to them. So we have to start from our own sense-data alone and build out simultaneously to other minds and the external world. But whether this can be done in a way that meets Russell's requirements for knowledge is doubtful.

Russell's discussion of these matters connects directly with the logical positivist program of the 1920s and 1930s (especially Carnap's *Aufbau*), though, as we shall see below, he himself had modified his approach in important respects by then. That change was an indirect result of a different problem, which caused him to substantially rethink his position even while he was still developing his logical atomist program. The background to this is his abandonment of Moorean propositions. The reason for this change of mind (in 1906) is that Russell ceased to find it credible that there are "objective falsehoods," false propositions that are ontologically on a par with true propositions, that is, facts. As we have seen, he continued to accept the existence of facts, and to that extent the old theory continued under a new name. But since propositions had been also taken to be the objects of judgment, false as well as true, Russell needed a new theory of judgment, which facts alone could not supply. His new theory, the "multiple-relation" theory, was that what had previously been conceived of as the constituents of a proposition that is the object of judgment should now be conceived of as terms of a new multiple-term relation conception of judgment; that is, instead of thinking of the Moorean proposition expressed by the sentence "Tom judges that A is larger than B" as having the logical form:

Judges (Tom, the proposition A is larger than B)

where this proposition is itself a complex entity somehow composed of A, B, and the relation of being larger than, we are to think of the same sentence (now regarded as a proposition itself, because it is to be the primary vehicle of truth and falsehood) as being such that, if true, it would correspond to a fact of the form:

Judges* (Tom, A, B, being larger than)

where "judges*" is the multiple-term relation that relates the subject of judgment (Tom) with certain objective terms (A, B, being larger than) in such a way that, together, they constitute a judgment that is true if and only if the objective terms constitute a fact – the fact that A is larger than B.

Russell never integrated this "no proposition" theory into his logical theory. Although it is stated in the introduction to *Principia Mathematica* (*PM*: 43–4), its implications for his conception of a propositional function, which has just been defined as a function whose values are propositions, are not worked through, nor are its implications for his substitutional treatment of quantifiers. Indeed it is flatly inconsistent with his theory of descriptions, since the complex interweaving of quantifiers and variables in that theory cannot be decomposed into "simple" constituents in the way required by the application of the multiple-relation theory to judgments involving descriptions.

Another difficulty comes from that old bugbear, the unity of judgment. For the theory in effect assumes that the objective terms of the multiple-term relation "judges*" can act as a surrogate for that which is judged, e.g. that A is larger than B. So the challenge that the terms by themselves do not constitute a complete judgment is one that cannot be avoided. Russell needs to explain how an appropriate specification of the right truth-making fact is determined simply by the objective terms of the multiple-term relation. One standard objection is that there is no basis for the distinction between judging that A is larger than B and that B is larger than A. It is arguable that this can in fact be handled simply by attending to the order in which the objects occur as terms of the relation "judges*"; but, as with descriptions, this strategy will not cope with general judgments involving multiple quantifiers – e.g. the judgment that all elephants are larger than all mice – where the bound variables obstruct the decomposition into simple constituents essential for Russell's approach. Wittgenstein's objection to Russell concerns a related point, that the theory permits one to judge nonsense. For unless some constraints are placed upon the terms of "judges*" there seems nothing to rule out a simple permutation of terms to generate, say, Judges* (Tom, being larger than, A, B)

which would have to be a surrogate for Tom's "judgment" that being larger is A than B (see WITTGENSTEIN). Russell might seek to rule this out by placing type-restrictions on the terms of the multiple-term relation; but since type distinctions were explained in terms of the capacity of things to form a complete proposition Russell cannot appeal to them once he has embarked on his "no-proposition" perspective.

Russell was shattered by this objection. He had spent the months of May and June 1913 working at tremendous speed and in high spirits on a book about judgment and knowledge; but once he grasped Wittgenstein's point he abandoned the book (now published in *Papers*, 7) and fell almost into despair. Three years later he wrote to Ottoline Morrell about the crisis this event had induced:

> Do you remember that at the time when you were seeing Vittoz I wrote a lot of stuff about Theory of Knowledge, which Wittgenstein criticized with the greatest severity? His criticism, tho' I don't think you realized it at the time, was an event of first-rate importance in my life, and affected everything I have done since. I saw he was right, and I saw that I could not hope ever again to do fundamental work in philosophy. (1968: 57)

Later philosophy

Russell was prevented from lapsing into silence by the pressure of prior commitments at this time, most notably the Lowell lectures, which he delivered at Harvard in spring 1914 (published as *Our Knowledge of the External World*). Indeed his productivity during 1914 is a remarkable testament to his strength of will. Once the First World War began he turned with some relief from the need to rethink his philosophy to public opposition to the war, though by 1918 he was keen to return to philosophy. (At the very time in early 1918 that he was standing trial for his anti-war propaganda and then appealing against the terms of his sentence of six months' imprisonment he was also delivering the lectures on the philosophy of logical atomism (*Papers*, 8) I have referred to above).

Imprisonment, under the comfortable conditions permitted to him, turned out to provide the conditions of relative isolation that Russell needed to achieve a fresh start in his philosophy (though he also wrote his *Introduction to Mathematical Philosophy*, which is a lucid informal discussion of the main themes of *Principia Mathematica*). The starting point for this new work, which was published as *The Analysis of Mind* (1921), was William James's "neutral monism." Russell had already been thinking about this for some time; James's claim had been that the traditional opposition between mind and matter could be transcended by somehow conceiving of them as just different ways of thinking about something intrinsically neutral, which James called "experience." Russell, noting the similarities between this approach and his own account of the external world, develops a similar account based upon "sensations," which are like his old sense-data, except that he now holds that the fact that they are physical is no reason not to hold that they are not also mental (1995b: 143–4). The details of the constructions of the mind and of the physical world that follow are complex and unpersuasive; but what is nonetheless striking in the light of contemporary philosophy of mind is that

Russell sets out a position that combines ontological monism concerning mind and matter with an insistence that the natural laws regulating them are not reducible either way (1995b: 104–5), so that the position is not a reductive monism.

A central feature of Russell's analysis of mind is his attempt to do justice to what he takes to be the insights of scientific psychology, and in particular the behaviorist psychology being propounded at this time by John B. Watson. Russell cannot endorse the full behaviorist position; he thinks that beliefs typically involve mental imagery in a way that is incompatible with behaviorism. But he does endorse a broadly behaviorist conception of desire, and indeed refines it into a position that is recognizable as a precursor of contemporary functionalism. Furthermore he offers a causal account of the content of the images that enter into beliefs and extends this into a generally causal account of meaning. So his analysis of mind, including mental content, is based quite generally upon causal considerations, and this then provides him with the materials for a new theory of judgment to replace that which Wittgenstein had overthrown. He does not, however, take full advantage of this opportunity largely because he still thinks that the meaning of a complete sentence is constructed from the independent meanings of its constituent words (1995b: 273). So it was left to Ramsey to think the matter right through and propose, soon after, a tentative functionalist theory of judgment.

On the subject of knowledge, however, Russell clearly grasps the potential of his new causal conception of the mind. He begins *The Analysis of Mind* by rejecting his old conception of acquaintance, and later in the book he reinforces this break with his past by denying that we can obtain self-evident, certain, knowledge either by perception or by a priori intuition (pp. 262–6). In place of his old conception of knowledge, which, he now thinks, cannot rule out such "logically tenable, but uninteresting" skeptical hypotheses as that the world was created, with all our putative memories, five minutes ago (pp. 159–60), he offers "a more external and causal view" (p. 270) of knowledge. This is indeed precisely the view that is now familiar as "externalist"; and Russell introduces it by means of the now-familiar comparison between an accurate thermometer and someone with reliably true beliefs (p. 253ff).

In *The Analysis of Mind* Russell's presentation of this externalist conception of knowledge is somewhat tentative. In his last major work of philosophy, *Human Knowledge: Its Scope and Limits* (1948), Russell is much more assured and sophisticated in his development of this conception. His general aim here is Kantian: he seeks to explain how scientific knowledge is possible, but (unlike Kant) to do so within a broadly scientific conception of human life. The topic on which he then directs much of his attention is induction. This is a topic he had discussed in *The Problems of Philosophy*, where he had argued, first, that scientific knowledge is dependent upon the validity of the inductive principle that the greater the experience of the association of properties A and B the larger the probability that A and B will be found to be associated in new situations, and, secondly, that since this principle is presupposed in all reasoning from experience, it must be regarded as a self-evident a priori truth comparable to fundamental truths of logic. In his later work Russell begins by arguing that this position is untenable, because the inductive principle is open to obvious counterexamples if no restrictions are placed upon the properties involved. His argument here is similar to that later made famous by Nelson Goodman (as "The New Riddle of Induction") (see GOODMAN) and he takes from it the conclusion that this is not an area within which self-evident truths are to be found. Instead, he proposes, it is essential to look to "scientific common sense" and discern the actual "postulates of scientific inference" (1948: 436).

In the last part of the book he undertakes this task in the context of a sophisticated conception of knowledge as something that comes in degrees: knowledge is not just true belief, but there are many kinds of warrant, some merely involving reliable connections, others involving understanding and reflection, which provide the higher degrees of knowledge. But in the end even the higher types depend on the existence of the connections that justify primitive types of knowledge. Russell's conclusion is that:

> Owing to the world being such as it is, certain occurrences are sometimes, in fact, evidence for certain others; and owing to animals being adapted to their environment, occurrences which are, in fact, evidence of others tend to arouse expectation of those others. By reflecting on this process and refining it, we arrive at the canons of inductive inference. These canons are valid if the world has certain characteristics which we all believe it to have. (1948: 514–15)

One could not ask for a clearer statement of the externalist's justification of induction, though Russell is under no illusion that this will altogether satisfy the philosophical skeptic.

This late work shows Russell still capable of originality at the age of 76. These late writings are often neglected today. But this is a mistake. For in these late writings, he practices the principle he had enunciated in 1924 that "we shall be wise to build our philosophy upon science" (1956: 339). As a result, his writings from this period connect directly with contemporary debates, since from the 1970s onwards the "naturalization" of analytical philosophy has introduced into philosophical debate the requirement of harmony with scientific knowledge that Russell had recognized fifty years earlier. Russell is still our contemporary.

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