CHAPTER ONE
INTRODUCTION AND OVERVIEW

1.1 THE IMPORTANCE OF MATHEMATICS IN FINANCE

Finance is an immensely exciting academic discipline and a most rewarding professional endeavor. However, ever-increasing sophistication of financial markets, design of financial securities, and computational technology has heightened the business practitioner's need for understanding of mathematics. In order to understand and participate in financial decision-making, students and practitioners of finance need to be familiar with the basic tools and techniques used in more formal analysis of financial problems. Mathematics is among the most important of these tools. In addition to being crucial to the analysis of financial problems, mathematics is a language that is most useful for communication of financial concepts, techniques, and results. Yet many of us approach mathematics very tentatively, feeling overwhelmed by its apparent complexity and insecure about our own preparation to understand and apply the variety of useful tools available to us. In many cases, our preparation for handling financial analysis merely requires a review of mathematics that we may have been quite comfortable with in the past; in others, we may need to learn new tools and techniques in mathematics. This book aspires to facilitate review and presentation of more elementary mathematics concepts sufficient to understand many of the most interesting concepts in financial analysis. The vast majority of students of finance require at least some review of mathematics tools, and most will be surprised at how quickly they can be grasped with relatively little dedication and effort.

A few students will question the need to understand mathematics and its application to finance. One challenge occasionally posed in the finance classroom, "Don't they have computers that do this?", can be difficult to confront convincingly. Nevertheless, computers cannot actually understand what types of problems can be solved with what types of technique. Computers are capable only of performing unthinking operations exactly as they are programmed, without any ability to criticize the suitability of solution technique or to understand the implications of the results that they generate. The computer user or financial analyst must formulate the problem, determine the solution technique, and interpret the computer output. This essentially relegates the role

of the computer to mindlessly performing routine and perhaps repetitive computations in exactly the manner specified by its user. The user should understand all of the aspects of the problem and its results, and must also be able to determine the usefulness of the results and apply them. Hence, an analyst, perhaps working in conjunction with other analysts and professionals, must understand the problem, formulate a solution, and interpret the results. In a sense, the computer cannot solve mush of the problem itself; not only must someone know enough to communicate to it exactly what to do, but most of the analytical work must be left to thinking and understanding humans.

Financial analysis is performed in intensely competitive and uncertain environments, bringing together many individuals, businesses, governments, and other institutions. These participants in markets interact with one another over many periods of time, which can be presumed to be infinitely divisible. Numerous sources of uncertainty abound and information derives from the unexpected as well as the expected. While this complex financial environment is most fascinating and exciting, its analysis can be most confounding, requiring application of many branches of mathematics, ranging from simple arithmetic, algebra, and calculus to stochastic processes, numerical methods, wave theory, and nonlinear dynamics. This book presents the most essential mathematical technique and its applications to financial analysis. The level of presentation of this book might be considered to be either prerequisite or parallel to that of an introductory business finance or investments course.

1.2 Mathematical and Computer Modeling in Finance

Financial analysis starts with the construction of *financial models*. A model is an artificial or idealized structure describing the relationships among variables or factors. All of the methodology in this book is intended to facilitate the development, implementation, and analysis of financial models to solve financial problems. For example, the elementary mathematics presented in chapters 2 and 3 is integral to almost any serious financial analysis. The valuation models in chapter 4 provide a groundwork for making investment and budgeting decisions, while the more sophisticated option pricing models in chapter 10 enable us to grasp the essentials for understanding derivative instruments. The use of models is important in finance because the direct analysis of actual markets is extraordinarily complex. For example, a thorough valuation of a stock should require us to understand everything about the economy, political arena, human psychology, and so on that could have an impact on that stock's value. This thorough understanding is simply impossible; it is much more practical to construct a valuation model that accounts for only the most important factors. Models provide the analyst the opportunity to simplify real-world circumstances to a construct that can be easily be manipulated and understood. Financial decision-makers frequently use existing models or construct new ones that relate to the types of decisions they wish to make.

The aim of a financial model is to simulate or behave like a real financial situation. Analysts who create financial models exclude "real-world" conditions that have only minor impact on the results of their decisions. Instead, analysts focus on those factors

that are most relevant to their situations. The most important conditions retained by analysts for model-building along with those ignored for sake of simplicity form the basis for the set of assumptions for the model. In most cases, analysts must make unrealistic assumptions in order to simplify their models and make them easier to work with. The simplest models building on more restrictive simplifying assumptions can be adapted to more life-like scenarios by relaxing the most unrealistic assumptions. The best financial models are those that appropriately account for the most significant factors affecting financial decisions, are simple enough be practical and easy to work with, and are useful for predicting actual financial results. Models that predict actual financial outcomes most accurately are most useful. Unfortunately, accuracy and simplicity in the construction of financial models often conflict with each other. Some degree of inaccuracy in the model must usually be tolerated in order to maintain its ease of use and analysis. The appropriate tradeoff between accuracy and simplicity is a problem constantly faced by the financial analyst.

This book is concerned with both theoretical and practitioner-oriented models. The book's primary focus is the mathematics and quantitative technique required to create and analyze financial models. Theoretical models are created to explain financial markets and scenarios; practitioner-oriented empirical models are intended to be applied to actual business situations. The financial analyst may construct and use a theoretical model to provide a framework for decision-making and then use a practitioner-oriented model to apply the theory. Analysts also use models to measure phenomena in financial markets and scenarios and to evaluate financial results. This book will emphasize practical applications of the most essential models.

In many cases, mathematics-based financial models are readily adaptable to computer spreadsheet programs. Computer spreadsheets and structured programs are most important in financial analysis because of frequent time-consuming and complex computations. At present, the most popular of the spreadsheet programs is Excel™. This spreadsheet appears on the user's computer screen as an array of columns and rows, where numbers, formulas, labels, or other code can be entered. Ease of use and ability to handle enormous numbers of computations are among the advantages of computer-based spreadsheet analysis over a calculator-based analysis. The authors recommend that the reader take every opportunity to use spreadsheets to aid to the understanding of the models presented here. Spreadsheet examples are developed in several chapter appendices in this text to illustrate financial applications.

1.3 Money, Securities, and Markets

Money is anything that is commonly accepted in exchange for goods and services. Money is a measure of value, a store of value and, as just suggested, a medium of exchange. Financial assets (including money) are contracts or certificates indicating financial claims. A *security* is a financial asset that can be purchased or sold in financial markets. For example, a corporate *bond* is a security that indicates a claim by its owner (the investor or bondholder) on a specified series of interest payments and principal repayment by the firm which issued the bond. The corporation is obligated to make payments to the bondholder as specified by the bond contract. A share of corporate *stock* is a residual

claim; that is, shareholders have a right to receive all cash flows (according to a schedule determined by the firm) which remain in the corporation after obligations to all other claimants have been satisfied. Shareholders also have the right to direct certain discretionary activities of the firm. Corporations and other business enterprises issue securities for the purpose of raising money to engage in business activities; investors may be willing to purchase securities in order to receive cash flows associated with these business activities. Markets exist for securities enabling investors to buy, sell, or trade securities. Among these markets are exchanges and over-the-counter markets for stocks and bonds and networks of financial institutions engaged in the trade of derivative instruments and other securities. An understanding of these markets is crucial to successful investing in the securities that trade in them.

Corporate issues are made either to specific investors (private placements, e.g., a bank note) or to the general public (public placements, e.g., publicly traded common stock). Primary markets for securities are the markets of original issue; by participating in these markets, corporations sell their own securities to raise money. Thus, a corporation may raise money by issuing or selling securities in primary markets. *Investment banks* are financial institutions whose function is to assist corporations in the placement of their securities to investors. Investment banks provide advice and other assistance to firms concerning the marketing and pricing new issues. Advertisements (sometimes called "tombstone ads") related to primary market offerings may be seen in financial publications such as the *Wall Street Journal*. Secondary markets exist for many previously issued securities. Secondary markets provide liquidity for primary market participants. That is, secondary markets provide the opportunity for original purchasers of securities to sell their securities before they mature, expire or cease generating payments from the corporations. The New York Stock Exchange and other exchanges provide secondary markets for many corporate issues. Issues not traded on exchanges are traded in the "over-the-counter" markets – markets for all publicly traded securities not listed on exchanges. The National Association of Security Dealers Automated Quotation System (NASDAQ) is among the most visible components of the over-the-counter markets.

In addition to the markets for corporate securities, there exist well-developed markets for a variety of instruments created by other types of issuers. For example, the federal government raises money through the sale of U.S. Treasury issues, including Treasury bills, Treasury notes, and Treasury bonds. These are all debt securities indicating that the United States government has borrowed money from their original purchasers. Such treasury instruments, particularly those of shorter term to maturity, are often regarded to be free of many of the risks associated with other issuers of debt instruments. The United States Treasury is a very reliable debtor and there is a very well developed market for treasury issues. State and local governments issue municipal bonds (the term "muni" does apply to state issues as well as to municipal issues) which often confer certain income tax advantages. A variety of other institutions offer securities, including nonprofit institutions, mutual funds, and government agencies. There even exist active markets for instruments representing claims on other securities (derivative instruments). *Derivative instruments* include *options* and *futures contracts*. The value of a derivative instrument is derived from the value of another security or asset. An option contract provides its owner the right to buy or sell another security or asset at a specified price on or before a given date. The owner of the option contract may choose whether

he wishes to exercise his right to buy or sell. A forward or futures contract obligates its participants to execute an agreed-upon transaction at a later date. All of these different types of instruments, securities, and markets will be discussed in greater detail later in this text.

1.4 Time Value, Risk, Arbitrage, and Pricing

Perhaps the single most important and difficult type of problem in finance is how to value or price an asset, particularly when its cash flow stream is unpredictable and/or lengthy. A variety of tools exists for these types of problems, providing us with different perspectives on value. Two of the most important valuation tools are based on the concepts of *present value* and *relative value*.

Present value recognizes that the worth to an investor of cash flows to be received at a later date is less than the worth of those to be received earlier due to the following:

1 *Inflation*. The purchase power of money tends to decline over time.
2 *Risk*. We never know with certainty whether we will actually realize the cash flow that we are expecting. In a sense, "the bird in the hand is worth two in the bush."
3 The option to either spend money now or defer spending it is likely to be worth more than being forced to defer spending the money.

Present-value methodologies typically involve predicting a cash flow or computing an expected or "most likely" cash flow and dividing it by a factor that accounts for the timing and risk of the cash flow, as well as other potentially important factors. Present-value models are emphasized in chapter 4.

The most important alternative to present-value methodologies is relative valuation. The relative valuation methodologies attempt to locate or devise other securities or combinations of securities whose values are already known and value the asset under consideration relative to those whose values are known. These methodologies make use of the "Law of One Price," which states that an asset whose cash flow structure is identical to that of another asset or combination of assets must be priced identically to the other asset or combination. This law derives from the activity known as arbitrage which, in its classic case, is the simultaneous purchase and sale of the same asset. For example, if gold is selling in Chicago markets for $400 per ounce and in New York markets for $410 per ounce, a classic arbitrage opportunity exists. That is, an investor has an opportunity to earn a riskless profit by engaging in arbitrage activities. In this case, the investor would purchase gold in Chicago for $400 per ounce and simultaneously sell it in New York for $410. This results in a $10 profit per "round trip" transaction. The transactions involve no risk, since both the selling and purchase prices are known. Furthermore, no initial net investment is required, because the transactions offset each other; the proceeds of the sale are used to finance the purchase. Thus, if a classic arbitrage opportunity exists, an investor will have the opportunity to make a riskless profit without investing any of his own money. If the laws of supply and demand are not impeded by market inefficiencies, investors will flock to exploit this opportunity. Their buying pressure in Chicago markets will force the Chicago price to rise; their

selling pressure in New York markets will force the New York price down. Buying and selling pressure will persist until the prices in the two markets are equal. Thus, classic arbitrage opportunities are not likely to persist long in unimpeded free markets. More generally, arbitrage might refer to the near simultaneous purchase and sale of portfolios generating similar cash flow structures. The accuracy of relative valuation models depends on the extent to which securities with known values can be located or combined to resemble the asset under consideration. Arbitrage and relative valuation models are emphasized in chapters 7 and 10.

1.5 The Organization of this Book

This text seeks to provide the reader a level of competence and confidence for the application of elementary mathematics to financial analysis. This book might be used as prerequisite or supplemental reading for an introductory M.B.A.-level finance or investments course. Chapters 2 and 3 provide reviews of topics of elementary mathematics and definitions required for the most basic quantitative applications to finance. These chapters might be regarded as reviews of mathematics at the high-school level with applications specific to business and finance. Particular applications of a particular topic will follow the description of the mathematical topic in this and in other chapters. Certain sections in these two chapters may contain material that is new to some readers. This is particularly true with the applications. Chapters 4, 5, and 6 cover time value of money and measures of return and risk for securities and portfolios. The emphasis of these chapters is relevant mathematics and the level of rigor in these chapters might be comparable to that of an introductory business finance course. Chapter 7 delivers an introduction to matrix algebra along with a number of applications in finance. This chapter and chapter 8 on differential calculus are likely to be most helpful to students enrolled or planning to enroll in an M.B.A.-level investments or portfolio management course. Chapter 8 also provides a large number of applications of differential calculus to finance. Chapter 9 discusses rudiments of integral calculus, differential equations, and a few simple applications to finance, which may be particularly helpful to students planning to enroll in a derivatives course. This chapter could be skipped for those students wishing to move directly into the elementary option pricing mathematics presented in chapter 10. There are appendices at the ends of many chapters and at the end of the text. Some of these appendices are intended to demonstrate spreadsheet applications of the mathematical models. Included in the end-of-text appendices are detailed solutions to end-of-chapter exercises, a z-table, and a list of notation definitions. A glossary of terms follows the text appendices.

This book is designed to enable a student to understand a given topic by reading specific background material without the need to start at the beginning and read all the material prior to a given topic. Generally, reading the previous section (except for the first section in each chapter) will be sufficient background for the reader to comprehend any given section or application unless additional "Background readings" are listed. In many instances, the reader will already be familiar with the material covered in the "Background readings" for a section or application. In some instances, listed "Background readings" may themselves require additional background reading.